TECHNOLOGY AND ORGANIZATION OF CONSTRUCTION

UDC 69.057.5:004

Voronezh State University of Architecture and Civil Engineering
Senior lecturer of Department of Urban Construction and Services
E. E. Burak
D. Sc. in Physics and Mathematics, Prof. of Department of High Mathematics
A. V. Loboda
Ph. D. in Engineering, Assoc. Prof. of Department of Building Production Technology
A. N. Tkachenko
Russia, Voronezh, tel.: +7(4732)71-52-49; e-mail: ekaterinaburak@yandex.ru

E. E. Burak, A. V. Loboda, A. N. Tkachenko

MATHEMATICAL MODELING OF THE OPTIMUM MODES OF CONCRETING WITH THE USE OF PNEUMATIC FORMWORK

Problem statement. The construction of a sufficient theoretical model of pneumatic formwork work and description of the dependence of strength properties of the concrete obtained on various parameters of the process of shotcreting are the problem of great importance.

Results and conclusions. Experimental-theoretical model of the technological process of fine-grained concrete mix shotcreting on horizontally located pneumatic formwork is presented. Recoil indicator and concrete strength dependence on such parameters, as the productive capacity of shotcrete-machine, the nozzle diameter, the distance from the nozzle to the formwork surface and tension of pneumatic formwork material. The comparison of theoretical dependences and experimental ones has shown their qualitative correspondence.

Keywords: pneumatic frame formwork, strength, mathematical model, modeling, optimum modes of concreting.

Introduction

The use of effective static type pneumatic formwork suggests concrete mix shotcreting on the upper surface of formwork.

As this takes place, dynamic interaction of a shotcrete stream with the compliant surface vigorously influences an erected building construction material quality. Rationalization of technological parameters of mixture application from the point of view of obtaining concrete design characteristics is possible during the elaboration and realization of the mathematical model describing the process presented. The theoretical approaches available are built upon the analysis of shotcrete concrete mixture concussion through contact with a rigid surface, which is unacceptable for ‘soft’ formwork. The construction of a sufficient theoretical model of pneumatic formwork work and description of the dependence of strength properties of the concrete obtained on various parameters of the process of shotcreting is an urgent problem.

There are numerous approaches to the construction of a similar model. Experimental and theoretical description of the process is given. It is based on the application of already familiar models of physical phenomena connected with the processes of shotcreting and construction of a theoretical formula which would express the sought-for dependence the obtained shotcrete concrete strength of on the technological parameters of its application.

The theoretical bases for the construction of the suggested shotcrete model were the experimental data on the strength of the obtained concrete depending on the following parameters:

1) the capacity of a shotcrete machine (Q);
2) the diameter;
3) formwork material tension(N);
4) the distance between the shotcrete machine nozzle (D) and the surface on which the concrete is applied (H).

1. Bounce hypotheses and parameters of the shotcreting process

The main hypotheses offered is that strength of the obtained shotcrete concrete to a large extent is determined by the phenomenon of the bounce of concrete particles off formwork deflection surface.

The presence of the bounce effect is obvious when shotcrete concrete is applied on a vertical surface. As the placement of concrete takes place, accumulation of particles kicked due to the oscillations of the lower part of the formwork surface. The concrete content and consequently its strength change as well.

The characteristics of the process of concrete placement at a vertically placed surface of formwork and optimization of these processes (i.e. providing a decrease in concrete losses and an enhance of its strength) were investigated in the study [1]. Basically, the present paper includes the development of the ideas from the above mentioned research. The study of 'hidden' bounce effect during the process of placement of concrete on a horizontal surface is also combined with the analysis of velocities of concrete particles and of the surface during its oscillation. The ratio

$$\xi = \frac{v_2}{v_1}$$

where υ_2 is the reentrant speed of a pneumatic formwork module surface to the initial speed υ_1 of this surface flying up to a concrete particle is introduced as a bounce index. The larger the value of this index is, the more possible it is that concrete particles detachment will occur.

During a horizontal surface shotcreting suc a detachment does not cause losses of concrete. Nonetheless, its strength characteristics are impaired. By the analogy to the study [1] instead of spatial oscillation of the module surface we are dealing with plane oscillations of a thinline of this surface, a string per se. We are substituting the simplest model connected to a string linear triangular deflection form by a separate particle by more complex models.

A shotcrete machine nozzle is placed on a fixed height over the pneumatic formwork surface. Its total overall capacity is made up of two constituents:

$$Q^* = Q_0 + Q,$$

where Q is the concrete feeding capacity; Q_0 is the compressor capacity.

Concrete particles flying out of a shotcrete machine nozzle have different mass and velocity. During fixation of different parameters of the process introduced above, it is essential to discuss some averaged values of these quantities and instead of a stream of heterogeneous particles study a statistically average one.

For instance, the average speed and mass of a concrete particle flying out of a shotcrete machine nozzle may be calculated depending on the values of the parameters Q, D. Over a small amount of time Δt, the volume of concrete 'shooting' out of a shotcrete machine is equal to the product of a shotcrete machine Q^* into a time interval. This volume is equal to a small cylinder volume flying out of a nozzle and proportional to the particles velocity v_0, to the nozzle sectional area

$$S = \frac{\pi D^2}{4}$$

and time Δt. This proves the following formula correct:

$$\frac{\pi D^2}{4} v_0 \Delta t = Q^* \Delta t. \tag{1}$$

Reducing both parts of the equation by Δt, we get the expression for the velocity of a particle v_0 flying out of a shotcrete machine nozzle:

$$v_0 = \frac{4Q^*}{\pi D^2}. \tag{2}$$

Particle mass is proportional to its volume. Volume of average particle is proportional to the product of three linear measurements of the particle. Two measurements are related to cross sectional area S of the nozzle of the plant, and the third measurement («height» of outgoing particle) is proportional to the productivity of the plant. Therefore, we obtain the formula for averaged particle mass:

$$m = \gamma Q D^2, \tag{3}$$

where γ is some numerical coefficient.

Averaged particle does not change its mass in passing the distance from the nozzle of air placer to the impact on string. The velocity of the particle will change significantly during the flight. This is aided by the force of air resistance which is expressed by relationship

$$F = \alpha v^2,$$

where α is some coefficient. By solving corresponding differential equation, it is easy to obtain the law of change of velocity depending on passed distance:

$$v(t) = v_0 e^{-\eta s(t)}, \tag{4}$$

where coefficient η is equal to $\frac{\alpha}{m}$. So, we obtain following expression for the velocity of the particle approaching to the girder from height H:

$$\upsilon_1 = \upsilon_0 E, \tag{5}$$

where $E = \exp\left(-\beta \frac{H}{QD^2}\right)$, β is the positive numerical coefficient.

The formulas (2) and (3) allow the kinetic energy of an averaged particle flying up to a formwork to take the following form:

$$E_k = \frac{mv_1^2}{2} = \delta \frac{Q^3}{D^2} E^2, \tag{6}$$

where δ is some coefficient of proportionality.

2. Analysis of the string disturbance pattern

Particle interaction with a formwork surface will be described with the use of a known mathematical theory of string small oscillations [2], [3], [4]. It goes without saying that absolute conclusions about the formwork surface interaction with a stream of shotcrete concrete particles cannot be obtained on the basis of the examination of a single particle effect on a string. At the quality level, however, such an approach leads to a situation which seems fairly appropriate for a long-time process of shotcreting and is backed by experimental results.

While considering a single particle flying up to a string and possessing some kinetic energy E_k, let us discuss a string disturbance by a particle. Let us assume that at the expense of a particle energy conveyed to a string the latter departs from the horizontal equilibrium position and acquired some disturbed form.

This problem does not have a clear-cut solution in the context of the classic small-oscillations theory regardless of descriptions of various ‘ideal’ solutions for similar issues. For instance, in [2] there are formulas describe oscillations of a string disturbed by a point hammer blow and by one having some length.

In both cases the solutions suggested are of idealized non-physical nature. In their context certain point of the string are initially motionless, then they instantly transform into a new state and stay motionless afterwards.

We are making use of a ‘smoothed-out’ disturbance model where a string disturbance by a shotcrete concrete particle is performed in the string centerpoint. We will suggest here that the zero speed of all the points of a string in a disturbed state may be assumed.

In doing so we find a solution to the first boundary problem for the familiar string oscillations equation:

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}. \quad (7)$$

The following conditions are viewed as the problem initial data:

$$\begin{cases} u(x,0) = \varphi(x), \\ u'_t(x,0) = 0. \end{cases} \quad (8)$$

On the basis of the problem solution decomposition (7)—(8) into the Fourier multiple series:

$$u(x,t) = \sum_{k=1}^{\infty} A_k \cos\frac{ak\pi t}{l} \sin\frac{k\pi x}{l}, \quad (9)$$

the string oscillations may be examined and among other things traverse speeds of its certain points may be calculated.

The Dalamber principle [2], [3] may also be put to use according to which the solution (8) of the problem (6)—(7) at arbitrary initial form $u = \varphi(x)$ will look like the sum of two receding waves:

$$U(x,t) = \frac{\varphi(x+at) + \varphi(x-at)}{2}. \quad (10)$$

The traverse speed of a string arbitrary point thereby may take the following form:

$$v(x_0,t_0) = \frac{a}{2}(\varphi'(x_0 + at_0) - \varphi'(x_0 - at_0)). \quad (11)$$

As a derivative of a function at a point is equal to angle of inclination tangent of its plot at this point, the formula (11) means that the maximum reentrant speed of the string points initially looking like $u = \varphi(x)$ is equal to the maximum tangent of the angle of inclination of the function $u = \varphi(x)$ plot multiplied by the coefficient α.

In the study [1], the main hypothesis offered is that a string disturbance is in the form of a 45° right triangle, which means that

$$v_1 = v_2, \tag{12}$$

where v_1 is the velocity of a flying up concrete particle and v_2 is the reentrant speed of the string. The same property of speed balance is observed in a more complex parabolic model which we introduced as a kind of attempt to depart from the equation (12). For example, let us consider the parabola (fig. 1) located symmetrically relative to the string centre as the initial (disturbed) form of the string.

Taking into account the initial velocity of the flying up particle v_1 and the disturbance speed along the string equal to the parameter α value from the equation (7), it is natural to assume that the tangent of the angle of inclination of the parabola in question to the *OX* axis is

$$tg\vartheta = \frac{v_1}{a}. \tag{13}$$

Then, at a moment of time of 'the contact' of two receding parabola (fig. 2), the reentrant speed v_2 of the string centerpoint will be $atg\vartheta$, i.e. in this case the equality (12) is satisfied.

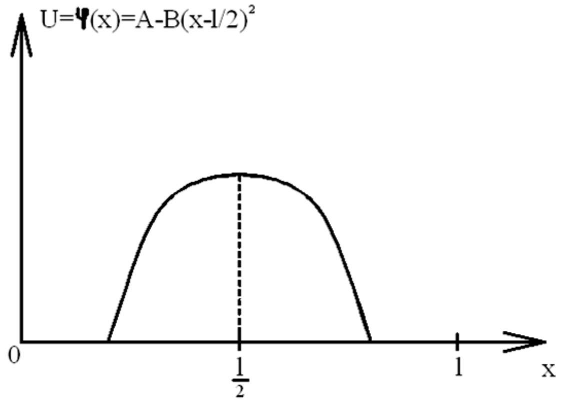

Fig. 1. Parabolic disturbance of the string

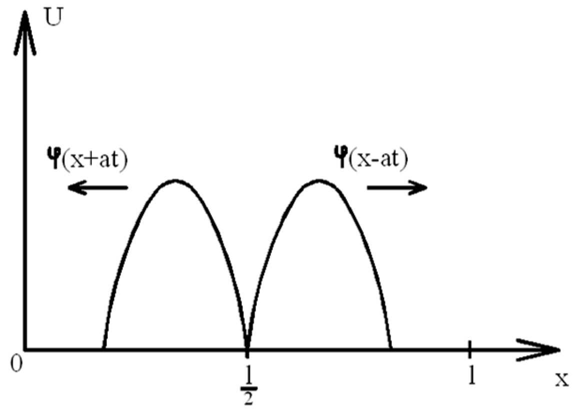

Fig. 2. Receding parabolic waves

It is this equation that makes us abandon the triangle and parabolic models of the string disturbance. The experimental data on the dependence of the strength of the shotcrete concrete obtained on various parameters of the process urges us to seek other models where the relation between the initial and reentrant speeds may deviate from the condition (12).

3. Gaussian disturbance of a string

Furthermore**,** let us assume that the principle case will be when a string under study is located on the intercept [0, π] and after the disturbance it looks like the Gaussian line with some parameter σ (fig. 3):

$$\varphi(x) = \frac{1}{\sqrt{2\pi}\sigma}\exp(-\frac{(x-x_0)^2}{2\sigma^2}), \quad x_0 = \frac{\pi}{2}. \tag{14}$$

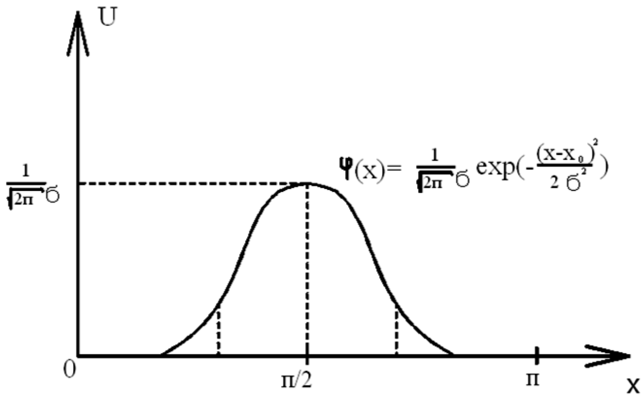

Fig 3. Gaussian disturbance of a string

By virtue of the probability theory three sigma rule we may assume that a disturbed string diverges from the horizontal position only in the area $[\frac{\pi}{2}-3\sigma, \frac{\pi}{2}+3\sigma]$. That certainly restricts the model suggested:

$$0 < \sigma < \frac{\pi}{6}. \tag{15}$$

As we know, the height of the curve (14) is determined by the parameter σ value and equal to $\frac{1}{\sqrt{2\pi}\sigma}$. The maximum value of the modulus of the derivative of function (14) is attained at the inflection point

$$x = \frac{\pi}{2} \pm \sigma$$

of this curve and is

$$\frac{1}{\sqrt{2\pi}\sigma^2}e^{-1/2} = \frac{1}{\sqrt{2\pi e}\sigma^2}. \tag{16}$$

The value of the parameter σ may be therewith related to string elastic disturbance energy:

$$E_B = \int_0^{\pi} k(\sqrt{1+(\varphi'(x))^2} - 1)dx = 2k(\int_{\pi/2}^{\pi} \sqrt{1+(\varphi'(x))^2}dx - \frac{\pi}{2}). \qquad (17)$$

Here *k* is the Hooke coefficient which may be considered proportional (with some coefficient *τ*) to the string tensile load *N*:

$$k = \tau N.$$

The integral for the function (14) can be calculated approximately only.

$$\int_{\pi/2}^{\pi} \sqrt{1+(\varphi'(x))^2}dx$$

Computer simulation technique offers to the following approximate relation between a disturbed string energy and the parameter *σ* value:

$$E_B = N(\frac{C}{\sqrt{\sigma}} - R), \qquad (18)$$

where *C* and *R* are some positive constants.

If the whole kinetic energy

$$E_k = \frac{mv_1^2}{2}$$

of a concrete particle flying up to the string without losses is considered to convert into the string elastic disturbance energy, the following relation between the observed parameters *Q*, *D, N, H* of shotcreting process and the parameter *σ*:

$$\omega\frac{Q^3}{D^2} = N(\frac{C}{\sqrt{\sigma}} - R), \qquad (19)$$

where ω is some numerical coefficient.

Besides, inserting a generalized parameter

$$\varsigma = \frac{Q^3}{D^2 N}$$

of this process, we get the following:

$$\omega\varsigma E^2 = (\frac{C}{\sqrt{\sigma}} - R)\,. \tag{20}$$

σ can be expressed from the equation. Although it is worth mentioning here that in view of the formula (16) we are more interested in the quantity σ^2 related to the string reentrant speed v_2. Simple calculations show that this speed looks like

$$v_2 = \frac{\delta}{\sigma^2} = \hat{C}(\mu + B)^4\,, \tag{21}$$

where $\hat{C}$ and B are next positive constants.

Thus, the ratio $\xi = v_2 / v_1$ takes the following form:

$$\xi = \frac{\hat{C}(\varsigma E^2 + B)^4}{v_1} = A\frac{D^2(\varsigma E^2 + B)^4}{(Q+Q_0)E} = A\frac{D^2(Q(Q+Q_0)^2 E^2 + BD^2 N)^4}{(Q+Q_0)N^4 D^8}\,.$$

Reducing the last fraction and using with the accurate result of multiplication on some constant, we get

$$\xi = \frac{(Q(Q+Q_0)^2 E^2 + BD^2 N)^4}{(Q+Q_0)N^4 D^6 E}, \quad E = \exp(-\beta\frac{H}{QD^2})\,, \tag{22}$$

where B, β are also positive constants.

4. Analytical model of shotcrete concrete strength

Taking into consideration that concrete strength to some extent is a quantity which is the reciprocal of a bounce index, it is appropriate to regard it as a smoothed-out inverse relationship:

$$f(Q, D, H, N) = \lambda \ln(1 + \frac{1}{\sqrt{\xi}})\,, \tag{23}$$

where λ is a selected proportionality coefficient.

The comparison of the plot of the function (23) section at

$$\lambda = 0.912,$$

$$B=3\cdot10^{-7},$$
$$\beta=9000,$$
$$Q_0=24$$

with the experimental dependence of shotcrete concrete strength on the four basic parameters is presented in the Fig. 4—7 (where 1 is the theoretical data, 2 is the experimental data).

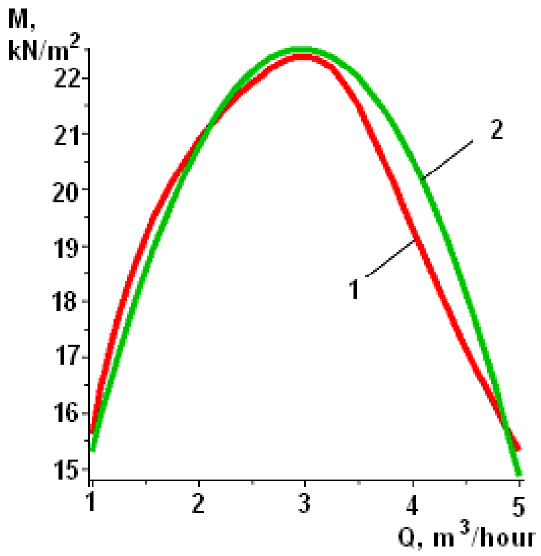

Fig. 4. The plot of the strength against the capacity *Q*

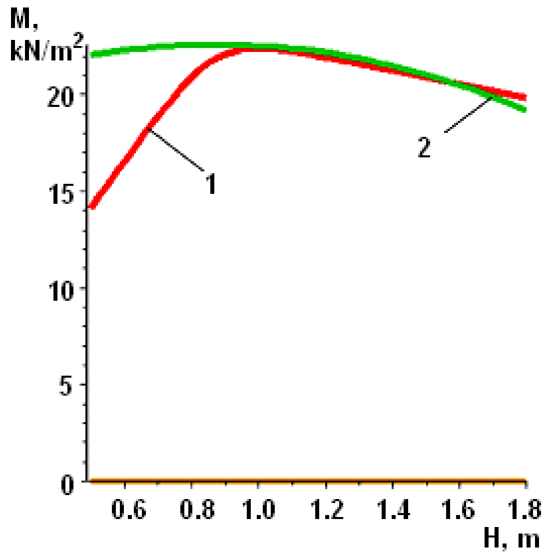

Fig. 5. The plot of the strength against the height *H*

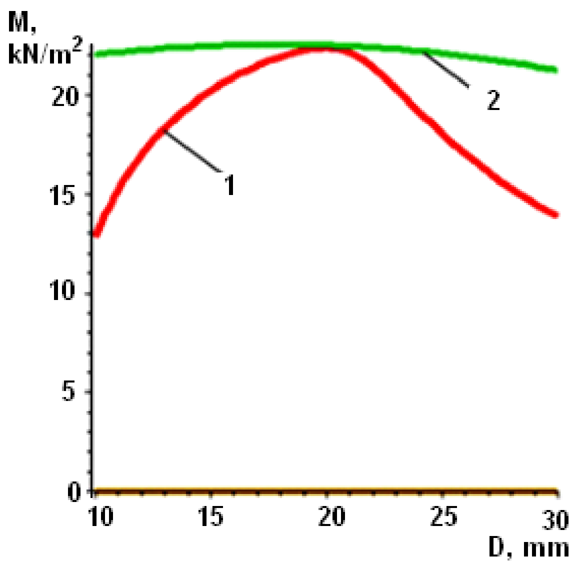

Fig. 6. The plot of the strength against the diameter *D*

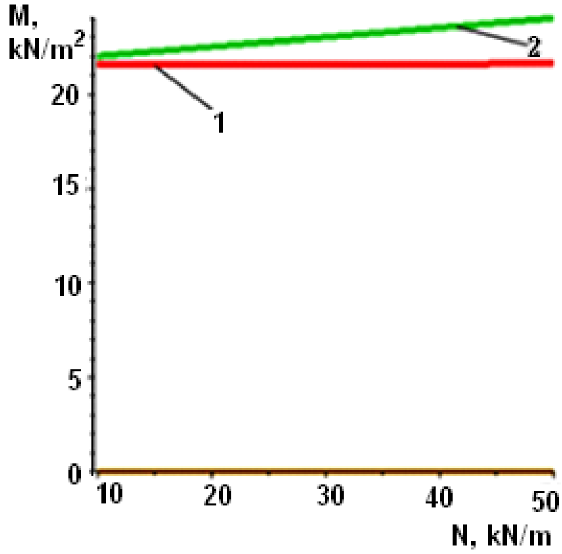

Fig. 7. The plot of the strength against the tension *N*

Conclusions

The analysis of experimental dependences and the theoretical models constructed showed that

1) the theoretical change in shotcrete concrete strength depending on a shotcrete concrete machine capacity almost completely coincides with in quantity and quality with the experimental data;
2) the theoretical dependence of shotcrete concrete strength on the distance between a nozzle and a formwork surface and the experimental curve all have similar in the form and clearly expressed extrema;
3) the analytical curve of the dependence of the shotcrete concrete strength on the nozzle diameter has a more clearly expressed extremum as compared with the experimental curve;
4) the theoretical influence of formwork material tension upon the shotcrete concrete strength almost coincides with the experimental data.

References

1. **Болотских, Л. В.** Технология торкретирования бетонной смеси на вертикальные поверхности пневмоопалубок: автореферат дис. … канд. техн. наук: 05.23.08 / Л. В. Болотских. — Воронеж, 2003. — 17 с. = **Bolotskikh, L. V.** Technology of shotcreting of concrete mix on the vertical surfaces of pneumatic formwork: Ph. D. in Engineering thesis’ abstract. Voronezh, 2003. 17 pp.
2. **Тихонов, А. Н.** Уравнения математической физики / А. Н. Тихонов, А. А. Самарский. — 3-е изд., испр. и доп. — М.: Наука, 1966. — 724 с. = **Tikhonov, A. N.,** Samarskiy A. A. The equations of the mathematical physics. 3rd edit., revised and added. Moscow, 1966. 724 pp.
3. **Смирнов, В. И.** Курс высшей математики: в 4 т. Т. 2 / В. И. Смирнов. — 21-е изд. — М.: Наука, 1974. — 656 с. = **Smirnov, V. I.** A course in higher mathematics. In 4 vol. Vol. 2. 21st edit. Moscow, 1974. 656 pp.