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NONLINEAR ANALYSIS OF REINFORCED SPATIAL SUSPENDED ROD ROOFS

Finite-element formulation for the matrix calculation of spatial suspended tube roofs is considered with regard to geometrical and physical nonlinearity. Technique of account of sagging of flexible strings because of dead load is offered. Control of development of plastic deformations has been carried out by parameter “measure of plasticity”. Laws of change of deflected mode of spatial suspended rod roofs have been studied for various nonlinear formulations.

Keywords: matrix calculation, nonlinear analysis, suspended tube roofs, plastic deformations.

Introduction

The key problem related to use of suspension structures in construction is their increased deformability. The most efficient method of suspension structures stabilization is increase in spatiality of such structures. Present paper deals with the problem of deflected mode of suspension spatial roof.

The finite element method in the form of deflection method is taken as basic calculation method. Flexible elements are approximated by idealized rectilinear rods when setting up calculation diagrams. In doing so carrying threads are broken up so that outline of polygonal line obtained was close to the form of reference flexible string.

Let us specify some features of torsion fibres behavior under loading [1], [3], [5]:

– flexible string cannot take compression forces considering small bending rigidity and is always in condition of central tension;
– flexible string takes chiefly nodal load which is loaded through terminal fastenings (exception is wind load and dead load).

Let us derive resolution equation of flexible string. In so doing let us consider that the string is flattened. The following dependences for terminal fastenings movements can be written from condition of compatibility for torsion:

$$\Delta = \frac{H \cdot l}{E \cdot A} - \frac{q^2 \cdot l^3}{24} \cdot \frac{1}{H^2}; \tag{1}$$

$$\Delta = \frac{H \cdot l}{E \cdot A} \cdot \left(1 - \frac{q^2 \cdot l^2}{24} \cdot E \cdot A \cdot \frac{1}{H^3}\right), \tag{2}$$

where H is the string thrust; E is the linear modulus of elasticity of string material; A is the string cross-section area; q is the dead load; l is the span of the string; Δ is the terminal fastenings movements.

The formula (2) shows that movements of the flexible string are combined from elastic elongations and kinematic (geometric) movements. The latter ones are the main reason for geometric nonlinearity of suspension systems.

Let us rearrange the dependence (2) to determine the thrust:

$$H^3 - E \cdot A \cdot \frac{\Delta}{l} \cdot H^2 - E \cdot A \cdot \frac{q^2 \cdot l^3}{24} = 0. \tag{3}$$

Let us note that absolute term of cubic equation (3) is the cube of string thrust under movements of its terminal fastenings, equal to 0.

$$H_0^3 = E \cdot A \cdot \frac{q^2 \cdot l^3}{24}, \tag{4}$$

where H_o *is the* string thrust under terminal fastenings, equal to 0.

Then the expression (2) can be written in the form:

$$\Delta = \frac{H \cdot l}{E \cdot A} \cdot \left(1 - \left(\frac{H_0}{H}\right)^3\right). \tag{5}$$

Let us introduce the following dimensionless parameters:

$$\delta = \frac{\Delta}{l};\ t = \frac{H}{E \cdot A};\ u = \frac{q \cdot l}{E \cdot A}. \qquad (6)$$

Let us note that parameter *t* describes longitudinal loading, *u* parameter *t* describes physical characteristics of the string. The dependence (3) in dimensionless form:

$$t^3 - \delta \cdot t^2 - \frac{1}{24} \cdot u^2 = 0. \qquad (7)$$

δ —*t* curve under *u* = *const* is shown in Fig. 1. The equation (4) may have three real roots or one real and two complex conjugate roots under different coefficients ratios. This correlates with two different branches to the left and to the right of vertical axis. Since flexible string cannot take compression force, the left side of the plot is fictitious; it is just formal solution of the equation (4).

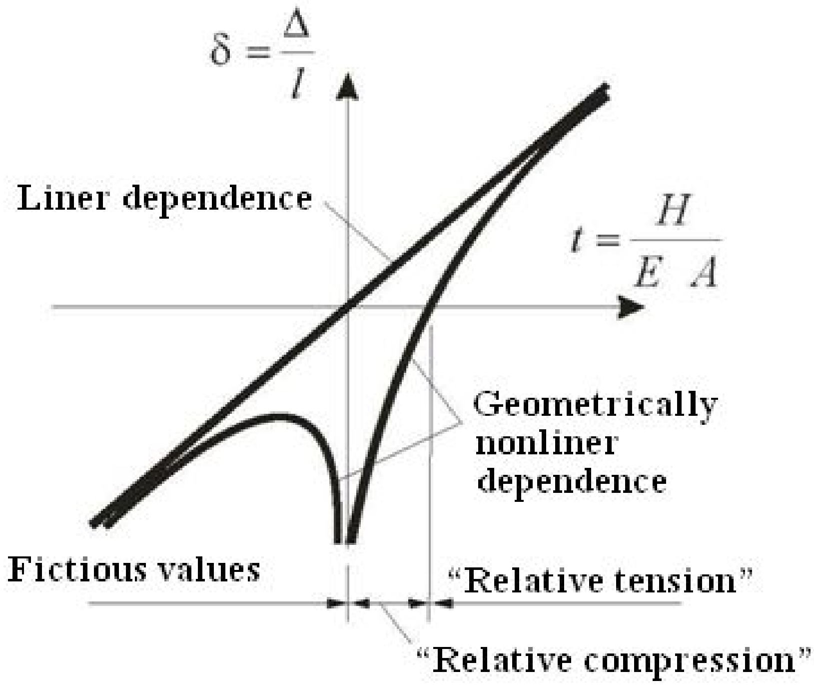


Fig. 1. Dependence of relative lengthwise movements of flexible strings on parameter of loading

The plot shows that flexible stretched string has two states during the process of deformation in geometrically nonlinear which differs by location of bearing fastenings. Let us designate the state of flexible string which correlates with separation of bearing fasteners as “relative tension”, and the state which correlates with shifting we designate as “relative compression”.

The expression (2) shows that nonlinearity of flexible string behavior under loading is described by multiplier in parentheses.

Let us designate this magnitude

$$K_{нел} = 1 - \frac{q^2 \cdot l^2}{24} \cdot E \cdot A \cdot \frac{1}{H^3}, \tag{8}$$

where $K_{нел}$ is the coefficient of string nonlinear behavior.

According to (5),

$$K_{нел} = 1 - \left(\frac{H_0}{H}\right)^3. \tag{9}$$

The plot of change of $K_{нел}$ against longitudinal loading of flexible string is shown in Fig. 2 based on dependences (8), (9). The left side of the plot ($K_{нел} > 1$) correlates with negative value of thrust, but since string is always stretched out, it does not implement physically (corresponds to empty values). Nonlinearity coefficient can be positive (this corresponds to "relative tension") or negative (this corresponds to "relative compression").

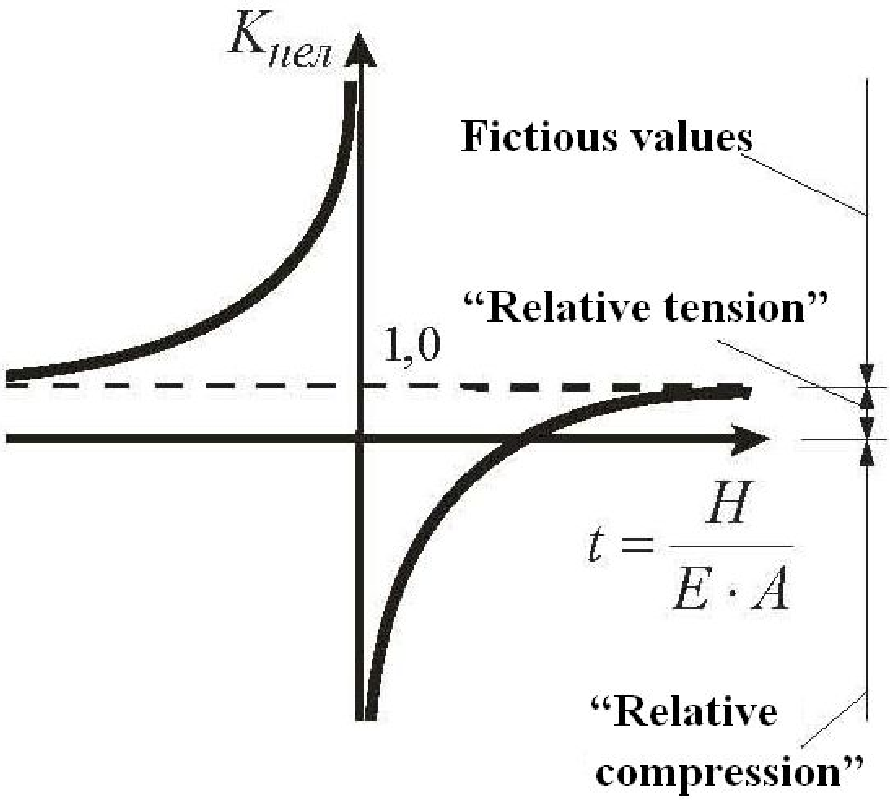


Fig. 2. Dependence of coefficient of string nonlinear behavior on parameter of loading

Dependences (3), (7) and plots in Fig. 2—3 led us to propose that basic parameter influencing the sagging of flexible strings is the thrust under zero movements of terminal fasteners. This factor of nonlinear behavior can be neglected under small values of H_0. In this case “elimination” of compressed strings from operation should be taken into account. In the case we have so-called constructively nonlinear calculation of suspension structures [1], [4], which is the special case of geometrically nonlinear calculation.

Proposed models of flexible string deformation in geometrically and constructively nonlinear statement are shown in Fig. 3.

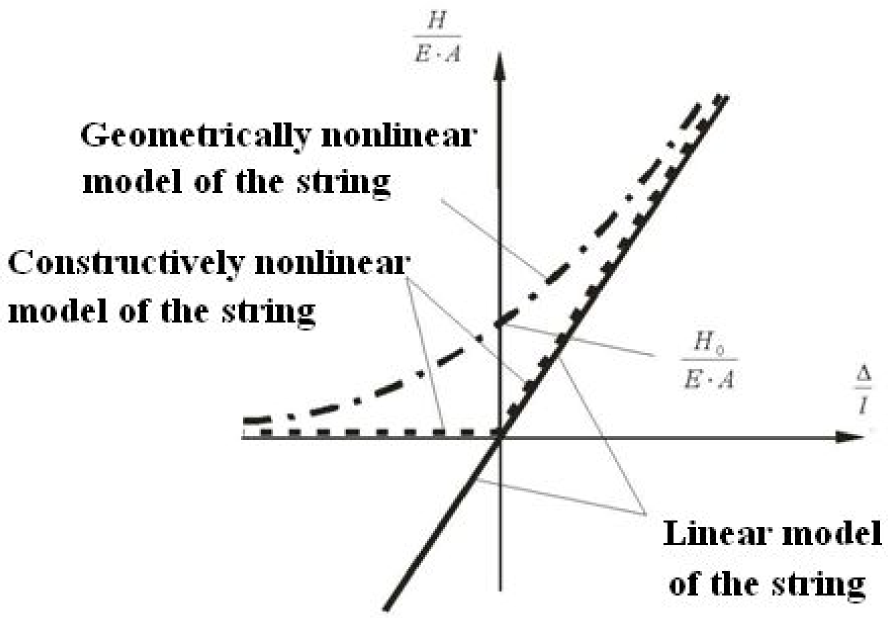


Fig. 3. Models of flexible string deformation

Since stiffness matrix expansion requires most of machine time cost at matrix calculation of multielement structures when solving the system of resolution equations, consequently, the method of elastic decisions was used [5].

Suspension spatial rod structure of industrial building covering with the use of cross bearing strings and double-inclined hangers is taken as system under investigation (Fig 4). Basic integration parameters: span L = 60 m, sag f = 9 m (1/8 L); column height to stiffening girder h = 18 m; gap between bearing strings and stiffening girder z = 1,5 m; pitch of columns and longitudinal beams is 12 m; the number of spans is 3; the number of pitches of columns is 6; canting angle of guy lines is 45^0 (suspension structure with exteroceptive perception of thrust); characteristic of longitudinal deformations n_0 = 5,3 10^{-5} [1]. Investigation of deflection mode has been carried out under load G_0 uniformly distributed over the whole area of central span and load P_0 concentrated in the middle cross-section of the middle stiffening girder of the central span. The system has been investigated with several values of permanent load, which was modeled as uniformly distributed over the whole area of the covering.

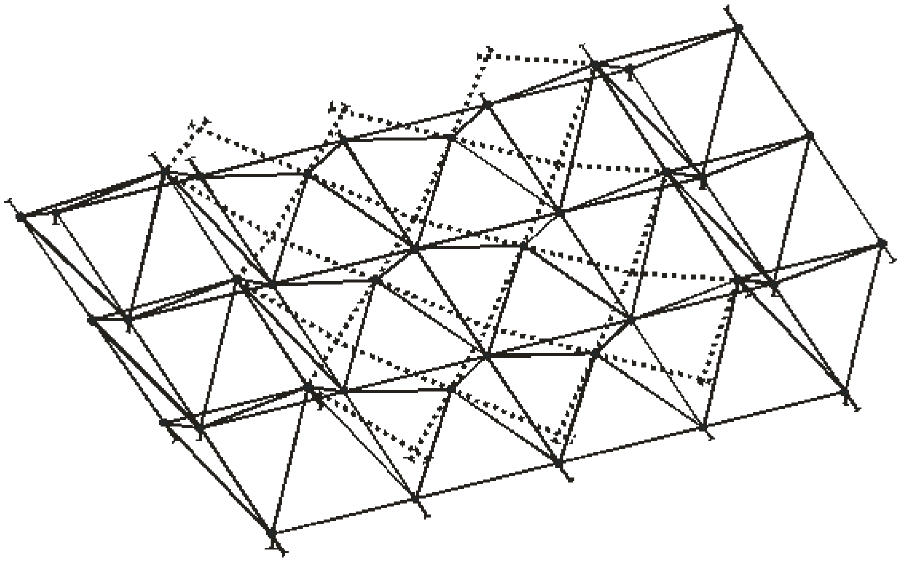


Fig. 4. Spatial suspension rod system:
▪▪▪▪▪ — bearing cable; ⤫ — element crossing core

It was found from the results of geometrically nonlinear calculation that suspension spatial multispan system has the following zones of operation of its flexible elements (Fig. 5a):

- zone A: loaded central span;
- zone B: half of the unloaded (extreme) span, which is adjacent to loaded one;
- zone C: half of the unloaded span, which is more distant from central span (adjacent to the system of guylines).

The graphs of nonlinear behavior of bearing strings of suspension spatial structure are shown in Fig. 5.

It was found out that suspension spatial structure shows following stages of deformation under increase in load intensity.

1. Initial stage. At this stage, temporary load is negligible, as a result, permanent load has the greatest influence on structure deflected mode, maximum flexures, correspondingly, are observed in extreme spans. In doing so changes in calculation model of the structure are due to elimination or inclusion of ascending hangers in system operation, which are adjacent to column leg of loaded span.

2. Conditional linear stage. At this stage, work model of the structure (design model of suspension system with “compressed” flexure elements eliminated from structure operation [1], [5]) becomes stable.

3. Nonlinear stage. Descending hangers are eliminated from operation of loaded spans, as well as bearing strings in end zones. In unloaded spans, as well as in half-spans, adjacent to central span, redistribution of load from hangers to bearing strings is observed; in half-spans adjacent to в guy lines system, ascending hangers are the most strained.

4. Quasi-linear stage. Change in load intensity does not affect calculation model of the structure. Flexible strings deflected mode does not undergo transformations.

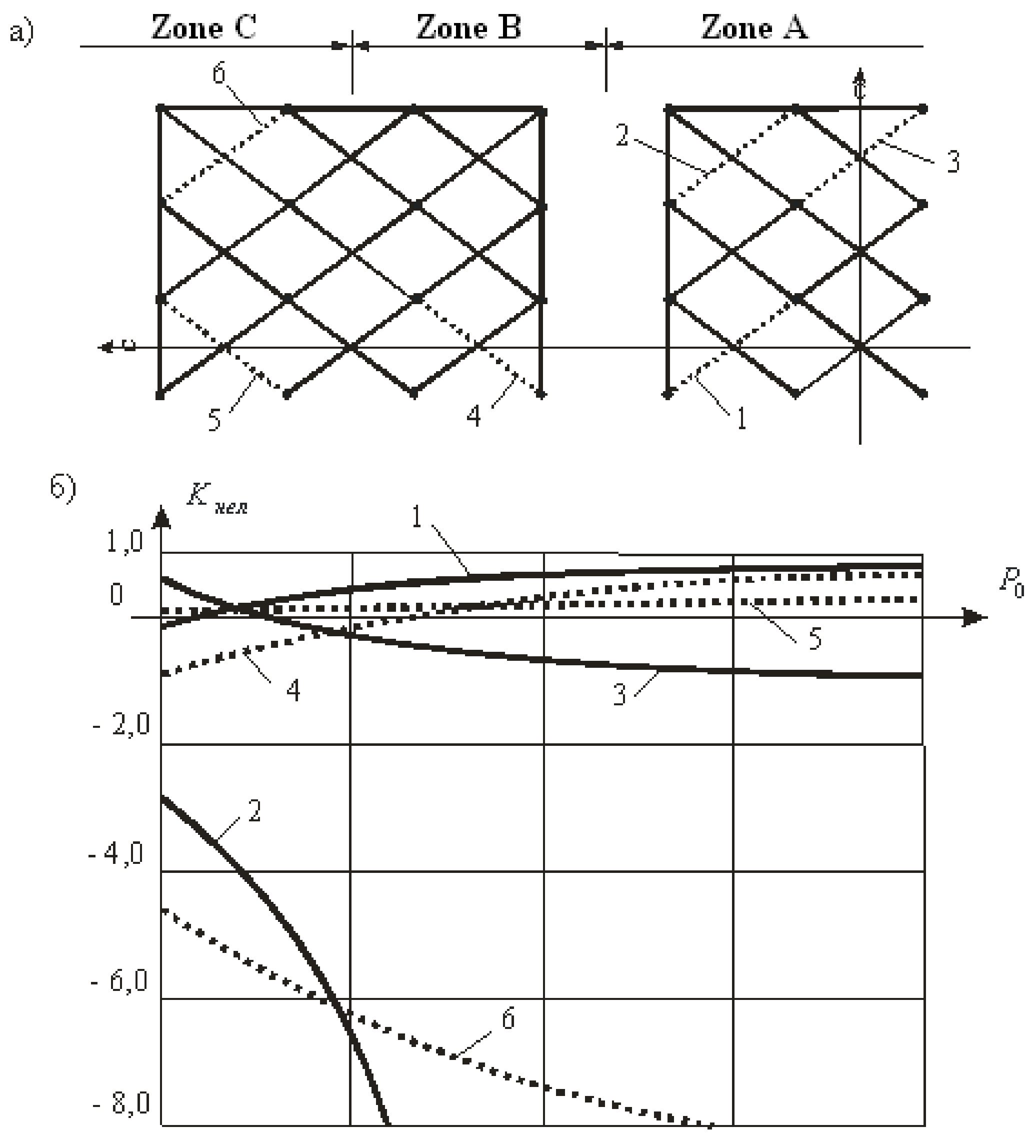
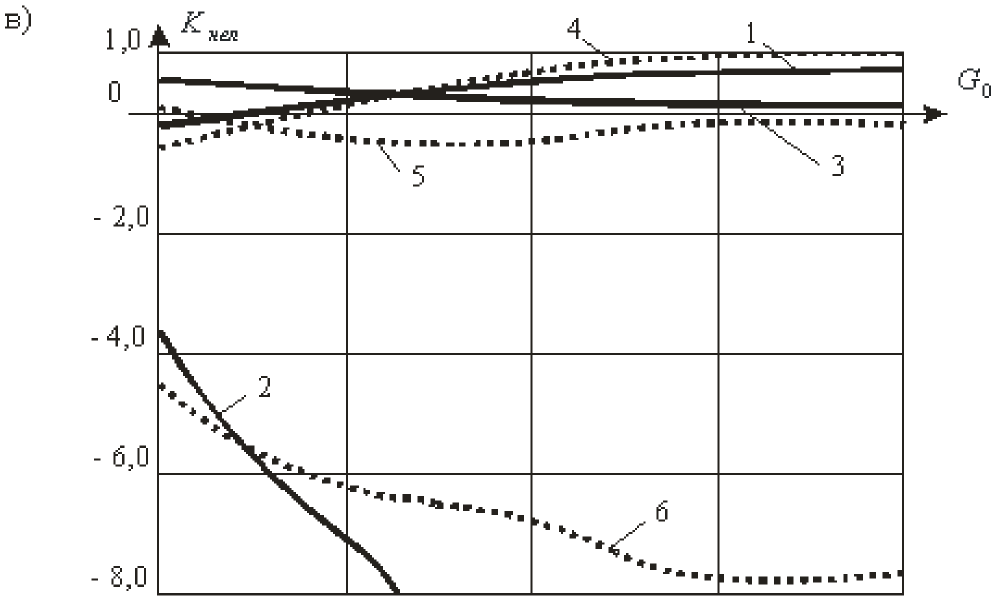


Fig. 5. Graphs of change in nonlinear behavior of bearing strings:
a — plan of arrangement of bearing strings; b, c — graphs of change in coefficient
of flexible strings nonlinearity in relation to intensity of concentrated and distributed load



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Initiation of elastic deformations has great effect on operation reliability of building structures. In the case of suspension system it is necessary to take into account geometric nonlinearity. In present paper development elastic deformations under single-pass loads are investigated.

Strain-deformation dependence in suspension structures material can be described in the form of line-segment function [4]:

$$\sigma = E \cdot \varepsilon \cdot (1 - \alpha_\varepsilon), \tag{10}$$

at $\varepsilon \le \varepsilon_t \;\; \alpha_\varepsilon = 1$; $\varepsilon > \varepsilon_t \;\; 0 \le \alpha_\varepsilon < 1$, where E, σ, ε are modulus of elasticity, strain, and relative elongation of flexible string material, respectively; α_ε is the coefficient of physical nonlinearity (the ratio of secant modulus of elasticity to initial).

Let us assume that sagging of flexible strings is negligible (let us use only flexible rods of small and middle length) [3], [5].

Model of deformation of elastoplastic string is shown in Fig. 6.

Taking into consideration that strings are always in conditions of central tension, longitudinal strains in elastoplastic string are determined by the following system of equations:

1. Zone "A" — zone of constructive nonlinearity:

$$\Delta \le 0; \;\; H = 0. \tag{11}$$

2. Zone "B" — zone of elastic deformations:

$$0 < \Delta \le \Delta_t; \;\; H = \frac{E \cdot A}{l} \cdot \Delta, \tag{12}$$

where Δ_t are movements of terminal fasteners.

3. Zone "C" — zone of plastic deformations:

$$\Delta_t < \Delta \le \Delta_u; \;\; H = \frac{E \cdot A}{l} \cdot \Delta \cdot (1 - \alpha_\varepsilon), \tag{13}$$

where Δ_u are movements of terminal fasteners.

4. Zone "D" — zone of destruction:

$$\Delta_u < \Delta; \;\; H = 0. \tag{14}$$

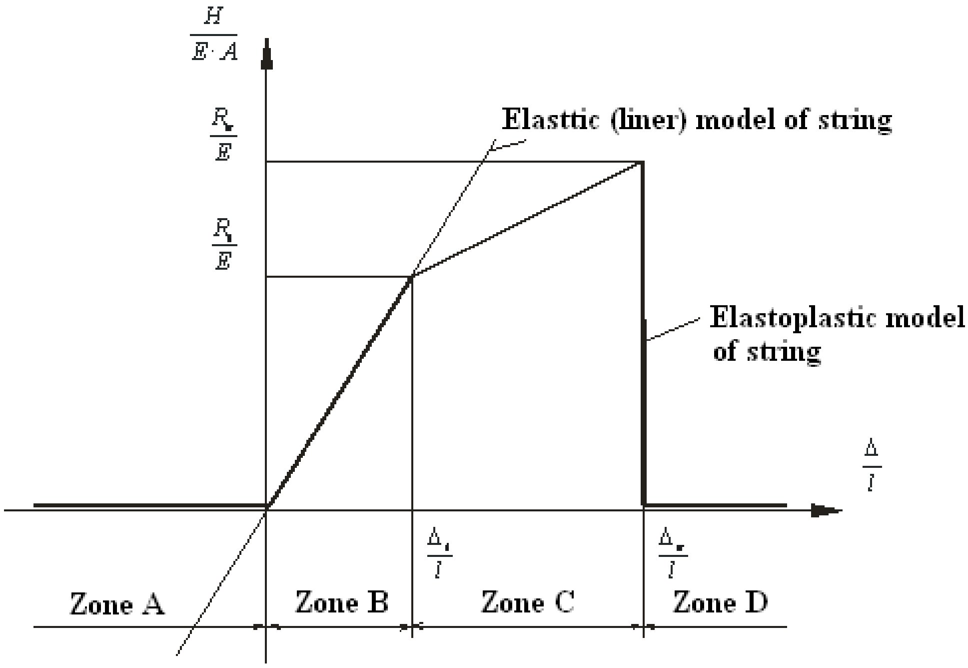

Fig. 6. Model of deformation of elastoplastic string

The degree of plastic deformations development of the group of elements was evaluated by measure of plasticity C_0, the level of strains was evaluated by index K_t:

$$C_0 = \frac{\sum_{i=1}^{m} \alpha_{\varepsilon i}}{m}; \;\; K_t = \frac{\sum_{i=1}^{m} \frac{H_i}{A \cdot R_t}}{m}, \tag{15}$$

where m is the number of elements in the group; R_t is the calculated yield point strength of the string material.

Suspension spatial rod structure of single-span industrial buildings roofs with the use of cross bearing strings and double-inclined hangers is taken into consideration.

Basic integration parameters: span L = 72 m, sag f = 9 m (1/8 L); column height to stiffening girder h = 12 m. Properties of flexible string material and beam system were taken in accordance with recommendations on design of suspension structures and the steel ropes tests in situ [6], [7], [8]: ultimate strength — 1400 MPa, conditional yield point — 750 MPa, relative elongation after rupture — 3 %, modulus of elasticity — $1{,}6\cdot10^5$ MPa. Asymmetric circuit with half-span loaded by uniformly distributed load is taken as circuit of loading by temporary load.

Fig. 7 shows graphs of change in maximum sags of suspension structure in relation to change in intensity of temporary load. Fig. 8 shows graphs of change in coefficients C_0 and K_t. The most typical operation calculated circuits of spatial system during the progress of its deformation are shown in Fig. 9.

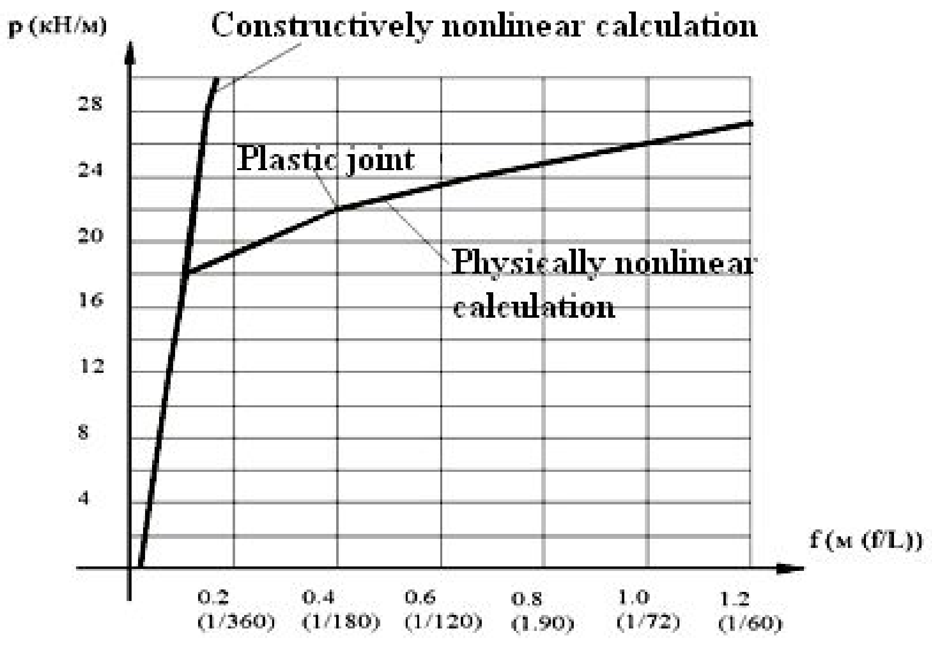

Fig. 7. Graphs of change in maximum sags

Analysis of data obtained showed that in the spatial suspension system at sags up to 1/500 of the span elastic deformations prevail. “Elastic” operation calculation circuit is observed at this stage. As this takes place, ascending inclined hangers are eliminated. The level of stresses at this stage does not surpass 10 % from the level of maximum stresses. The ending of elastic stage of operation of the structure is characterized by reformation of “elastic” operation circuit. In doing so, inclined hangers as well as bearing ropes begin to be eliminated.

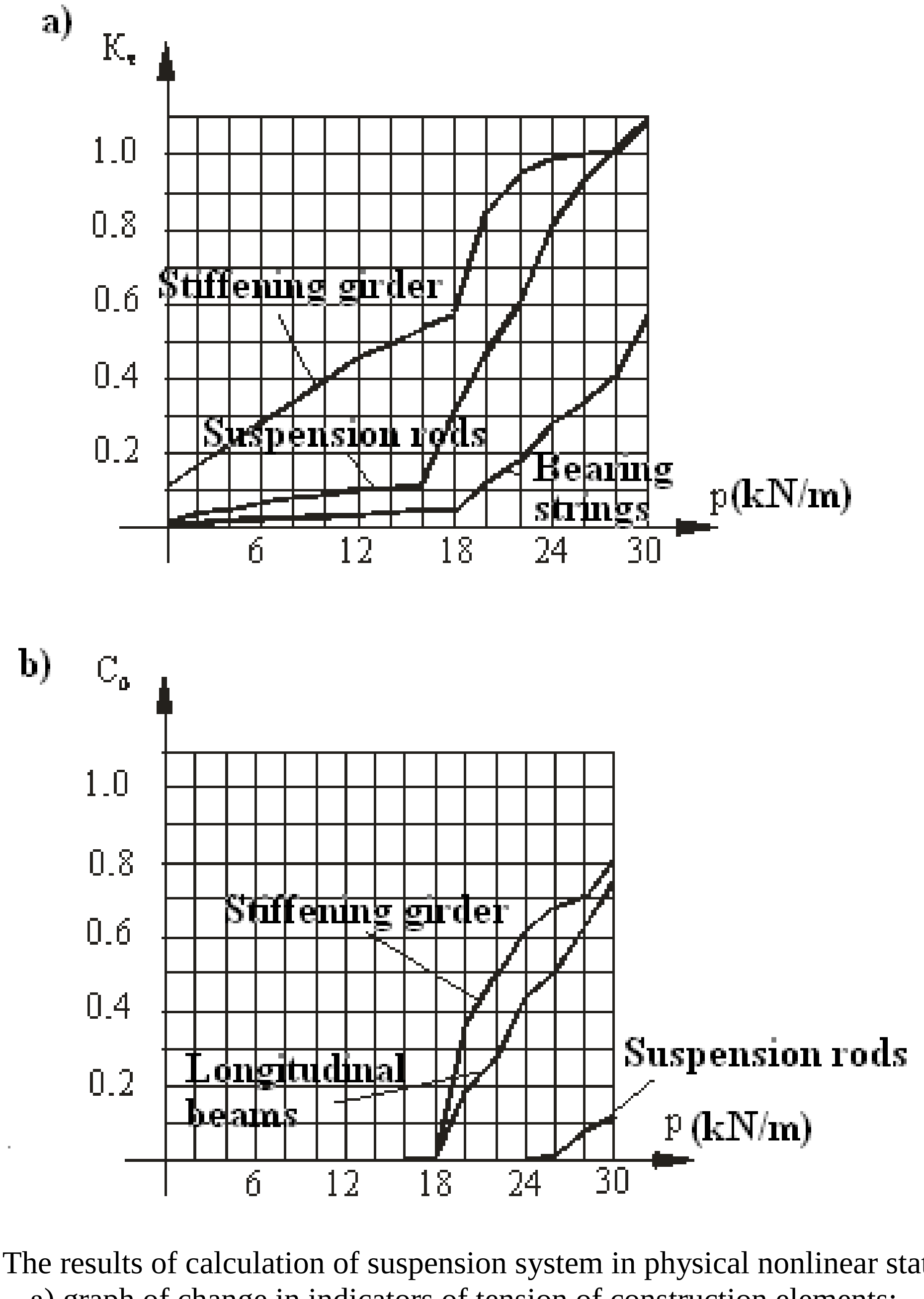

Fig. 8. The results of calculation of suspension system in physical nonlinear statement:
a) graph of change in indicators of tension of construction elements;
b) graph of change in indicators of plasticity of construction elements

Thereupon influence of plastic deformations decreases at practically invariant elastoplastic operation calculation circuit until sags reach value of 1/200 of the span. At this stage of deformation plastic elements are observed in beam system of the suspension structure.

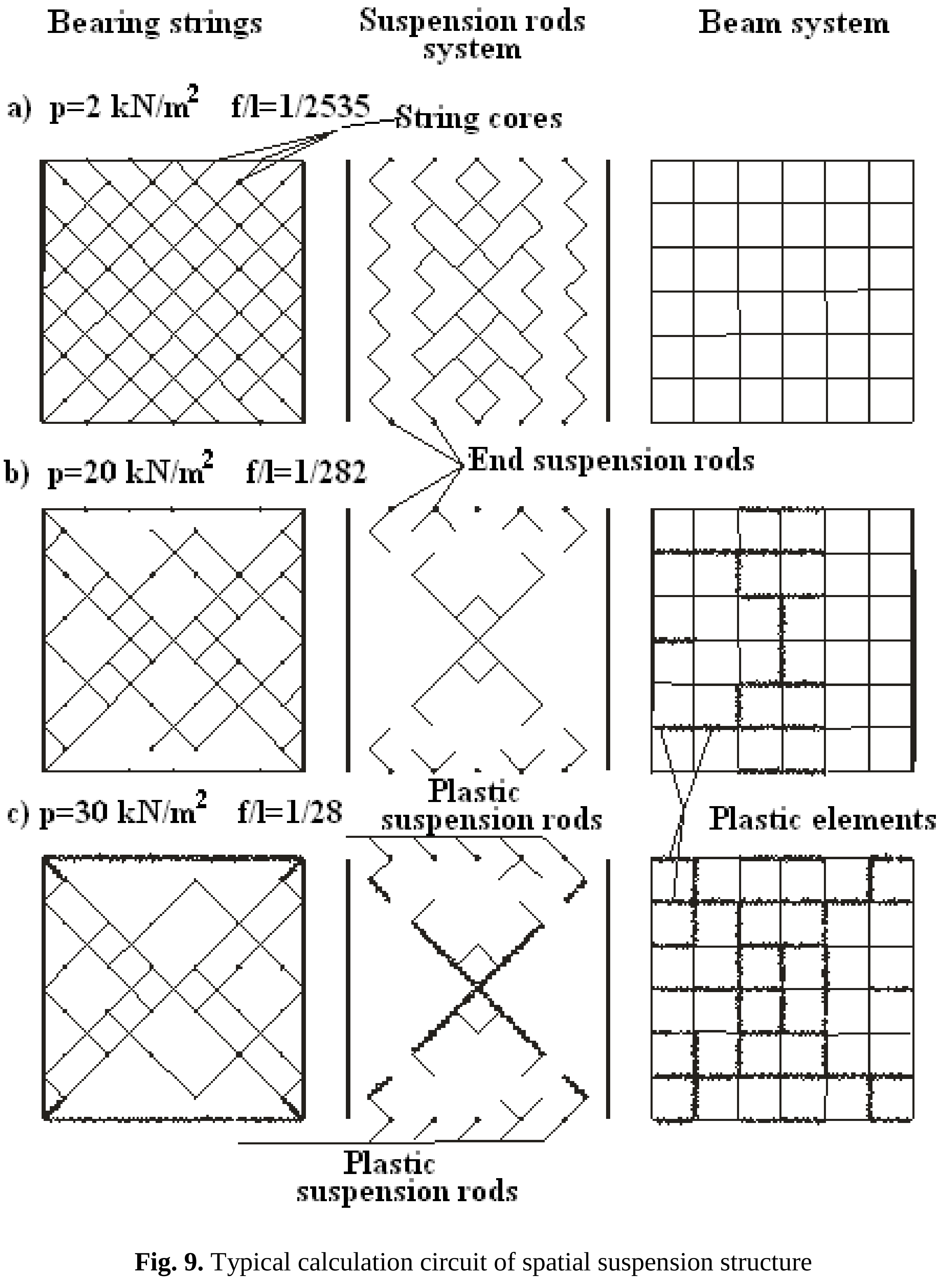

Fig. 9. Typical calculation circuit of spatial suspension structure

A plastic hinge occurs in suspension structure at sags of the order of 1/200 of a span. At this stage plastic deformations develop in both inclined hangers and in bearing slopes. As this takes place, repeated reformation of structure calculation circuit is performed. Elastoplastic operation calculation circuit of suspension structure is put in final form and retains up to the moment of destruction of a roof.

Based on the results stated above following stages of deformations of spatial suspension structures in elastoplastic stage can be distinguished:

1. *“Elastic”:* constructive nonlinearity of the rods of the system is observed;

2. *Transient “elastoplastic”:* the increase of plastic deformations in the elements of structure occurs after elastic reformation of structure calculation circuit (predominantly in the beam system);

3. *Final “elastoplastic”:* plastic deformations in hangers and structure bearing strings develops in beam system after repeated reformation of structure calculation circuit and plastic hinge formation.

Conclusion

From the results stated above following conclusion can be drawn: 1) spatial suspension rod structures exhibits geometric nonlinearity caused by flexible strings sag, 2) development of plastic deformations should be taken into account when analyzing deflected mode of such structures.

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