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THE USE OF THE METHOD OF CONFORMAL MAPPINGS TO DETERMINE VELOCITY FIELDS OF AIR FLOWS IN VENTILATION PROBLEMS

Problem statement. Modern informational technologies make it possible to solve various complex problems of aerodynamics including problems of ventilation at different dynamic characteristics of flows. The article deals with the problem of ventilation with the use of low-velocity irrotational air flows. Known numerical methods of calculation of electric current related to the solution of large systems of partial differential equations and are faulty with respect to reliability and calculation precision. The use of the method of conformal mappings to determine velocity fields in ventilation problems is substantiated.

Results and conclusions. The method for calculation of velocity fields and electric current lines in ventilated premises has been developed based on the method of conformal mappings and assumption of irrotational type of air flow motion. Application of this method allows one to calculate required fields using simpler algorithms without multiple computer calculations.

Keywords: conformal mapping, velocity field, electric current lines, ventilation.

Introduction

Nowadays, ventilation systems generating a horizontal one-way low turbulence air flow are more and more frequently used in industrial premises of various purposes. Air is delivered and removed at low speeds through supply and exhaust systems with a large surface (e. g. through perforated walls of the premise). It is essential that ventilation systems should generate an even air flow that circulates through the entire premise, just like a piston. The more even the air flow is, the more efficient and effective is ventilation.

Therefore, calculating fields of air rates in premises fitted with ventilation systems is a pressing problem.

The article looks into the evaluation of air speed and electric current fields in a routine (steady) ventilation mode. It is often the case that this task is accomplished by means of numerical methods utilizing a computer solution of a complex system of equations of mathematical physic [1—6]. A large amount of calculations involved in space tasks can be reduced due to a transition to two-dimensional models.

Natural two-dimension models of premises are rectangles with partitions. Thus, in [7] air flow movement is studied using the example of three partitions. The obtained description of air speed fields is quite credible, while the way these results were obtained involved step-wise numerical solution by means of grid methods for systems of differential equations turns out to be rather fragile when it comes to mathematical accuracy and thereby, validity. It is recommended that alternative methods and ideas should be applied to this kind of tasks.

Below, we discuss technical details of one well-known (see, e. g. [7]—[8]) approach to fluid and gas dynamics that is related to the method of conformal mapping of flat areas.

1. Conformal mapping related to the problem in question

The initial task of ventilating is studied in a two-dimensional (flat) setting, so that the main premise is a rectangle with sides parallel to the axes of X and Y coordinates. Its entire left side is the supply system, while the right one is the exhaust system that delivers and removes the air at a specified speed. In the simplest shape of such a premise, air flow is parallel to the horizontal axis (Fig. 1).

We are now proceeding to discuss a situation where the above premise is halfway split by a vertical partition that starts from the floor and does not reach the ceiling (Fig. 2). The problem we are going to engage in below deals with determining electric current lines in ventilation of such a premise.

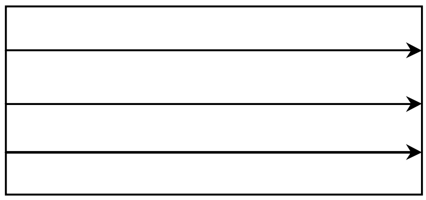

Fig. 1. Electric current lines in a rectangular premise

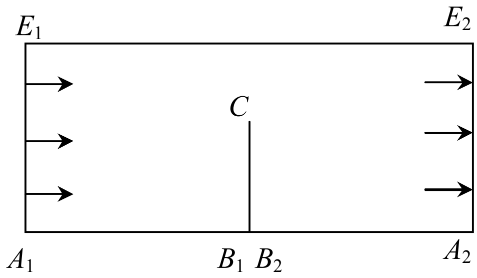

Fig. 2. Ventilation of premises with a partition

Note. For the sake of simplicity, we are studying a symmetrical premise with only one partition. However, conformal mapping is applicable to more complex areas as well. Ventilation from [7], in particular, can be calculated in the same manner. The technical task from below gets just a little bit more complicated.

The major assumption in the use of conformal mapping for the problem in question is a transition from a complete system of Navier–Stokes equations system that describes air movement in a premise to a simpler equation system of Koshi-Riman. This means that we consider a moving air flow to be noncondensable and its movement is nonrotational. In this case, the components of the speed vector v_1, v_2 of the moving air from a gradient of some potential function $\varphi(x, y)$:

$$v_1 = \frac{\partial \varphi}{\partial x},\ v_2 = \frac{\partial \varphi}{\partial y}.$$

Electric current lines (paths of moving air particles) are lines of the level of another function $\psi(x, y)$ associated with the function $\varphi(x, y)$. A complex fluidity potential, i. e. the function

$$f(z) = \varphi(z) + i\psi(z) \quad (z = x + iy), \tag{1}$$

is a holomorphic function in a flat figure *D* from Fig. 2. For the derivative $f'(z)$ of complex potential, we have

$$f'(z) = v_1 - iv_2. \tag{2}$$

Assuming that this vector at no point of the area *D* turns into null, we get $f'(z) \neq 0$. This means that mapping performed by the function $f(z)$ is conformal in the figure *D*. It can also be inferred that the projections of the area *D* under the action of $f(z)$ has ordinary geometric properties. In fact, the lower boundary of the figure *D*, i. e. the broken line $A_1B_1CB_2A_2$, as well as its upper boundary E_1E_2, can be regarded as electric current lines. Then, values of the function $\psi(z)$ on these boundaries are some constants, while the whole figure $D^* = f(D)$ can be assumed to be some rectangle in a plane of a new complex variable w (Fig. 3).

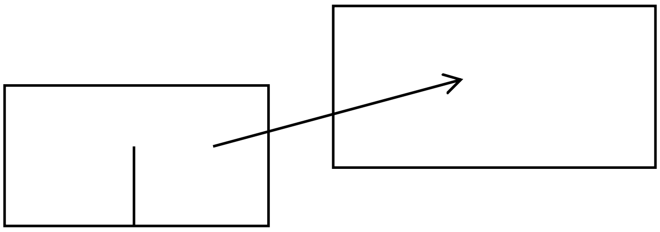

Fig. 3. Conformal mapping $f(z)$ of the initial figure *D* (a cut rectangle) onto a non-cut rectangle D^*

According to the main idea of the application of conformal mapping in gas dynamics, electric current lines in the figure *D* can be viewed as projections of the horizontal lines from $D^* = f(D)$ under the action of a projection inverse to $f(z)$ (which is also conformal).

2. Constructing a conformal projection

Let us give a more detailed description of the projection f and f^{-1}. For that, we introduce another assumption about the air moving in the figure *D*. To be more precise, using the symmetry of the figure, we assume that electrical current lines are also symmetrical on the left and right sides as opposed to the partition BC.

The above assumption only enables us to discuss the left half of the figure *D* and the left part D^* respectively. Conformal mapping $g(z)$ that unites these two halves according to the well-known symmetry principle (see, e. g. [7]) goes on till initial conformal mapping $f(z)$. It is also to be noted that from the assumption about symmetry of electrical current lines in the figure *D*, it can be inferred that they are perpendicular to the partition BC (or its continuation, to be more exact). This note completes formalization of 'external' properties of the initial conformal mapping.

So, we need to construct (to describe using formulas) a conformal projection $g(z)$ that comes (Fig. 4) from the rectangle $MNPQ$ of the fixed size into some rectangle $M'N'P'Q'$. The left side MQ is expected to transfer into the left side $M'Q'$ (air moves from left to right), while a part PR of the right side PN (through which the air flows into the symmetrical rectangle) is to correspond to the entire right side $P'N'$ of the rectangle $M'N'P'Q'$ in the projection $g(z)$.

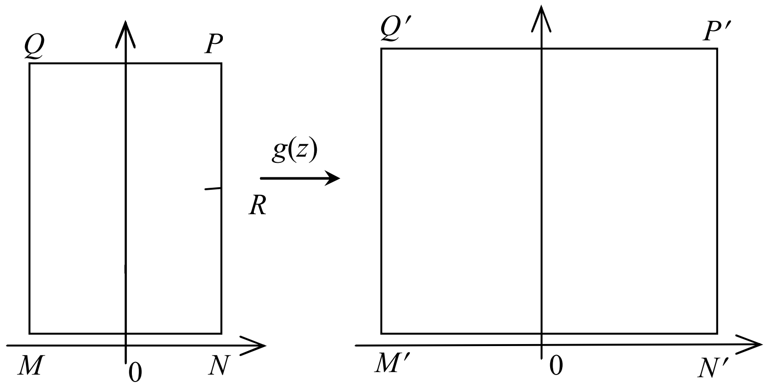

Fig. 4. Projection of the halves of the figures from Fig. 3

Let us now move on to a constructive description of this projection. It relies on the properties of the well-known (see e. g. [2]) of the elliptic integral of the first type

$$F(z,\alpha)=\int_0^z \frac{d\varsigma}{\sqrt{(1-\varsigma^2)(1-\alpha^2\varsigma^2)}} \quad (0<\alpha<1) \tag{3}$$

and its inverse function called elliptic sine. In the study of these functions, the upper subplane is used as an intermediate area where the function (3) is specified. Under the action of this function, the subplane turns into the rectangle $M'N'P'Q'$, when

$$0\to 0, \quad 1\to N', \quad \frac{1}{\alpha}\to P',$$

while the rest of the construction is performed using symmetry considerations. The coordinates of the vertexes N', P' are determined using the formulas

$$\begin{aligned} K' &= \int_0^1 \frac{dt}{\sqrt{(1-t^2)(1-\alpha^2 t^2)}}, \\ K'' &= \int_1^{1/\alpha} \frac{dt}{\sqrt{(t^2-1)(1-\alpha^2 t^2)}}. \end{aligned} \tag{4}$$

so N' has the coordinates (K',0), P' has the coordinates (K', K'').

Note. The parameter $\alpha \in (0.1)$ that is contained in determining the function $F(z,\alpha)$ allows for the variation of the ratio of the sides of the rectangle $M'N'P'Q'$ from null to infinity.

Most of this construction is connected with the action of the function inverse to (3), i. e. elliptic sine $sn(z,\beta)$. The value of the parameter $\beta \in (0.1)$ is chosen based on the ratio of the sides of the left half of an actual premise. This rectangle can be projected onto the upper subplane using the elliptic sine. Under the influence of this 'unfolding', projections of the vertex Q, M, N, P go over into the points $-\frac{1}{\beta}, -1, 1, \frac{1}{\beta}$ of the real line respectively. The point *R* goes into some point *S* from the interval $\left(1, \frac{1}{\beta}\right)$.

Now we note that using the arbitrary fixed $\alpha \in (0,1)$, we can construct a linear fractional automorphism *h* of the upper subplane that transforms three points $\left(-\frac{1}{\beta}, -1, \frac{1}{\beta}\right)$ into three points $\left(-\frac{1}{\alpha}, -1, \frac{1}{\alpha}\right)$. Accurate formulas are cumbersome for this projection, but its construction is described in any textbook on the theory of the functions of a complex variable (see, e. g. [9]).

Under the influence of such mapping, the already well-known point

$$S \in \left(1, \frac{1}{\beta}\right)$$

goes into some point of the real axis. Finally, we need to select a suitable value of the parameter α that ensures that the point *S* under the action of fractional linear projection *h* becomes one.

The superposition of the mappings

$$F(w,\alpha) \circ h \circ sn(z,\beta) \tag{5}$$

is then the initial conformal mapping *g*, as points Q, M, R, P on the sides of the rectangle $MNPQ$ (see Fig. 4) go into the vertexes Q', M', N', P' of the second rectangle in the same Figure respectively.

3. Numerical construction of electrical current lines

Let us discuss some details of the suggested method using a numerical example.

The left half of the premise under investigation is 4 m high and 6 m long (see Fig. 2) with a 3 m high partition in the middle has the 4/3 ratio of the rectangle sides. Then, the ratio K'' / K' is 8/3. This ratio is attained for a rectangle that is obtained from the subplane by the mapping $F(\zeta, \beta)$ with the parameter value $\beta \approx 0.06$. Ordinary computer calculations according to the formulas (4) yield $K'' \approx 4.2,\ \ K' \approx 1.57$. The upper point C of the partition is at the height of $\approx 3{,}15$ of this rectangle. Mapping by means of the elliptic sine applied to the rectangle under investigation transforms this point into the point 10.4 lie in the axis OX.

The above-mentioned fractional linear mapping that transforms three points $\left(-\frac{1}{\beta},\ -1,\ \frac{1}{\beta}\right)$ into three points $\left(-\frac{1}{\alpha},\ -1,\ \frac{1}{\alpha}\right)$ is specified by the formula

$$z^* = c\frac{z-a}{z-b},$$

where

$$a = \left(-\frac{1}{\beta}\right) \cdot \frac{\frac{1}{\alpha} - \frac{1}{\beta}}{1 - \frac{1}{\alpha} \cdot \frac{1}{\beta}},$$

$$b = \left(-\frac{1}{\beta}\right) \cdot \frac{1 - \frac{1}{\alpha} \cdot \frac{1}{\beta}}{\frac{1}{\alpha} - \frac{1}{\beta}},$$

$$c = \left(\frac{1}{\alpha}\right) \cdot \frac{1 - \frac{1}{\alpha} \cdot \frac{1}{\beta}}{\frac{1}{\alpha} - \frac{1}{\beta}}.$$

In this example, we can approximately assume the mapping

$$w = \frac{8(\zeta - 5)}{50 - \zeta},$$

for which

$$-\frac{1}{\beta}\to-\frac{28}{11},\ -1\to-\frac{16}{17}\approx-1,\ \frac{1}{\beta}\to\frac{44}{17}\approx\frac{28}{11},\ 10\to1.$$

This means that the parameter α that governs the last component of the mapping (5) in this case acquires the value that is close to $\frac{11}{28}\approx0.37$.

We have therefore reduced the problem of constructing a needed conformal mapping to the superposition of a practically calculated fractional linear mapping with the classic function $sn(z,\beta)$ and the function (3) that is inverse to it. The parameters α,β are determined by a specified initial area according to the below scheme.

In practical calculations, the values of two 'complex' components of the mapping (5) can be borrowed from the tables compiled with any degree of accuracy (e. g. using integration). The results of the calculations of electric current lines are given in Fig. 5.

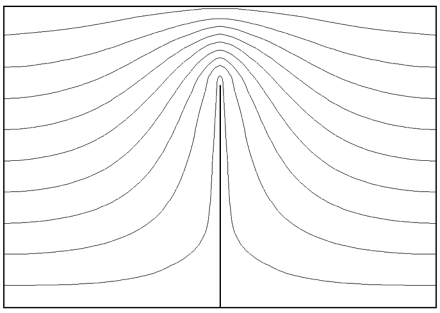

Fig. 5. Electric current lines in a premise with a partition and a horizontal one-way air flow

Conclusions

The suggested method for the calculation of speed fields and electric current lines of air in a premise is believed to be simpler as compared to integration of differential equations. In cases where errors associated with a transition from Navier–Stokes equations to Kosh-Riman's conditions are acceptable, this approach can be favored above numerical solutions of equations in a partial derivative. This method has proved to be valid in the course of the comparison of its electric current lines (see Fig. 5) with the results obtained by others.

The application of the obtained technique allows one to calculate electric current lines of air in a premise with a horizontal one-way air flow and a partition and also to evaluate the sizes of perturbation zone of an air flow. These sizes are applicable to the height of a system (partition).

The use of this technique in the calculation of speed fields and electric current lines of air allows one to calculate electric current lines of air in different air exchange schemes as well as to select the most efficient one, thus improving the efficiency of microclimate systems.

References

1. V. N. Melkumov , et al., “Nonstationary Processes of Formation of Air Flows in Premises by Ventilation Systems”, Proceedings of Orel State Technological University. Construction and Transport, N 3—15 (2007), 36—42.
2. V. N. Melkumov, et al., “Nonstationary Field of Concentrations of Natural Gas in Well at Its Leak from Subterranean Gas Pipeline”, Privolzhsky Scientific Journal, N 4(8) (2008), 98—103.
3. V. N. Melkumov, et al., “Estimation of Accumulating Power of Ventilated Volumes for Lowering the Required Air Exchange in Premises”, Proceedings of Voronezh State Technological University, vol. 3, N 1 (2007), 205—207.
4. V. N. Melkumov, et al., “Formation of Convective Air Flows at the Presence of Heat Sources in Premise”, Proceedings of Volgograd State University of Architecture and Civil Engineering. Construction and Architecture, N 12 (2008), 76—80.
5. V. N. Melkumov, S. N. Kuznetsov, “Dynamics of Formation of Air Flows and Temperature Fields in Premises”, Scientific Herald of Voronezh State University of Architecture and Civil Engineering. Construction and Architecture [in Russian], N 4 (2008), 172—178.
6. V. N. Melkumov, S. N. Kuznetsov, “Interaction of Ventilation Air Flows with Convective Flows from Heat Sources”, Proceedings of Universities. Construction, N 1 (2009), 63—70.
7. K. A. Sklyarov, A. V. Cheremisin, S. P. Pavlyukov, “Two-Dimensional Stationary Movement of Air Flows in Premises with Partitions”, Scientific Herald of Voronezh State University of Architecture and Civil Engineering. Construction and Architecture [in Russian], N 9 (2008), 118—123.
8. M. A. Lavrentyev, B. V. Shabat, *Methods of the Theory of Functions of a Complex Variable* (Moscow, 2002) [in Russian].
9. B. V. Shabat, *Introduction to Complex Analysis,* in 2 vols, vol. 1., Moscow, 2004 [in Russian].
10. N. I. Akhiyezer, *Elements of Elliptic Function Theory*, 2nd ed. (Moscow, 1970) [in Russian].