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DESTRUCTION OF LARGE NETWORKS

Background. Interest in the problem of network reliability increased rapidly owing to the new field of application, including computer and transport networks, as well as social and innovation networks. Models and algorithms of analysis of network destruction dynamics are not sufficiently advanced. In present paper, the simplest — epidemic — type of destruction is considered.
Results and conclusions. Epidemic type of network destruction is observed when destruction of the series of network nodes in certain step causes destruction of all the adjacent nodes in the next step of the process. The simple algorithm of calculation of step-by-step destruction of networks in the form of graphs and directed graphs was constructed with the aid of adjacency matrices. General recommendations on efficient network protection by transition from the initial graph to the frame are given on the basis of the analysis of matrix solution and graph structure. It is established that efficiency of operation and reliability of the network are interdependent and cannot be provided simultaneously. The relation between large network reliability and problem of globalization is remarked.

Keywords: network, destruction, model, graph, adjacency matrix, protection, globalization.

Introduction

Networks, from distribution to water-supply pipeline ones, are typically representative of a wide range of complex systems where binary connections play an extremely important role. They serve for transmitting resources between communications centers. Networks are exposed to external and internal factors, thus they are very likely to break down. This may lead to a wide range of consequences that may undermine the whole system in a number of ways. The general characteristics of destruction types of networks is given in [1—3]. At an earlier stage of network reliability and destruction research, the focus was mainly on applications to electrotechnical chains and communications systems [4, 5]. The problem of network reliability has been given more prominence since the expansion of new application fields including computer, transport, social, and innovation networks [6—10]. At the same time, models and algorithms of analysis of network destruction dynamics have not been entirely developed. A more detailed analysis is thus required both of general patterns of network destruction and of ways of their protection with the consideration to some certain peculiarities of their different types as well.

In the present paper we will limit our study to destructions of a simpler type which is of epidemic nature. In this case, destruction of any element at some moment leads to destruction of neighboring elements at the next stage. Let us assume that initially there is only one destroyed network node, at each following stage destruction of nodes neighboring a destroyed one occurs. If an initial node is not the only one, an additional fictitions node connected with the nodes of the initial destruction stage is introduced, thus an initial state of a network will occur only after the first stage. The further destruction development is easily visually traced on a graph provided it is not of a large size. Such a task becomes too complicated for direct analysis in large systems and the solution can be found through an adequate mathematical apparatus which is necessary to develop. The ultimate goal of the research is to determine the destruction time for the whole system, i. e. a number of steps to its complete destruction, time of loss of network coherence and finally to outline efficient protective measures.

1. A mathematical model

In the example under consideration destruction goes through adjacent nodes. Each step in time means addition of another step to each way on a graph of length t coming out of the first destroyed node. This allows to use the well-known theorem [11] for description of network destruction. According to the theorem, if an adjacency matrix of a graph C is raised to power t, an element $A_{ij}(t)$ of a matrix $A(t) = C^t$ is equal to a number of ways of length t from node V_i to node V_j. The null elements $A_{ij}(t)$ indicate that node V_j cannot be reached in steps t from node V_i.

Recall that the initial matrix C is valid and symmetrical for non-oriented graphs. The same is true for all its powers. Hence matrix $A(t)$ contains all the inforamtion about the network destruction dynamics on step t. As matrix C is valid and symmtrical, it can take the form $C = BLB^{-1}$ for non-oriented graphs where L is a diagonal matrix made up of the proper values of matrix C:

$$L = \begin{pmatrix} \lambda_1 & 0 & \dots & 0 \\ 0 & \lambda_2 & & \\ \dots & & \dots & \\ 0 & \dots & & \lambda_n \end{pmatrix}. \tag{1}$$

Then
$$A(t) = C^t = \left(BLB^{-1}\right)^t = BL^tB^{-1}, \tag{2}$$

where
$$L^t = \begin{pmatrix} \lambda_1^{\,t} & 0 & \dots & 0 \\ 0 & \lambda_2^{\,t} & & \\ \dots & & \dots & \\ 0 & \dots & & \lambda_n^{\,t} \end{pmatrix}. \tag{3}$$

In any case a diagonal element $(\lambda_{\max})^t$ dominates at long periods of time. Let us assume that $\lambda_{\max} = \lambda_1$. Then at long periods of time

$$L^t \to \lambda_1^{\,t} \begin{pmatrix} 1 & \dots 0 \\ 0 & \dots 0 \end{pmatrix} \tag{4}$$

and
$$C^t \to \lambda_1^{\,t} B \begin{pmatrix} 1 & \dots 0 \\ 0 & \dots 0 \end{pmatrix} B^{-1} = \lambda_1^{\,t} \left\| G_{ij} \right\|, \tag{5}$$

where $G_{ij} = B_{i1}\left(B^{-1}\right)_{1j}$. If matrix G has null elements, this means that an initial network is not coherent and destroyed in no period of time.

If we construct a matrix

$$T(t) = I + C + C^2 + \dots + C^t, \tag{6}$$

its nonzero elements fix network nodes destroyed in t steps. Hence, matrix $T(t)$ describes all the destructions that occurred in a network within time t. Complete network destruction after time t at the start from node V_i means that all the elements became nonzero in a correspondent line i of matrix $T(t)$. Selecting a line which will be the first to become nonzero in this sequential process, we can determine the minimum time of network destruction $t_{\min}$ and an initial height V_i which corresponds to such a pattern.

By finding the last line which became non-zero, we will determine the maximum time of network destruction $t_{\max}$ and a corresponding starting node. An isomorphic double graph can be constructed for flat networks by replacing nodes by arcs and arc by nodes. This will finally enable us to consider the task of sequential network destruction using the method above.

2. Network protection

Various strategies of network protection can be proposed depending on a goal. One of such goals can be maximum destruction slowdown by increasing the length of the ways, i. e. the time of complete destruction t. In terms of the structure of matrix $A(t)$ protection means removing fastest filling lines, i. e. complete protection of corresponding elements from destruction, which will make the ways longer and increase the time of complete destruction.

Examining the limit specific cases can help to clarify possible variants of network protection. Thus, it is extremely difficult, next to impossible, to protect a fully connected network, since any node is reached within one step. In a ring star graph presented in Fig. 1 protecting a central node is most reasonable, which will also increase the time of network destruction by $\sim N/2$ times where N is a number of nodes on a loop.

Such a structure is, for instance, typical for the Moscow underground. An example of destruction being examined may be realized during water flooding of single-level stations with the increase of a water level in one of them. It should be borne in mind that if protection will set the central node out of order, the time common for sequential access between the network elements also increases from 2 to $\sim N/2$. A ring star network structure is common for centralized management systems. Its topological peculiarities play the most significant part in boosting high-efficiency at the normal operation and a rapid system breakdown in case of a central node failure. Thus, the reason for the Soviet Union's collapse, for example, are unmistakenly of economical, social and political nature. Its rapid character is closely associated with a centralized management model in the country.

In another specific case, when a network takes the form of a tree, whose ways of destruction are nowhere interwoven making a complete network destruction rather long in time at any initial destruction point. A covering graph corresponding to an initial one presented in Fig. 1 is illustrated in Fig. 2. In this case, obviously the time of destruction significantly increases.

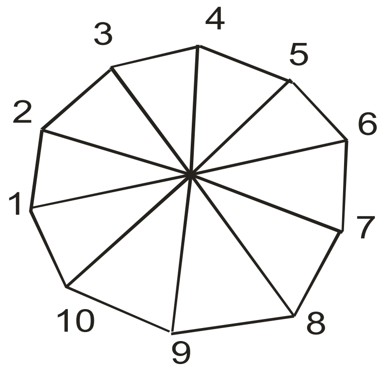

Fig. 1. Graph with a ring star structure

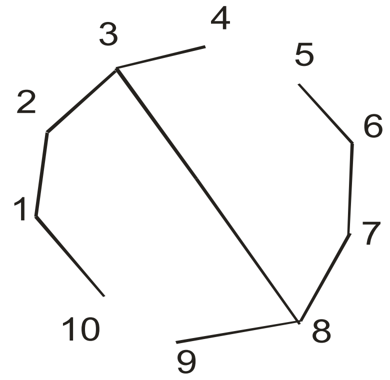

Fig. 2. A covering tree which is a subgraph of an initial graph

It becomes clear that some contradictory conditions are to be followed for perfect functioning. On one hand, a way between two nodes can be maximum short. This provides for a high-speed resource motion along a network. On the other hand, a high-speed communication installation at the same time makes rapid network destruction highly probable. This contradiction can in a way be eliminated if two networks are considered: one for a normal functioning and the other, functioning in the protection mode, is a subgraph of the first one. In other words, it should be accpted that a highly coherent network in danger of destruction immediately transfroms into a tree. As covering trees are not limited in number, it would be reasonable to select the strongest one. Destruction in tree reminds a branching character process. Its complete time of destruction corresponds to the longest branch from an initial network. Each tree has its center, i. e. each node maximum distant from the ends. In order to find the longest way from a given node i to all possible dangling vertexes j . Then out of all the vertexes i we select the one for which this value is maximum:

$$t^k = \min_i \max_j \left(t_{ij}^k\right). \tag{7}$$

Index k labels different covering graphs. Further, out of all the graphs we select the one for which the value obtained is maximum:

$$\tilde{t} = \max_k \min_i \max_j \left(t_{ij}^k\right). \tag{8}$$

The centre of the tree obtained needs maximum protection. This approach to network protection organization reduces the protection expenses and enhances the reliability.

Analyzing very large networks, it is reasonable to use the matrix limit. In this case, adjacency matrix C_{ij} is substituted by symmetrical integral nucleus $C(x, y)$, while sequential steps of destruction are described by iterations

$$\begin{aligned}\tilde{C}(x,y,t) &= C(x,y) + \int C(x,y_1)C(y_1,y)dy_1 + \ldots \\ &+ \int C(x,y_1)C(y_1,y_2)\ldots C(y_{t-1},y)dy_1\ldots dy_{t-1}.\end{aligned} \tag{9}$$

The following integral equation is valid for a limit distribution

$$\tilde{C}(x,y) = C(x,y) + \int C(x,y_1)\tilde{C}(y_1,y)dy_1. \tag{10}$$

It can be solved by iterations, which is equivalent to summing up a row in Equation (9). It should be noted that function $C(x, y)$ for a model examined is discontinuous. Thus, integration in formulae (9) and (10) is symbolic, while real calculation procedures presuppose a direct summing up.

Conclusions

It is thus revealed that operating efficiency and network reliability are mutually dependent and can not be provided simultaneously.

Simple and efficient matrix methods of modelling and analysis of a stepwise process of network destruction have been discovered. They allow to make simultaneous calculations of all possible destruction examples depending on a number of initial destroyed nodes. If necessary, each single destruction example can be easily visualized.

This paper looked into the simplest models of determined destruction occurring at a constant speed without restoration. In real systems it may become necessary to consider the factors earlier neglected. A probabilistic nature of destruction may be taken into consideration by introducing a jump matrix supposing that destruction has Markov behavior. Varying longevity of neighbouring nodes destruction can be described by introducing fictitious intermediate nodes.

The results obtained allow to take a fresh look at globalization as a problem of efficiency and reliability of global information networks, finance and epidemic spreads. Even a simple model examined in this paper indicates that communications, trade and transport efficiency rise owing to globalization simultaneously leads to a greater risk rise during the drestruction of corresponding networks. Nowadays, we are getting acutely aware of these problems and most importantly, practical conclusions and recommendations should be in agreement with the network destruction laws discovered. Creation of fully connected global networks without effective mechanisms of their protection may have a disastrous outcome. Further development of theoretical models of network protection and, most importantly, their practical realization are imperatives of the years to come.

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