DESIGNING AND CONSTRUCTION OF ROADS, SUBWAYS, AIRFIELDS, BRIDGES AND TRANSPORT TUNNELS

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MODELING OF CONTACT STRESSES IN VIBRATION PLATES AND HOT ASPHALT MIX FOR ROAD SURFACING REPAIRS

Statement of the problem. The quality of works on repair of road surfacings of a flexible type depends on the properties of the used material and parameters of the machines for sealing. Operational requirements are achieved by a combination of stresses in the zone of contact of a working body of the machine and strength characteristics of the material. Justification of effective choice of parameters of the slab with consideration of the strength characteristics of the material is impossible without the analytical dependence of influence of constructive and force parameters of vibrating slabs on the magnitude of stresses.

Results. Obtained the analytical dependence of the influence of constructive and force parameters of vibrating slabs on the magnitude of stresses in the zone of contact of the slab with a working body.

Conclusions. The obtained dependence of the influence of constructive and force parameters of vibrating slabs on the amount of contact stresses allows one to scientifically substantiate the choice of the parameters of vibrating slab with account of the strength characteristics of sealing materials and provide a higher quality of road surfacing repairs.

Keywords: vibrating plate; contact stresses; balance equation; road surfacing repairs.

Introduction. Higher traffic intensity accompanied by a higher load carrying capacity of vehicles results in larger loads on road surfacings and affects changes in transport and operating characteristics and an increasingly deteriorating condition of highways. The cause of worse transport and operating characteristics of highways can be many other things that depend on the quality of a material being applied and commitment to road surfacing requirements.

There can be plastic deformations and failures such as potholes, subsidence, cracks and wavy bumps. This results in lower speeds, higher dynamic transmission loads and compromises traffic safety. The analysis of road surfacing condition showed that there is a need for routine maintenance works of 2—3 % of the total area of a highway after a short period of time [1]. Road potholes compromise traffic safety the most. A material used to address these defects should be identical and in accordance to the requirements for density, strength, water-tightness, evenness and roughness of the major part of surfacing. Therefore potholes are repaired if the surface in small parts of a surfacing fails to comply with the above requirements. There are guidelines set out for the size of potholes and periods for addressing these defects on road surfacing [2].

The technologies used for maintaining and repairing road surfacings vary. A choice of a certain technology is determined by construction parameters of the roads, surfacing material, weather conditions, amount of works to be carried out, types of defects and tools available [3].

Hot asphalt mixes are most common in pothole repairs of non-rigid surfaces. The quality of works depends on weather conditions, properties of a material and commitment to temperature modes as well as a well-informed choice of the parameters of compaction machines.

The analysis of the use of vibration plates for compaction of asphalt concrete mixes in pothole repairs of road surfaces suggests that they are chosen with no regard to the effect of construction and strength parameters of the plate on its compaction capacity which is dependent on contact stresses between the plate and material.

Compaction machines are known to run smoothly when contact stresses over the operating body of the machine agree with the strength parameters of a compaction material. The effect of the parameters of vibration plates on compaction is investigated by V.N. Vladimorov and N.Ya. Kharkhut [4].

The obtained dependencies for calculating contact stresses were based on the experimental material during the compaction of subbases and allow the calculation of compression stresses only. They do not help identify shear stresses and take no regard of how construction parameters of vibration plates and rigidity of the lower base layer. No connection was revealed between geometric parameters of plates and contact stresses.

The fact that there are no analytical dependencies to calculate contact stresses under the vibration plate is not helpful in the right choice of their parameters considering construction and strength characteristics of a material being compacted.

1. Statement of the problem. In order to establish the dependence of the effect of construction and strength parameters of vibration plates on contact stresses, the interaction of the vibration plate and material being compacted was modelled (Fig. 1).

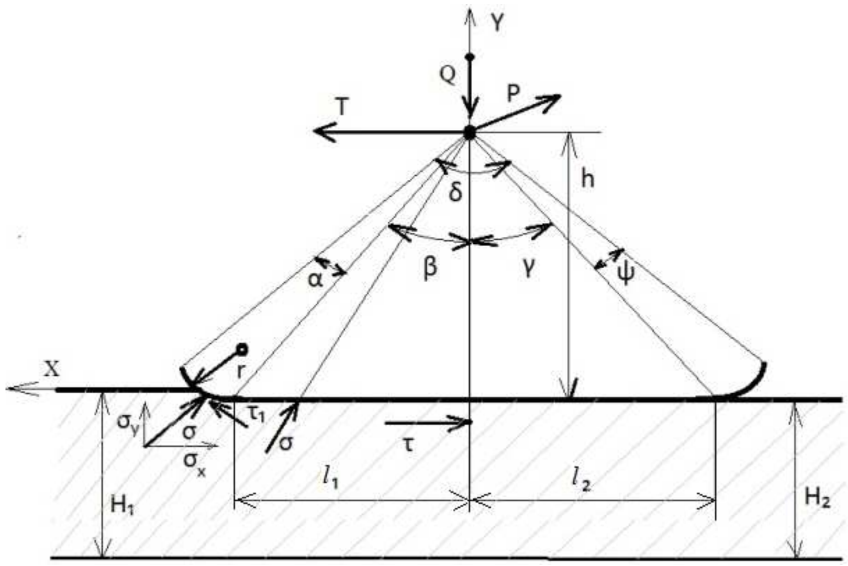

Fig. 1. Schematic of the interaction of the vibration plate and the material being compacted

The calculation scheme uses the following designations: Q is the mass of a vibration plate; B is the width of the bottom of the vibration plate; T is pushing effort; δ is a contact angle between the plate bottom and the material; σ are normal stresses; τ are shear stresses; φ is the angle of this point at the contact area of the roller; X and Y are coordinate axis; P is constraining force; ω is the oscillation frequency; t is the time; α, β, γ, ψ are angles describing the geometric parameters of the vibration plate; φ is the variable of the angle describing stresses at the plate-material interaction area; H_1 is the thickness of the material being compacted prior to compaction; H_2 is the thickness of the material following the compaction; h is the distance from the bottom of the plate to the application of the pushing force; l_1 and l_2 is the distance from the center of gravity of the vibration plate to a curve in the bottom of the plate.

The following assumptions were made in the modeling of the interaction of the vibration plate and the material being compacted:

– the center of gravity of the vibration plate and the point of the application of the constraining force of the vibroexciter are at the same point;
– contact stresses over the plate are towards the point of the application of the exterior forces;
– contact stresses over the plate are even which allows the calculation of stresses to be identical to those in flat tasks.

Generally, the stress condition of the material particle in the plate-material area is given by

$$\overline{\sigma} = \overline{\sigma_x} + \overline{\tau_y} \text{ , MPa,}$$

where σ_x, τ_y are normal and shear stresses under the effect of the exterior forces along X and Y.

2. Designing an analytical model based on balance equations. Based on a balance equation for the stress state of a particle in the plate-material area considering the initial and boundary conditions, the way the plate and material interact is given by a system of equations. Let us accept that there is a dependence between normal and tangential stresses

$$\tau = \mu\sigma + C \text{ , MPa,}$$

where μ is the interior friction coefficient; C is the adhesion coefficient. Depending on the location of the gravitational centre in relation to the bottom of the plate and its symmetry axis the numerical value of the angles β and γ is determined from the expressions

$$\beta = arctg\left(\frac{k \cdot l}{h}\right); \quad \gamma = arctg\left(\frac{(1-k) \cdot l}{h}\right),$$

where k is the coefficient that depends on the location from the gravitational centre of the vibration plate the numerical value of which ranges from 0 to 1; l is the total length of the bottom of the vibration plate. Let us accept that $l_1 = l_2 = r$.

Based on the condition of the stressed state balance, the interface of the plate and the material can be given by the following system of equations considering the initial and boundary conditions:

$$\Sigma X = 0;\quad \int_0^{\alpha} \sigma Br \cdot \sin(\phi-\alpha)d\phi + \int_0^{\alpha} \tau Br \cdot \sin(\phi-\alpha)d\phi + \int_{\alpha}^{\beta} \sigma Bl_1 \cdot \sin(\phi-\beta)d\phi +$$
$$+\int_{\beta}^{\gamma} \sigma Bl_2 \cdot \sin(\gamma-\phi)d\phi + \int_{\gamma}^{\psi} \sigma Br \cdot \sin(\phi-\psi)d\phi + \int_{\gamma}^{\psi} \sigma Br \cdot \cos(\phi-\psi)d\phi -$$
$$-\int_{\alpha}^{\beta} \tau Bl_1 d\phi - \int_{\beta}^{\gamma} \tau Bl_2 d\phi = P \cdot \cos(\omega \cdot t) - T;$$

$$\Sigma Y = 0;\quad \int_0^{\alpha} \sigma Br \cdot \cos(\phi-\alpha)d\phi + \int_0^{\alpha} \tau Br \cdot \sin(\phi-\alpha)d\phi + \int_{\alpha}^{\beta} \sigma Bl_1 \cdot \cos(\phi-\beta)d\phi +$$
$$+\int_{\beta}^{\gamma} \sigma Bl_2 \cdot \cos(\gamma-\phi)d\phi + \int_{\gamma}^{\psi} \sigma Br \cdot \cos(\psi-\phi)d\phi + \int_{\gamma}^{\psi} \tau Br \cdot \sin(\psi-\phi)d\phi =$$
$$= Q - P \cdot \sin(\omega \cdot t).$$

The initial conditions look like

$$\sigma_x = \tau_y = 0 \text{ if } \phi = 0 \text{ or } \phi = \delta .$$

The boundary conditions are

$$\tau_{X2} - \sigma_{X1} = \tau_{X3} - \sigma_{X3};\ \tau_{Y2} + \sigma_{Y1} = \sigma_{Y3} \text{ if } \phi = 0 ;$$

$$d\sigma_Y / dt = 0; \quad \tau_{X3} = \tau_{X5} \text{ if } \phi = \alpha + \beta ;$$

$$\tau_{X5} + \sigma_{X4} = \tau_{X5} + \sigma_{X6} - \tau_{X6}; \quad \sigma_{Y4} = \tau_{Y6} + \sigma_{Y5} \text{ if } \phi = \alpha + \beta + \gamma .$$

The following function is an approximation function of the stress function

$$\sigma(\phi) = a_0 + a_1\phi + a_2\phi^2 + a_3\phi^3, \text{ MPa.}$$

Considering the initial conditions, we have the coefficient a_0: $a_0 = 0$.

Let us transform the system of equations. In order to do so, let us insert the approximation function under the integrals and divide the projections of the exterior forces into the contact surface of the plate and introduce the designations for the numerical values of the integrals:

1) $\phi \in [0;\alpha]$:

$$K_1 = \int_0^{\alpha} \phi \cdot \sin(\phi-\alpha) d\phi = \sin\alpha - \alpha,$$

$$U_1 = \int_0^{\alpha} \phi \cdot \cos(\phi-\alpha) d\phi = 1 - \cos(\alpha),$$

$$M_1 = \int_0^{\alpha} \phi^2 \cdot \sin(\phi-\alpha) d\phi = -\alpha^2 + 2(1-\cos(\alpha)),$$

$$V_1 = \int_0^{\alpha} \phi^2 \cdot \cos(\phi-\alpha) d\phi = 2(\alpha - \sin(\alpha)),$$

$$N_1 = \int_0^{\alpha} \phi^3 \cdot \sin(\phi-\alpha) d\phi = 6(\alpha - \sin(\alpha)) - \alpha^3,$$

$$W_1 = \int_0^{\alpha} \phi^3 \cdot \cos(\phi-\alpha) d\phi = 3\alpha^2 + 6(\cos(\alpha) - 1);$$

2) $\phi \in [\alpha;\beta]$:

$$K_2 = \int_{\alpha}^{\beta} \phi \cdot \sin(\phi-\beta) d\phi = \alpha\cos(\alpha-\beta) - \beta + \sin(\beta-\alpha),$$

$$M_2 = \int_{\alpha}^{\beta} \phi^2 \cdot \sin(\phi-\beta) d\phi = 2 - \beta^2 + (\alpha^2 - 2)\cos(\alpha-\beta) + 2\alpha\sin(\beta-\alpha),$$

$$N_2 = \int_{\alpha}^{\beta} \phi^3 \cdot \sin(\phi-\beta) d\phi = 6\beta - \beta^3 + (\alpha^3 - 6\alpha)\cos(\alpha-\beta) + (3\alpha^2 - 6)\sin(\beta-\alpha),$$

$$U_2 = \int_{\alpha}^{\beta} \phi \cdot \cos(\phi-\beta) d\phi = 1 - \cos(\alpha-\beta) + \alpha\sin(\beta-\alpha),$$

$$V_2 = \int_{\alpha}^{\beta} \phi^2 \cdot \cos(\phi-\beta) d\phi = 2\beta + (\alpha^2 - 2)\sin(\beta-\alpha) - 2\alpha\cos(\beta-\alpha),$$

$$W_2 = \int_{\alpha}^{\beta} \phi^3 \cdot \cos(\phi-\beta) d\phi = 3\beta^3 - 6 + (\alpha^3 - 6\alpha)\sin(\beta-\alpha) + (6 - 3\alpha^2)\cos(\beta-\alpha);$$

3) $\phi \in [\beta; \gamma]$:

$$K_3 = \int_{\beta}^{\gamma} \phi \cdot \sin(\phi - \gamma) d\phi = \beta \cos(\gamma - \beta) - \gamma + \sin(\gamma - \beta),$$

$$M_3 = \int_{\beta}^{\gamma} \phi^2 \cdot \sin(\phi - \gamma) d\phi = 2 - \gamma^2 + (\beta^2 - 2)\cos(\gamma - \beta) + 2\beta \sin(\gamma - \beta),$$

$$N_3 = \int_{\beta}^{\gamma} \phi^3 \cdot \sin(\phi - \gamma) d\phi = 6\gamma - \gamma^3 + (\beta^3 - 6\beta)\cos(\gamma - \beta) + (3\beta^2 - 6)\sin(\gamma - \beta),$$

$$U_3 = \int_{\beta}^{\gamma} \phi \cdot \cos(\phi - \gamma) d\phi = 1 - \cos(\beta - \gamma) + \beta \sin(\gamma - \beta),$$

$$V_3 = \int_{\beta}^{\gamma} \phi^2 \cdot \cos(\phi - \gamma) d\phi = 2\gamma + (\beta^2 - 2)\sin(\gamma - \beta) - 2\beta \cos(\gamma - \beta),$$

$$W_3 = \int_{\beta}^{\gamma} \phi^3 \cdot \cos(\phi - \gamma) d\phi = 3\gamma^3 - 6 + (\beta^3 - 6\beta)\sin(\gamma - \beta) + (6 - 3\beta^2)\cos(\gamma - \beta);$$

4) $\phi \in [\alpha; \beta]$:

$$K_4 = \int_{\gamma}^{\psi} \phi \cdot \sin(\phi - \psi) d\phi = \gamma \cos(\psi - \gamma) - \psi + \sin(\psi - \gamma),$$

$$M_4 = \int_{\gamma}^{\psi} \phi^2 \cdot \sin(\phi - \psi) d\phi = 2 - \psi^2 + (\gamma^2 - 2)\cos(\psi - \gamma) + 2\gamma \sin(\psi - \gamma),$$

$$N_4 = \int_{\gamma}^{\psi} \phi^3 \cdot \sin(\phi - \psi) d\phi = 6\psi - \psi^3 + (\gamma^3 - 6\beta)\cos(\psi - \gamma) + (3\gamma^2 - 6)\sin(\psi - \gamma),$$

$$U_4 = \int_{\beta}^{\psi} \phi \cdot \cos(\phi - \psi) d\phi = 1 - \cos(\gamma - \psi) + \gamma \sin(\psi - \gamma),$$

$$V_4 = \int_{\gamma}^{\psi} \phi^2 \cdot \cos(\phi - \psi) d\phi = 2\psi + (\gamma^2 - 2)\sin(\psi - \gamma) - 2\gamma \cos(\psi - \gamma),$$

$$W_4 = \int_{\gamma}^{\psi} \phi^3 \cdot \cos(\phi - \psi) d\phi = 3\psi^3 - 6 + (\gamma^3 - 6\gamma)\sin(\psi - \gamma) + (6 - 3\gamma^2)\cos(\psi - \gamma);$$

5)

$$K_5 = \int_{\alpha}^{\beta} \phi d\phi = \left(\beta^2 - \alpha^2\right)/2;$$

$$K_6 = \int_{\alpha}^{\beta} \phi^2 d\phi = \left(\beta^3 - \alpha^3\right)/3;$$

$$K_7 = \int_{\alpha}^{\beta} \phi^3 d\phi = \left(\beta^4 - \alpha^4\right)/4;$$

$$U_5 = \int_{\beta}^{\gamma} \phi d\phi = \left(\gamma^2 - \beta^2\right)/2;$$

$$U_6 = \int_{\beta}^{\gamma} \phi^2 d\phi = \left(\gamma^3 - \beta^3\right)/3;$$

$$U_7 = \int_{\beta}^{\gamma} \phi^3 d\phi = \left(\gamma^4 - \beta^4\right)/4.$$

Considering the accepted designations for the numerical values of the integrals and following their grouping in the coefficients a_1, a_2 and a_3 the system of equations is as follows

$$\begin{aligned}
\Sigma X = 0; \quad & a_1\left[K_1 + K_2 + K_3 + K_4 + K_5 + U_5 + \mu\left(U_1 + U_4\right)\right] + \\
& + a_2\left[M_1 + M_2 + M_3 + M_4 + K_6 + U_6 - \mu\left(V_1 + V_4\right)\right] + \\
& + a_3\left[N_1 + N_2 + N_3 + N_4 + K_3 + U_7 + \mu\left(W_1 + W_4\right)\right] = F_1; \\
\Sigma Y = 0; \quad & a_1\left[U_1 + U_2 + U_3 + U_4 + \mu\left(K_1 - K_4\right)\right] + \\
& + a_2\left[V_1 + V_2 + V_3 + V_4 + \mu\left(M_1 - M_4\right)\right] + \\
& + a_3\left[W_1 + W_2 + W_3 + W_4 + \mu\left(N_1 - N_4\right)\right] = R,
\end{aligned}$$

where $F = P \cdot \cos(\omega t) - T/\left(Bl\right);\ R = Q - P \cdot \sin(\omega t)/\left(Bl\right);\ l = l_1 + l_2.$

We will designate

$$\begin{aligned}
L_1 &= K_1 + K_2 + K_3 + K_4 + K_5 + U_5 + \mu\left(U_1 + U_4\right); \\
L_4 &= U_1 + U_2 + U_3 + U_4 + \mu\left(K_1 - K_4\right); \\
L_2 &= M_1 + M_2 + M_3 + M_4 + K_6 + U_6 - \mu\left(V_1 + V_4\right);
\end{aligned}$$

$$L_5 = V_1 + V_2 + V_3 + V_4 + \mu\left(M_1 - M_4\right);$$
$$L_3 = N_1 + N_2 + N_3 + N_4 + K_3 + U_7 + \mu\left(W_1 + W_4\right);$$
$$L_6 = W_1 + W_2 + W_3 + W_4 + \mu\left(N_1 - N_4\right).$$

Considering the accepted designations and initial conditions, finding the stress function σ(φ), i. e. finding the coefficients a_1, a_2 and a_3 can be reduced to solving the linear algebraic system of three equations:

$$\begin{cases} L_1 a_1 + L_2 a_2 + L_3 a_3 = F, \\ L_4 a_1 + L_5 a_2 + L_6 a_3 = R, \\ \delta a_1 + \delta^2 a_2 + \delta^3 a_3 = 0. \end{cases}$$

The above system is solved using Kramer's method based on calculating the determinants of the matrix Δ, Δa_1, Δa_2, Δa_3:

$$\Delta = \delta\left(L_2 L_6 - L_3 L_5\right) - \delta^2\left(L_1 L_6 - L_3 L_4\right) + \delta^3\left(L_1 L_5 - L_2 L_4\right);$$
$$\Delta a_2 = \delta\left(F L_6 - L_3 R\right) + \delta^3\left(L_1 R - F L_4\right);$$
$$\Delta a_1 = -\delta^2\left(F L_6 - L_3 R\right) + \delta^3\left(F L_5 - L_2 R\right);$$
$$\Delta a_3 = \delta\left(L_2 R - F L_5\right) - \delta^2\left(L_1 R - F L_4\right).$$

The numerical solutions Δ, Δa_1, Δa_2, Δa_3 can be given in the following way:

$$\Delta = \beta\left(L_2 L_6 - L_3 L_5\right) - \beta^2\left(L_1 L_6 - L_3 L_4\right) + \beta^3\left(L_1 L_5 - L_2 L_4\right);$$
$$\Delta a_2 = \beta\left(F L_6 - L_3 R\right) + \beta^3\left(L_1 R - F L_4\right);$$
$$\Delta a_1 = -\beta^2\left(F L_6 - L_3 R\right) + \beta^3\left(F L_5 - L_2 R\right);$$
$$\Delta a_3 = \beta\left(L_2 R - F L_5\right) - \beta^2\left(L_1 R - F L_4\right).$$

Therefore, the values of the coefficients of the approximation function are

$$a_1 = {\Delta a_1}/{\Delta} = \frac{-\beta^2\left(F L_6 - L_3 R\right) + \beta^3\left(F L_5 - L_2 R\right)}{\beta\left(L_2 L_6 - L_3 L_5\right) - \beta^2\left(L_1 L_6 - L_3 L_4\right) + \beta^3\left(L_1 L_5 - L_2 L_4\right)};$$

$$a_2 = {\Delta a_2}/{\Delta} = \frac{\beta\left(F L_6 - L_3 R\right) + \beta^3\left(L_1 R - F L_4\right)}{\beta\left(L_2 L_6 - L_3 L_5\right) - \beta^2\left(L_1 L_6 - L_3 L_4\right) + \beta^3\left(L_1 L_5 - L_2 L_4\right)};$$

$$a_3 = \Delta a_3 / \Delta = \frac{\beta\left(L_2R - FL_5\right) - \beta^2\left(L_1R - FL_4\right)}{\beta\left(L_2L_6 - L_3L_5\right) - \beta^2\left(L_1L_6 - L_3L_4\right) + \beta^3\left(L_1L_5 - L_2L_4\right)}.$$

Generally, the stressed state of the particle of the material at the interface of the plate and material is given by

$$\sigma = \phi\frac{-\delta^2\left(FL_6 - L_3R\right) + \delta^3\left(FL_5 - L_2R\right)}{\Delta} + \phi^2\frac{\delta\left(FL_6 - L_3R\right) + \delta^3\left(L_1R - FL_4\right)}{\Delta} + $$
$$+\phi^3\frac{\delta\left(L_2R - FL_5\right) - \delta^2\left(L_1R - FL_4\right)}{\Delta}.$$

The above dependence suggests that the stressed state of the particle of the material under the bottom of the vibration plate depends on the effect of the force factors *R* and *F* and the location of the particle described by geometric angles.

Let us transform the obtained expression considering the effect of the force factors *R* and *F*. Finally, the dependence for the calculation of compressive and shear force under the vibration plate is

$$\sigma_x = \frac{\phi\delta^2 R\left(L_3 - L_2\beta\right)}{\Delta} + \frac{\phi^2\delta R\left(L_1\beta^2 - L_3\right)}{\Delta} + \frac{\phi^3\delta R\left(L_2 - L_1\beta\right)}{\Delta};$$

$$\tau_y = \frac{\phi\delta^2 F\left(L_5\beta - L_6\right)}{\Delta} + \frac{\phi^2\delta F\left(L_6 - L_4\beta\right)}{\Delta} + \frac{\phi^3\delta F\left(L_4\beta - L_5\right)}{\Delta}.$$

The analysis of the equations shows that at φ =0 and φ =δ the components of the total stress are zero. The obtained dependencies are in agreement with the particular interfaces of the plate and the material being reinforced.

Therefore, the obtained dependencies allow one to identify the stresses (compressive and shear ones) in any point of the interface of the plate and the material depending on technical characteristics of the vibration plate.

Conclusions

1. The dependence was identified for the first time of the effect of construction and force parameters of vibration plates on the distribution and values of stresses in the interface of the plate and the material being reinforced.

2. The obtained dependence allows a scientific substantiation of the parameters of vibration plates considering their construction parameters and strength characteristics of the material being reinforced.
3. The use of vibration plates employing the hot method provides for a better quality of road repair works.

References

1. **Dorokhin, S.** Jamochnyjj remont dorogi. Tradicionnye i novye metody / S. Dorokhin. — /http://www. os1. ru/article/road_equipment/2006_05_A_2006_09_29-18_11_55. — (01.11.2012).
2. **GOST R 50597-93.** Avtomobil'nye dorogi i ulicy. Trebovanija k ehkspluatacionnomu sostojaniju, dopustimomu po uslovijam obespechenija bezopasnosti dorozhnogo dvizhenija. — M.: Izd-vo standartov, 1994. — 8 s.
3. **Zubkov, A. F.** Analiz primenenija vibracionnykh plit pri remonte dorozhnykh pokrytijj nezhestkogo tipa / A. F. Zubkov, E. Ju. Evseev, R. V. Kuprijanov // Mekhanizacija stroitel'stva. — 2011. — № 6. — S. 28—31.
4. **Kharkhuta, N. Ja.** Mashiny dlja uplotnenija gruntov / N. Ja. Kharkhuta. — L.: Mashino-stroenie, 1973. — 173 s.