HEAT AND GAS SUPPLY, VENTILATION, AIR CONDITIONING, GAS SUPPLY AND ILLUMINATION

UDC 669.86: 536.21

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MODELLING OF HEAT TRANSFER PROCESSES IN THE ZONE OF HEATING PIPE LAYING AND IN INHOMOGENEOUS BUILDING ENVELOPES WITH THE USE OF CONFORMAL MAPPING METHOD

Statement of the problem. Metal elements used in building envelopes increase durability, but noticeably change value and nature of heat losses. The problem is to show the influence of metal ribs inside a flat wall on heat transfer process. The problem of determination of the temperature fields around heat conductor partially embedded in soil is also considered.

Results. The calculation of heat transfer processes around heat conductor partially embedded in the wall (body) and in the inhomogeneous building envelopes with metal ribs was performed with the use of conformal mapping method. Simple and efficient method for modelling steady state processes of heat transfer in the area of heat conductor partially embedded in body and in inhomogeneous building envelopes with metal ribs.

Conclusions. Obtained dependencies enabled us to calculate the most important feature of the product, namely coefficient of thermal and technical homogeneity. The tackled problems of heat transfer reveal the advantages of the conformal mapping method.

Keywords: temperature field, heat flows, heating networks, inhomogeneous building envelope, heat transfer, conformal mapping method.

Introduction

An efficient way of tackling heat transfer problems is the conformal mapping method. One of the features that make this method stand out above the others is analytical functions of a complex variable. The values of analytical functions in a complex domain are largely determined by the values of the functions at the boundary of this domain. This exhibits the role of conformal mappings in addressing the heat transfer problems: it will suffice to find conformal transformations of the boundary of a complex domain for these transformations to map one domain onto the other.

Analytical formulas which determine a conformal correspondence between the boundaries of the domain in question define the correspondence between heat flows and heat boundary conditions.

Heat transfer in the below domains of complex configuration can be calculated numerically. However, a more simple solution is obtained using the conformal mapping method. The examples of its use are discussed, in particular, in [1—3] and etc.

1. Heat pipe partially embedded in the wall (body)

In the process of laying heat networks, sometimes there arises a need to define a temperature field surrounding a part of a long heat pipe embedded into the wall (body). A problem like this might sound as follows: a part of the long pipe with the radius R is embedded into the body whose boundary has a zero temperature. The angle between the boundary of the body and the tangent line to the circle of the pipe is β (Fig. 1).

The temperature of the surface of the pipe T is kept stable. Between the surfaces there are isolating points iR_o $-iR_o$. It is necessary to find a stationary temperature field around the part of the pipe submerged into the body.

The linear fractional function is like

$$W = \frac{Z - iR_o}{Z + iR_o}, \tag{1}$$

where

$$Z = x + iy = r\cos\phi + ir\sin\phi,$$

$$W = u + iv = \rho \exp i\theta, \quad R_o = R\cos\beta,$$

maps the dashed area of the plane Z into the inside of the angle $(-\beta)$ of the plane W (Fig. 2).

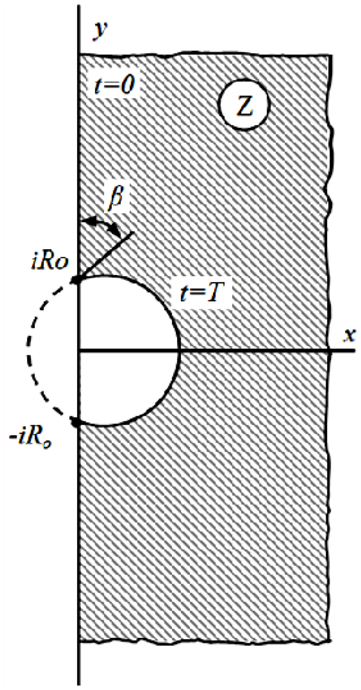

Fig. 1. Physical model and the coordinate system

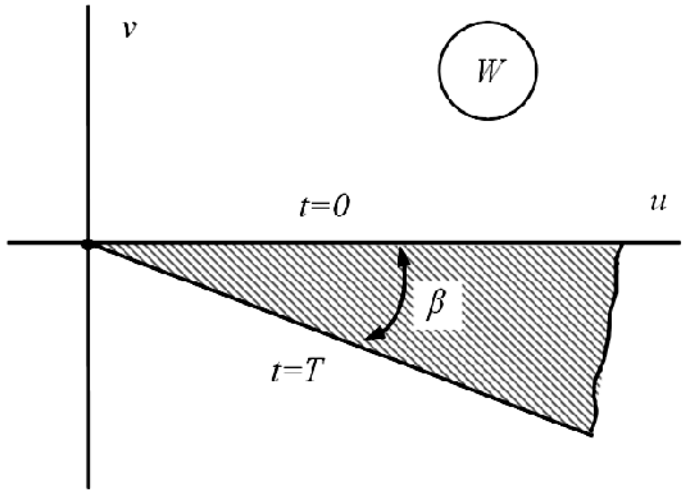

Fig. 2. Mapping of the examined area of the plane *Z* into the inside of the angle of the plane *W*

Meanwhile zero isotherms and these of T of the plane Z are mapped into the plane W respectively into an actual positive semiaxis and ray $\theta = -\beta$.

The temperature distribution inside this angle is given by the equation

$$t(W) = t(\rho, \theta) = -\frac{T}{\beta}\theta, \tag{2}$$

where

$$\theta = \operatorname{arctg}\frac{v}{u}. \tag{3}$$

Substituting (2) into (1) and removing the imaginary part of the denominator we get

$$u = \frac{x^2 + y^2 - R_o^2}{x^2 + y^2 + 2yR_o + R_o^2}, \tag{4}$$

$$v = -\frac{2xR_o}{x^2 + y^2 + 2yR_o + R_o^2}. \tag{5}$$

Combining the equations (3), (4) and (5) we write

$$\theta = \operatorname{arctg}(-)\frac{2xR_o}{x^2 + y^2 - R_o^2}.$$

Born in mind that $\operatorname{arctg}\beta = -\operatorname{arctg}(-\beta)$ ultimately we get

$$t(z) = t(x, y) = \frac{T}{\beta}\operatorname{arctg}\frac{2xR_o}{x^2 + y^2 - R_o^2}$$

or

$$t(z) = t(r, \phi) = \frac{T}{\beta}\operatorname{arctg}\frac{2r\cos\phi R_o}{r^2 - R_o^2}.$$

If the temperature of the body which is far enough from the surface of the pipe and the temperature of the proper surface of the pipe is maintained respectively at t_1 and t_2 $(t_1 < t_2)$, then the temperature field has the form

$$\frac{t(r, \phi) - t_1}{t_2 - t_1} = \frac{1}{\beta}\operatorname{arctg}\frac{2r\cos\phi R\cos\beta}{r^2 - (R\cos\beta)^2}. \quad (6)$$

The use of a conformal mapping makes the problem solution a great deal easier. Using the expression (6) we can find the temperature field around the pipe partially embedded into the body.

A similar approach to thermal calculation of heat pipes built into a slab was utilized in [4].

2. Structure with metal ribs

The introduction of metal parts into heat insulation products improves the strength of the structure but metal elements can also largely contribute to the magnitude and nature of heat losses. Below is the effect of metal ribs inside a flat wall with the thickness *H* have on heat transfer. The structure under investigation is borrowed from [5, 6].

Thin metal ribs are assumed to be highly heat conductive and be only of *h* height, therefore their temperature can be considered equal to that of metal sheets the wall is sheathed with (the ribs are attached to both sheets). The space between the sheath and ribs is filled with a heat insulation material with the heat conductivity coefficient of λ. The temperature of the environment t_2 and of the surface of the wall from the side of the ribs t_1 is known.

Fig. 3 shows isotherms and heat current paths inside the ribbed wall.

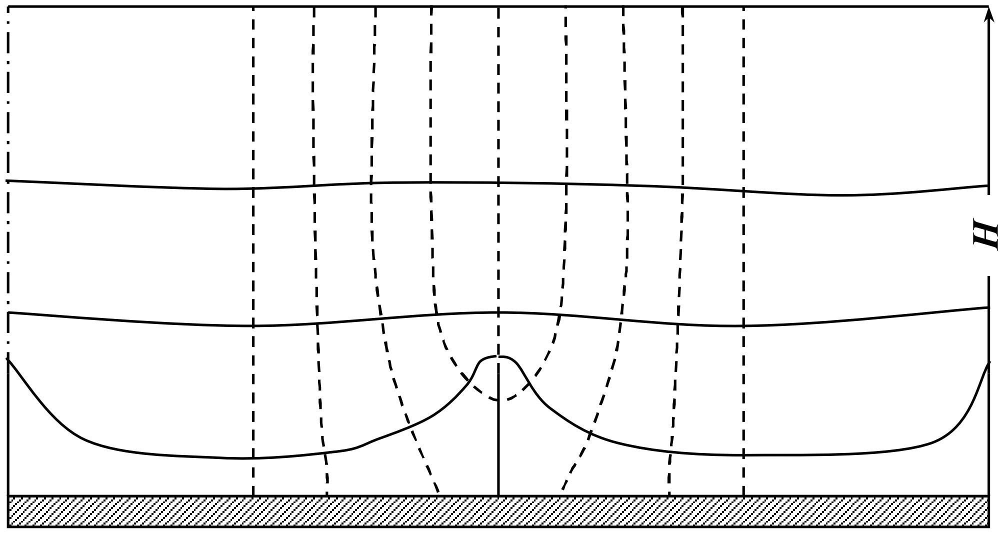

Fig. 3. Isotherm and heat current paths

For the time being, following [7] external thermal resistance can be assumed to be a substitute for the value H with the sum

$$\left(H+\frac{\lambda}{\alpha}\right)=L,$$

where α is the heat emission coefficient in the environment.

Then the heat transfer problem can be dealt with using first order boundary conditions for when the temperatures t_1 and t_2 are kept constant on the surface of a heterogeneous wall with the thickness L.

The problem posed is efficiently solved by means of conformal mappings. The function that maps a ribbed flat wall into a homogeneous (Fig. 4) one has the form [3]

$$W=\frac{a}{\pi}\arccos\left(\frac{1}{\operatorname{ch}\frac{\pi h}{a}}\cdot\cos\frac{\pi Z}{a}\right). \tag{7}$$

Here

$$Z=x+iy, \tag{8}$$

$$W = u(x, y) + iv(x, y)\,, \tag{9}$$

where a is the distance between the ribs.

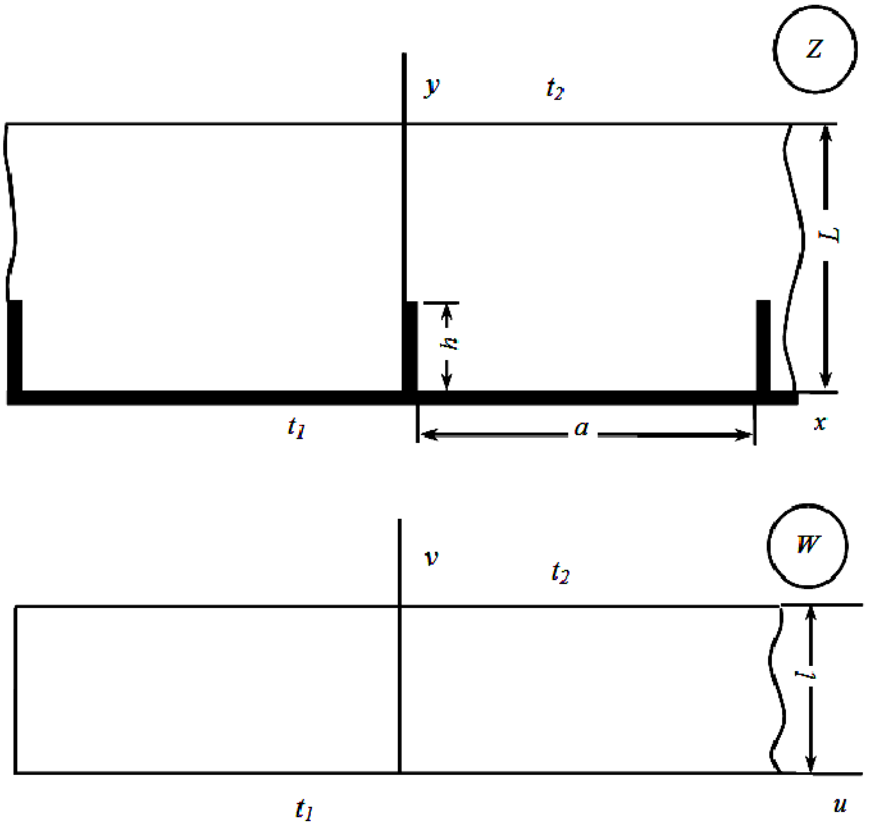

Fig. 4. Conformal mapping of the ribbed wall into a homogeneous one

The transformation (7) maps the isotherm ($y = L$) of the plane Z into the straight line $v = l$ of the plane W.

Using the fundamental properties of conformal mappings we conclude that densities of heat flows passing through the areas between the isotherms t_1 and t_2 on both planes are identical. Then

$$q = \frac{\lambda}{l}(t_1 - t_2)\,. \tag{10}$$

Now substituting (7)—(9) and for the sake of simplicity assuming x equal to zero following the transformations we get

$$l = v_{\substack{x=0 \\ y=L}} = \frac{a}{2\pi} \ln \frac{1}{\operatorname{ch}\dfrac{\pi h}{a}} \left(\operatorname{ch}\frac{\pi}{a}L + \sqrt{\operatorname{ch}^2 \frac{\pi}{a}L - \operatorname{ch}^2 \frac{\pi h}{a}} \right)^2. \tag{11}$$

If we assume $h = 0$ and apply the known formulas

$$\sqrt{\mathrm{ch}^2\frac{\pi}{a}L-1}=\mathrm{sh}\frac{\pi}{a}L,$$

$$\left(\mathrm{ch}\frac{\pi}{a}L+\mathrm{sh}\frac{\pi}{a}L\right)^2=\exp\left(\frac{2\pi}{a}L\right)$$

as a particular case of the equation (11), we have $l = L$.

Let us now compare the densities of heat flows passing through the ribbed and homogeneous walls of the same thickness equals L.

The density of the heat flow for the ribbed wall is determined by the equations (10) and (11), for the homogeneous wall without ribs it is defined by the formula

$$q^*=\frac{\lambda}{L}(t_1-t_2).$$

Then

$$\Delta q=q-q^*=\lambda(t_1-t_2)\left(\frac{L-l}{Ll}\right)=\frac{\lambda(t_1-t_2)}{l}\cdot$$

$$\cdot\left[1-\frac{a}{2\pi L}\ln\frac{1}{\mathrm{ch}\frac{\pi h}{a}}\left(\mathrm{ch}\frac{\pi}{a}L+\sqrt{\mathrm{ch}^2\frac{\pi}{a}L-\mathrm{ch}^2\frac{\pi h}{a}}\right)^2\right] \tag{12}$$

or

$$\delta=\frac{q-q^*}{q}100\ \%. \tag{13}$$

The thermal and engineering integrity of the structure can be determined using the formula

$$r=\frac{q^*}{q}. \tag{14}$$

Therefore the equations (12) and (13) allow one to evaluate heat modes of the ribbed wall compared to the smooth one (depending on the height of the ribs h and distances between them a).

In conclusion let us give a concrete example of that.

The geometrical sizes of the ribbed wall are

$$L = 0.20 \text{ m}; \; h = 0.06 \text{ m}; \; a = 0.07 \text{ m}.$$

The heat conductivity coefficient of the isolation layer is

$$\lambda = 0.07 \text{ Watt/m}^0\text{C};$$

$$t_1 = 20\ ^0\text{C and } t_2 = -25\ ^0\text{C}.$$

Then $\Delta q = 1.98$ Watt/m^2:

$$\delta = \frac{q - q^*}{q} 100 = 11{,}2\ \%, \quad r = 0{,}89\ .$$

Based on the numerical research, we obtained dependencies of the coefficient of thermal and engineering integrity *r* on the height of the ribs *h* and distances between them *a* (Fig. 5).

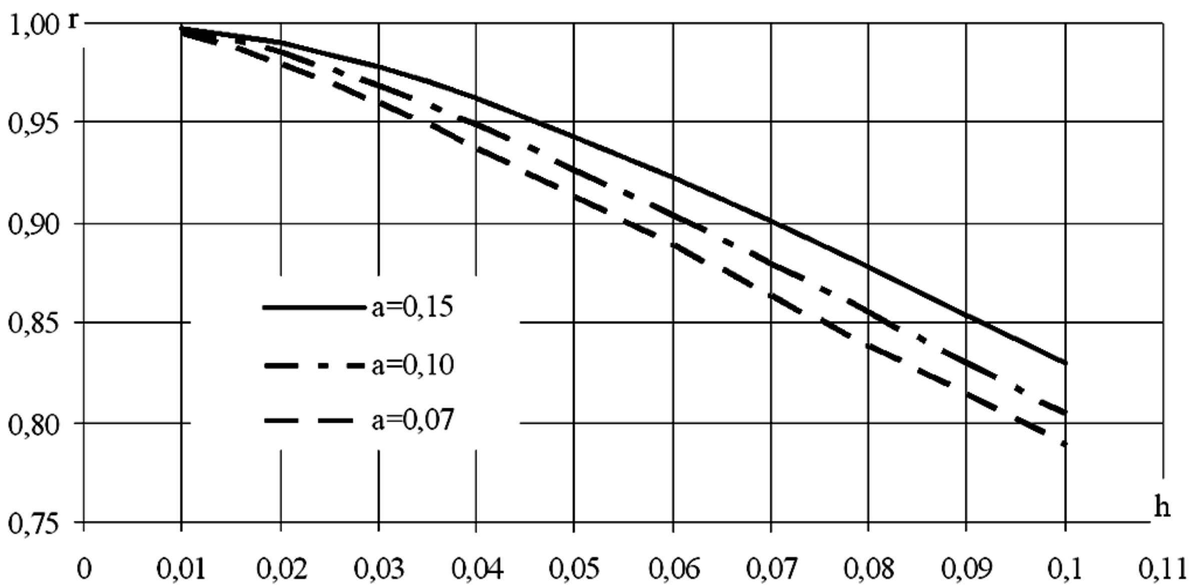

Fig. 5. Dependency of the coefficient of thermal and engineering integral *r* on the height of the rib *h*

Conclusions

1. We have sought to present a simple and efficient method of modeling stationary heat transfer processes within a heat pipe partially embedded into the body and within a

non-homogeneous structure with metal ribs. We have designed the equations that describe temperature fields in the body surrounding a heat pipe partially embedded into the ground.

2. The use of the conformal mapping method allowed research into the effect metal ribs within heat insulation products have on the magnitude and nature of change in temperature and heat flows.
3. The resulting dependencies enabled us to calculate the critical characteristic of the product. This is the coefficient of thermal and engineering integrity of the structure. The graph was presented that exhibits the dependency of the coefficient of thermal and engineering integrity on the height of the ribs and distances between them.
4. The heat transfer problems discussed describe the strengths of the conformal mapping method. The solution of other problems associated with transfer in other boundaries can be found in the identical fashion since the methods for designing solutions of this type are not fundamentally different.

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