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THEORETICAL ESTIMATION OF FIBER DISTRIBUTION IN FIBER REINFORCED CONCRETES

Problem statement. Theory of fiber reinforcement of concrete materials calls for the construction of appropriate mathematical models. The article describes the mathematical basis for calculating the density of intersections of reinforcing fibers in cross section of fiber concrete element as well as average cross sectional area of fibers.

Results and conclusions. The problems of geometric probability are taken as the basis for construction of the model of reinforcing fiber distribution in concrete. The function of density of reinforcing fiber distribution and formula for determining reinforcing fiber concentration in concrete are obtained. The functions and formulas obtained are valid for different geometrical characteristics of reinforcing fibers.

The results obtained are presented in such form for the first time. At present, work on mathematical planning of the experiment which will enable us to check the correctness of obtained theoretical conclusions is underway.

Keywords: fiber, concrete, fiber reinforcement, geometric probability.

Introduction

In the middle of the previous century a new field connected with designing efficient composite materials began to develop in structural material science. These days it is widely used in various industries. Construction is one of the most resource-demanding industries. Concrete is a most widely used and available material. Having all necessary plastic properties in structures, it is used in designing efficient varieties of composite materials for industrious and civil construction.

Dispersion suggests a significantly increasing interfacial area between dispersed phase and disperse medium. Under some conditions this process can lead to qualitative changes in the physical state of various multicomponent systems. It is important to determine dispersion boundaries efficient for armored concrete. To assess dispersion efficiency, we need to analyze corresponding changes in physical and firstly mechanical characteristics of concrete matrix (strength, crack growth resistance, etc.). These are all caused by largely decreasing diameters of reinforcing elements and their growing dispersion within concrete. The studies show that the optimal levels of disperse distribution of reinforcing elements within concrete are closely connected with the concrete structure parameters.

The development of the theory of disperse reinforcement of concrete materials particularly requires adequate structural models of disperse reinforced systems. For thin steel fibres a diameter-length ratio is about 10^{-2}. It is most important here to find out the spatial location of reinforcement. The studies show that the optimal levels of disperse reinforcement within concrete are closely connected with the concrete structure parameters.

Many researchers studied the prediction of fiber distribution in concrete matrix. The work [1] gives an in-depth analysis of the problem and suggests its possible solution. Here another attempt at finding the solution to it is made.

Most of the mathematical issues arising in the study of fiber distribution in concrete are so-called issues of geometrical probability [2, 3].

Our aim is to construct a model suitable for various correlations of concrete and steel fiber parameters with further testing of their validity in experimental studies.

The main assumption made in the reinforcement choice is that geometrical characteristics of reinforcing elements, their diameters and dispersion should be related to concrete flaws and above all by discontinuity of the upper level of the concrete structure. The basic characteristics in describing the disperse distribution of reinforcing elements within concrete are volume content, section area, diameter, quantity, surface area and length of reinforcing fibres (μ_f, A_f, d_f, N_f). In reinforcing row concrete the optimal distance between reinforcing elements is 20 mm and more dependent on a filler fineness. The diameter is about 3 mm, which is necessary to increase concrete efficiency [1].

Modelling is not possible without taking into account the whole concrete pattern. Thus the basic modelling object is a concrete nucleus. It is important to find out the homogeneity conditions for a concrete nucleus. The sizes of an elementary concrete nucleus should significantly exceed all the internal structural heterogeneities of concrete determined mostly by the size of a filler. Designing a reinforcement model, a lot of attention should be given to the effect a concrete structure has on the geometrical characteristics of a system. It should also be borne in mind that heterogeneities in concrete are absolutely randomly dispersed and the system should thus be viewed as statistically homogenous. There are different forms of heterogeneities. These are mostly spheres or polyhedrons all helping to get a geometrical image of the structure. A suitable structure makes the reinforcement process much easier. However, such quasi-crystalline systems can have differential properties of disordered and crystal structures, which is well known in solid state science. What is more, these days all the technologies of obtaining disperse reinforced concrete are based on agitation of mix for even distribution of reinforcing elements within concrete, the distribution being random.

1. Function of fiber distribution in concrete

Let us assume that the fiber volume density is n, length l, orientation random.

We will locate the concrete section plane and will examine the spherical coordinate system where the vertical is perpendicular to the plane and the angles range within $0 \le \phi < 2\pi$, as shown in Fig. 1. In even distribution along the directions, i. e. if it is isotropic, the density function of angle distribution takes the following form

$$f(\theta,\phi) = \frac{1}{4\pi}\sin\theta\,. \tag{1}$$

The function of distribution density is standardized per unit, i. e.

$$\int_0^{2\pi}\int_0^{\pi} f(\theta,\phi)\,d\theta\,d\phi = 1\,. \tag{2}$$

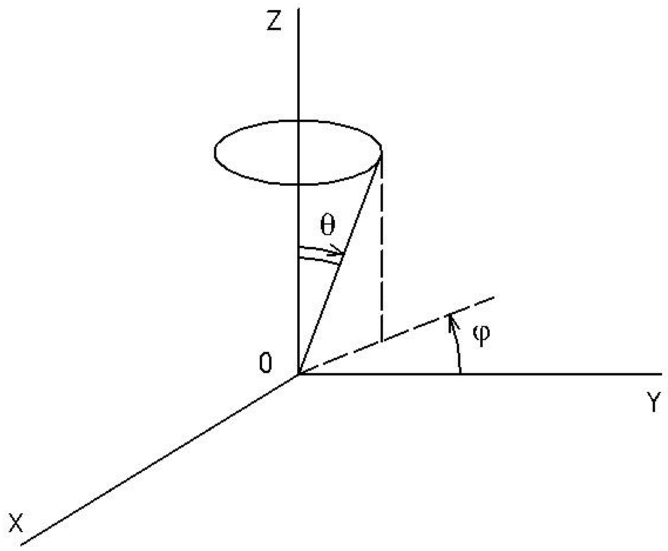

Fig. 1. Spherical coordinate system

2. Determining fiber concentration in a finished pattern based on the obtained function of fiber density distribution

Obviously, intersection of fibres with the planes does not depend on polar angle φ. Thus, in order to find out the intersection distribution, let us pass on to the distribution function in angle θ. Then as a result of integration over angle φ we get a new distribution function

$$F(\theta) = \int_0^{2\pi} f(\theta, \phi) d\phi = \frac{\sin\theta}{2}. \quad (3)$$

For fibres inclined in the angle interval $(\theta, \theta + d\theta)$,

$$\frac{dn}{n} = F(\theta) d\theta. \quad (4)$$

Let us count the number of fiber intersections at angle θ with the section plane. Let us compare each wire located at angle θ with a segment of length $l_\theta = l|\cos\theta|$ which is perpendicular to the section plane. Obviously, a number of intersections with such segments will be the same as with an initial fiber. Let us thus examine an equivalent calculation of density surfaces dn_θ of fiber intersections of length l_θ with the plane per surface area unit.

There are intersections with wires whose centers are located at distance of no more than $l_\theta/2$. Let us thus choose a rectangular parallelepiped of height l_θ and base area S to calculate intersection densities dn_s.

The volume of such a parallelepiped $V=Sl_\theta$, and a number of fibres in it equal to a number of fiber intersections with the plane is

$$dN_\theta = dn_\theta V = dn_\theta S l_\theta . \quad (5)$$

Then the intersection density is

$$dn_\theta = \frac{dN_\theta}{S} = dn_\theta l_\theta = nF(\theta)d\theta l\left|\cos\theta\right| = n\frac{\sin\theta}{2} l\left|\cos\theta\right| d\theta . \quad (6)$$

To obtain a total surface intersection density, we should integrate the formula (6) over angle θ.

Then $$n_s = \int dn_\theta = \int_0^\pi \frac{nl}{2}\sin\theta\left|\cos\theta\right| d\theta = 2\int_0^{\pi/2} \frac{nl}{2}\sin\theta\cos\theta d\theta = nl . \quad (7)$$

For non-isotropic distribution there is another factor considering predominant orientation of reinforcing fibres. Tasks considering spatial constrains caused by boundaries particularly belong to this type of tasks.

Calculating the average section area and relations of fiber section area respectively to the total concrete section area, we should remember that the area increases in oblique section (Fig. 2).

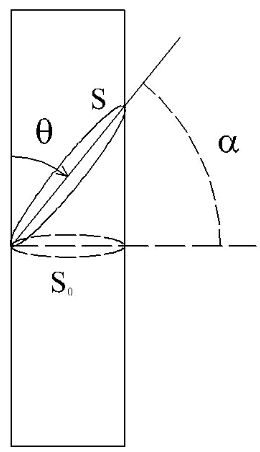

Fig. 2. Fibre section with a plane

There are elements whose area S is proportional to value $1/\sin\theta$:

$$S_0 = S\cos\alpha = S\sin\theta . \quad (8)$$

Formally with $\theta \to 0$ the function is $1/\sin\theta \to \infty$. There is, however, a limit angle θ_0 (for a long fiber it is $\approx d/l$), which constrains a domain of variation θ, since fibres have both the ultimate length l and diameter d. Let us calculate the relative area ρ_S of fiber sections similar to the above calculations of the intersection density. For this purpose, the intersection density dn_θ at angle θ is multiplied by the factor $\pi d^2/4\sin\theta$ with further calculation of an integral over angles θ.

Then
$$\rho_S = \frac{\pi d^2}{4}\int_0^{\pi}\frac{dn_\theta}{\sin\theta} = \frac{\pi d^2}{4} nl \int_0^{\pi/2}\cos\theta d\theta = nl\frac{\pi d^2}{4}. \quad (9)$$

Since n is a fiber volume concentration within concrete, value p_s is dimensionless.

Conclusions

In order to find out the fiber concentration in a finished pattern, we need either to measure the fiber intersection density with the section plane or calculate the relation of the fiber section area to the total section area and finally calculate the concentration with the formulas (7) or (9) respectively. A hand-made calculation makes it easier to count the intersections. An automatic computerized calculation is more suitable for measuring the total fiber section area, since fresh cuts of steel fiber have a much higher reflectivity and are easily detected by computer contrast programs. In any case to calculate the fiber n concentration in a concrete pattern it is necessary to know their parameters, i. e. diameter and length. In case of calculating n by a number of intersections only the length of fibres needs also to be known. To calculate n by the section area, we should also know their diameter. On the other hand, if we need to determine the volume concentration of fibres in concrete n_v, we can get the solution measuring $p_s = n_v$ according to the formula (9).

The obtained results in this form are presented for the first time. Presently, an experiment to test the validity of the theoretical conclusions is being mathematically planned.

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