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DETERMINATION OF VELOCITY FIELDS OF AIR STREAMS IN VENTILATED ROOMS WITH CONFORMAL MAPPINGS

Statement of the problem. To improve the accuracy of the aerodynamic calculation of ventilation and air conditioning systems in different rooms methods based on modern computing and information technology are becoming more common now. The most commonly used technique is related to the solution of systems of partial differential equations. In this paper we propose an analytical method for determining the geometric characteristics of the air flow in the ventilation of rooms of fairly complex form, based on the method of conformal mappings, and virtually free of natural errors of computer calculations.

Results and conclusions. The article shows the possibility of solving the problem of ventilation of rooms of rather complex shapes based on the method of conformal mappings. The resulting solution is analytical and therefore in some cases it may be more accurate than the solution of a system of differential equations. With the help of the software package MAPLE the scheme of movement of considered streams is constructed. The method of constructing streamlines in a room with partitions allows to consider geometrical characteristics of the room rather accurately at the organization of displacement ventilation.

Keywords: velocity field, streamlines, ventilation, conformal mapping, ventilation industrial and public-owned buildings.

Introduction

Currently, displacement ventilation receives prevalent in the organization of air exchange in the areas of public and industrial use. It is proved that displacing ventilation has a number of advantages compared with the mixing. Typically, it is used in the following areas: meeting rooms, dining rooms, classrooms, conference rooms, and so on. The main principle of this method of air distribution is the uniform supply and venting at low speeds. The effectiveness of such schemes of ventilation increases with increasing the degree of uniformity of distribution of air flow.

The main problem in the design of displacement ventilation is a complex configuration of rooms and the presence of partitions between them. In this case a precise calculation of the streamlines of air flows, their direction and speed is required.

To solve this problem, as a rule, numerical methods of modeling of the processes of gas dynamics [1-4], based on the solution of equations are used:

— The continuity equation:

$$\frac{\partial}{\partial x_i}(\rho u_i) = 0, \tag{1}$$

where ρ — density of air, kg/m^3; x_i — spatial coordinate, m; u_i — i-component of the velocity of air, m/s.

— Equation of Navier — Stokes, averaged according to Reynolds [3]:

$$\frac{\partial(\rho u_i u_j)}{\partial x_i} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_i}\left(\Gamma^u_{eff}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial u_i}\right)\right) - \frac{2}{3}\frac{\partial}{\partial x_i}\left(\Gamma^u_{eff}\frac{\partial u_j}{\partial x_j}\right) - \frac{2}{3}\frac{\partial}{\partial x_i}(\rho k) - \delta_{i3}\rho g, \tag{2}$$

where p — pressure, Pa; Γ^u_{eff} — diffusion coefficient for the variable u, kg/(m·s); k — kinetic energy of turbulence, m^2/s^2; g — acceleration of gravity, m/s^2.

The results obtained in [3] Examples of numerical simulation of ventilation (when changes are made to the geometric characteristics of the premises and some fixed values of the parameters in equation (1), (2)) are shown in Fig. 1.

In the situation under discussion air is supplied uniformly to one side of the room and removed from the opposite side at a speed of 0.3 m/s. The height of all the premises is accepted equal 5 m, the height of partitions in the first case is 2 m, in the second — 3 m in the third — 4 m. The distance between the partitions is 5 m. Results of the study [3] shows that with increasing the height of partitions, the speed of movement of air flow in the upper part of rooms increases. At the same time, the height of the partitions practically does not affect the movement of air in the fenced-off cells of ventilated rooms. From this the conclusion about possibility of using for rooms with partitions the systems of ventilation designed for rooms without partitions is drawn.

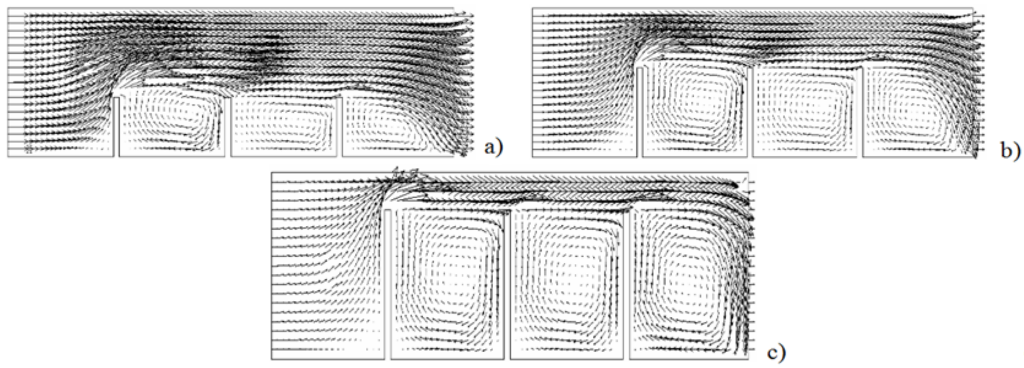

Fig. 1. The direction of air flow in a room with partitions of different height

The validity of this conclusion and the absolute figure 1 are not indisputable. It is clear that the values of the parameters in equation (1) and (2) affect the process of ventilation. The whole pattern of the current lines, in general, can vary, depending on the speed of delivery and removal of air, on its density, and other characteristics of the ventilation process. Besides, the research of equations (1) and (2) assumes considerable volumes of computer calculations. Errors of such calculations may also affect the accuracy of a final picture of the current lines and on its qualitative nature as a whole.

In this regard, it is advisable to use different approaches to the problem, as close as possible to the analytical methods [5]. Below a calculation of the flow lines at the ventilation of similar premises with partitions based on the theory of conformal mappings is offered.

The authors do not exclude that in some cases, such calculations can be more accurate than the results of work [3].

One of the arguments here is the argument that at low speeds of the incoming air even in the presence of partitions in ventilated rooms all parts of the premises are involved in the ventilation process (in the model given below such situation takes place).

We note at the same time that on Fig. 1 «stagnant» zones between the partitions almost not affected by ventilation are clearly visible. Apparently, clearly defining the difference between the two models of ventilation should make natural experiments.

1. The construction of flow lines using conformal mappings

The general principle of application of conformal mapping in relation to the problem of ventilation was introduced in article [5].The description of display is based on the properties of the elliptic integrals of the first kind [5—7]:

$$F(z,\alpha)=\int_0^z \frac{dt}{\sqrt{(1-t^2)(1-\alpha^2 t^2)}}(0<\alpha<1) \tag{3}$$

and the return function to it called an elliptic sine. Considering these functions as an intermediate area the top half-plane, in which the function is set, is used (3). As mentioned in work [5] the symmetrical room with a partition located in the center was considered.

However, in practice such arrangement of the premises is extremely rare, so the solution of the problem with several partitions evokes great interest. As an example, we will consider a room with three partitions (Fig. 2).

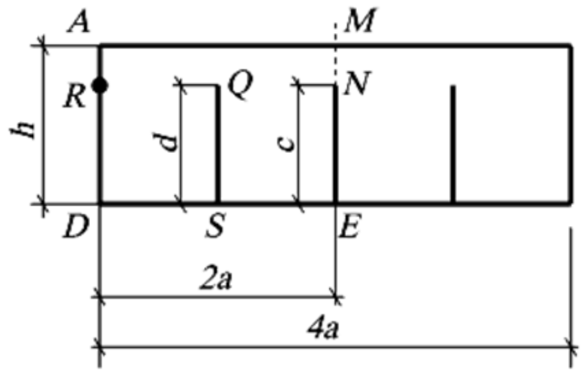

Fig. 2. Considered room with three partitions

For the construction of the flow lines in a room with three partitions it is necessary to construct a conformal mapping F of the initial rectangle to some other rectangle so that the upper horizontal and lower broken line of the figure $ABCD$ passed, respectively, in the upper and lower horizontal part of a new rectangle (Fig. 3).

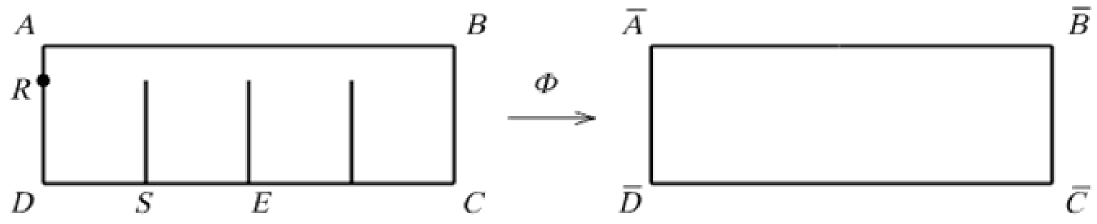

Fig. 3. Conformal mapping F of the initial figure

As it was stated in work [5], the main assumption for the application of conformal maps in the problem discussed is the condition of incompressibility of moving medium and its laminar movement.

Taking into account known in the theory of functions of a complex variable of the principle of symmetry it is enough to construct conformal display of a half of an initial rectangle with *MN* «window» (we will call it Π_1) on any rectangle of the unknown sizes Π_3 with «floor-to-ceiling window» (fig. 4).

We will denote such a map by Φ_1 and call it a solution of problem number 1. In the presence of display Φ_1 display Φ demanded in an initial task coincides with Φ_1 on the left half of then initial figure with three partitions, and on the right half display Φ_1 proceeds symmetrically.

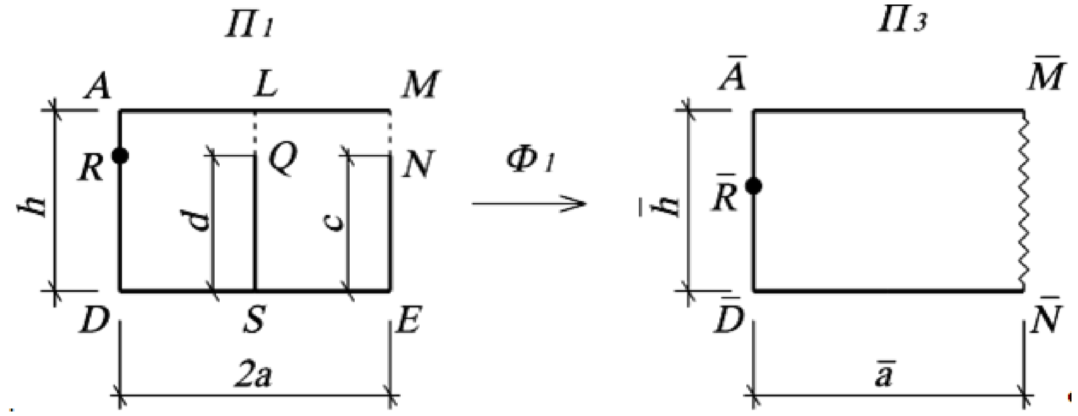

Fig. 4. Conformal mapping Φ_1 of (left) half of the initial rectangle: *h* — height, *2a* — width, *d* — height of the central partition, c — height of the second partition (for the case *c*=*d*)

In its turn the creation of display Φ_1 can be reduced to simpler display using the same principle of symmetry. In this case, to solve the problem number 1 we should consider an auxiliary task 2, which consists in the construction of conformal mapping of rectangle Π_2 with a «window» LQ (h — height of the rectangle, a — width, d — height of the «sill») at any Π_3 rectangle with a «floor-to-ceiling window» $\overline{L}\overline{Q}$ (Fig. 5).

We will construct such a map below, denoting it by $\Phi_2 = \Phi_2^{(h,a,d)}$, and the sides of a rectangle received at such display Π_3 by $\overline{h}, \overline{a}$.

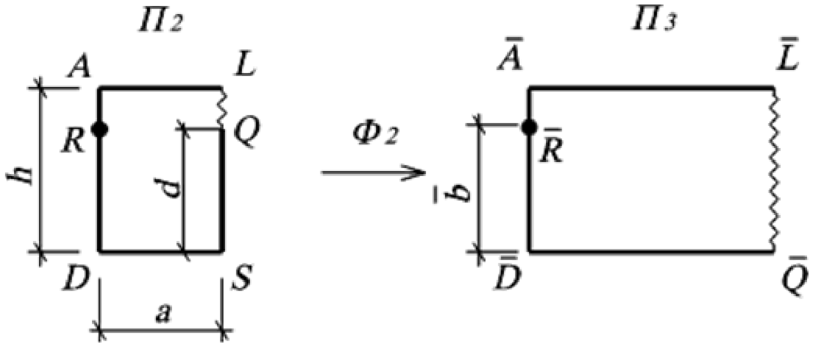

Fig. 5. Conformal mapping Φ_2 of rectangle Π_2

In figure Π_1 we are still interested in the point R, being at height «c» on the left vertical side of Π_1 (this point is symmetric to a point N about the central partition of Π_1). Since the angular points of the left vertical side of the figure Π_2 remain angular in the construction of the mapping Φ_2 the point R which interests us will pass to a point R on the left side of the rectangle Π_3. We denote the height of the point with $\overline{b}$.

Now, using the principle of symmetry, it can be argued that there is a conformal mapping of the interior part of Π_1 on some new figure, the latter contains a rectangle Π_3, «floor-to-ceiling window» of the rectangle, and the figure, which is symmetric Π_3 concerning this «window». Constructed map is denoted with $\hat{\Phi}_2$ keeping in mind that it is a symmetric extension of the mapping Φ_2. Mapping $\hat{\Phi}_2$ continuously proceeds to the boundary of Π_1, besides couples of points of a figure П1symmetric concerning to the cut pass to couples of points symmetric concerning «floor-to-ceiling window».

From the above features follows that the image of the top «window sill» of the initial figure Π_1,which was located initially at height c, under the influence of map $\hat{\Phi}_2$ will pass to a point on the right side of the double rectangle $\hat{\Phi}_2$ (Π_1),being at height $\bar{b}$. Thus, $\hat{\Phi}_2$ conformally displays an initial figure Π_1 on a rectangle Π_2 with «a new window sill» with height $\bar{b}$.

The next step is the examination of the pass-through (complex) display represented in the form of:

$$\Phi_2^{(\bar{h},2\bar{a},\bar{b})} \circ \hat{\Phi}_2^{(h,a,d)}, \tag{4}$$

meaning as the second applied of its components the solution Φ_2 of the second task for the figure Π_2.

It clear, that such mapping is also conformal and solves the initial problem 1. Hence, it can be considered as the desired map Φ_1. Recall that in our task the interest is the construction of the flow lines, which are the images of the horizontal lines under the influence of the reverse mapping

$$\Phi_1^{-1} = \left(\hat{\Phi}_2^{(h,a,d)}\right)^{-1} \circ \left(\Phi_2^{(\bar{h},2\bar{a},\bar{b})}\right)^{-1}, \tag{5}$$

2. Determination of the size of a figure in the pass-through display

In this paper, on the example of the concrete ventilated room, we will illustrate the above display. Let's consider the initial room as a rectangle with the sizes of 4x12, divided by three partitions. All partitions are 3m away from each other and have height which is also equal to 3 m (fig. 2).

Then a half of such room represents in our designations a figure Π_1 with sizes h=4; $2a$=6; b=3; c=3. Respectively, the (left) fourth part of the initial room can be considered as a figure Π_2 with sizes h=4, $2a$ =3, c=3. R point on the left vertical side Π_2 is also located at the height of 3 m.

In the future, we need Φ_2 mapping with two different sets of the initial parameters. The first of them associated with the figure Π_2 (4, 3, 3), we will discuss in detail, the second map of the same type is described more briefly.

As both figures discussed in Problem 2 are rectangles naturally we use the display (3) in its solution. At fixed α function from the formula (3) carries out conformal mapping of the upper half-plane onto the interior (symmetric about the imaginary axis) of the rectangle of the complex plane with vertices in points K', $(K' + iK'')$, (-K '+ iK") , -K ' (Fig. 6). Thus, this vertex K' and $(K' + iK'')$ are images of the points 1 and $1/\alpha$ of real axis and point O remains stationary at display (3) and symmetrical in respect to it points $-1/\alpha$ -1 and pass under the action of (3) to the vtrtices $(-K' + iK'')$ and $-K'$.

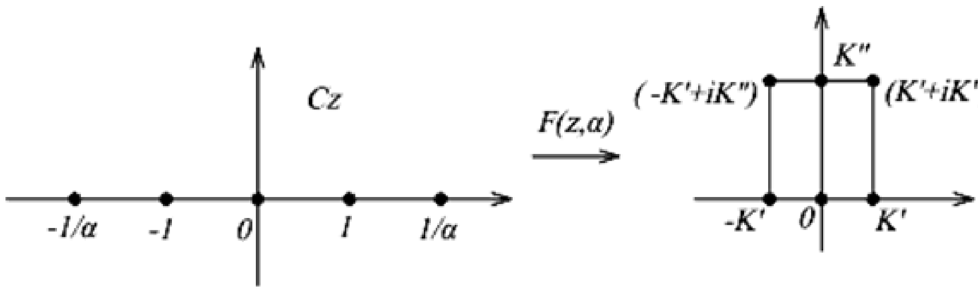

Fig. 6. Conformal mapping of the upper half-plane to the "interior" of the rectangle

The inverse to (3) map, that is the elliptic sine, transfers the rectangle into the upper half-plane.

Remark 1. It is known [8], that the ratio (K''/K') in dependence upon a parameter α is monotonically and continuously decreases from ∞ to zero. That means that to an approximation of tension (compression) rectangle of any size $h \times d$ (h — height, d — width) is the image of the upper half-plain under the action of display (3) with some α.

Remark 2. In [5] the rectangle of the 4x3 size transformed by stretching into a rectangle with dimensions of 4.2 x 3.15. The latter is the image of half-plain under the action of display (3) with the parameter $\alpha \approx 0.06$. For this parameter, $K' \approx 1.57$, $K'' \approx 4.2$. Solving the problem 2 for a rectangle Π_2 (4, 3, 3), we will immediately assume it given to more convenient form Π_2 (4.2; 3.15; 3.15).

Conformal mapping of the figure Π_2 (h; a; c) on a rectangle Π_2 «with a floor-to-ceiling window» will build (taking into account remark 1) as a superposition of three maps:

$$\Phi_2 = F(w,\beta) \circ g \circ F^{-1}(z,\alpha), \tag{6}$$

where *F(w, β)* — is a mapping which is realized by elliptic integral with a parameter *β*, the value of which is necessary to determine; *w=g(ζ)* is a fractional-linear automorphism (the map on itself) of the upper half-plane; $\zeta = F^{1}(z,\alpha)$ is an elliptic sine with known parameter *α*.

3. Description of the first constituent of the main display

For the figure *П₂* (4.2; 3.15; 3.15), as it was noted above, *α*≈0.06. It means that the four vertices of the rectangular-figure pass under the action of F^{1} *(z; 0.06)* into the four pixels (1/0.06, -1.1, 1/0, 06) of the real axis.

The image of the top point of the «sill» (located at the height of 3.15) at the display is calculated by the formula (6). So on the real axis, there is one more point with co-ordinate ≈ 10.3.

Fractional-linear mapping of the formula (6) can be written as:

$$w = g(\varsigma) = \frac{A\varsigma + B}{\varsigma + D}, \tag{7}$$

where *A, B, D* — some real coefficients. Geometric requirements to this display are that the four of points (-16.7, -1, 10.3, 16.7) have to pass to the four of numbers of the form $\left(-\frac{1}{\beta}, -1, 1, \frac{1}{\beta}\right)$ with any $\beta \in (0,1)$.

We can prove that, generally, such mapping of the upper half-plane exists and is defined uniquely. In [5] it is shown that in the discussed situation *β* ≈ 0.37, and the (specified) formula for *g(ζ)* is of the form:

$$g(\varsigma) = \frac{-9{,}19\varsigma + 48{,}91}{\varsigma - 57{,}11}, \tag{8}$$

It means that the solution of the task 2 for the figure *П₂* (4.2; 3.15; 3.15) is the superposition (6) with *α*≈0.06, *β*≈0.37.

The image of the figure is a rectangle «with a floor-to-ceiling window» with height *K"*≈2.431. The width of the rectangular image of half-plane is 2K"≈2·1.629=3.258.

Remark. Recall that in the above solution we are interested in the image of $\overline{R}$ of the point R, located on the left vertical side of the figure Π_2 at the height of 3.15. Applying the three-step mapping to it (6), we can calculate its position in the rectangle Π_3 (2.431; 3.258). The height of the point $\overline{R}$ that is also on the left side of Π_3, is about 1.61.

Passing in this situation from the constructed display Φ_2 to its symmetric continuation, we receive a figure Π_1 (4.2; 3.15; 3.15; 3.15) in the prototype of Φ_2 and the doubled rectangle Π_3 in the image of (fig. 7). This doubled rectangle has height 2.431 and width 2·3.258=6.516. Besides, on its right vertical side there is a «window» beginning at the height of 1.61.

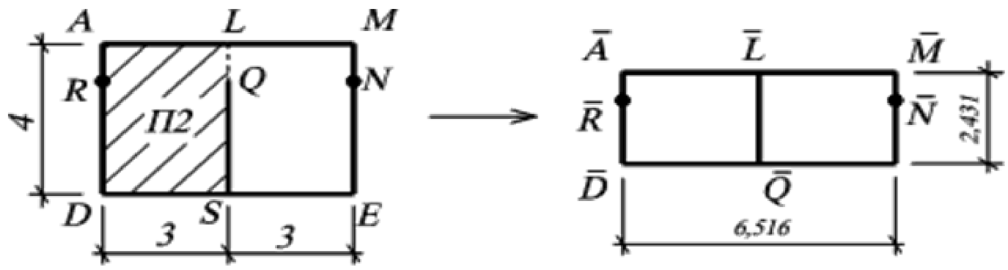

Fig.7. The conformal mapping of P1 onto a rectangle

4. Description of the second constituent of the main display

To construct the mapping Φ_1, solving the original problem 1 it is enough to solve one more problem 2 for the figure Π_2 (2.431; 6.516; 1.61).

By analogy with the above solution firstly we determine the ratio K"/ K' for the new figure Π_2 (located symmetrically in respect to the imaginary axis). Here

$$\frac{K''}{K'} = \frac{2{,}431}{3{,}258} \approx 0{,}75$$

Such ratio corresponds to a rectangular image of a half-plane under the action of the elliptic integral $F\ (z,\ \gamma)$ with the parameter $\gamma \approx 0.88$. For such value of γ $K'' \approx 1.67$ and $K' \approx 2.2$. Consequently, for the construction of a new mapping Φ_2 it is necessary, first of all, to compress the right figure to the designated size. In the compressed figure the «window» on its right side starts at 1.1, and the whole figure is represented as Π_2 (1.672; 4.44; 1.1).

Elliptic sine $\zeta = F^{1}(z;\ 0.88)$ transfers the tops of the figure into four points (-1/0.88, -1, 1, 1/0.88) and the upper point *N* of the «sill» transfers to the point *N* of real axis with the coordinate 1.1. Further according to the scheme described above, it is necessary to use one more fractional-linear map:

$$w = g(\varsigma) = \frac{57{,}11\varsigma + 48{,}91}{\varsigma + 9{,}19}, \tag{9}$$

It gives us four points of the real axis that after one more elliptical mapping *F(w, 0.937)* has to turn into the four vertices of the next rectangle Π_3.

Dimensions of this figure are K'≈2.485 and *K"*≈1.622, and the whole rectangle («with a floor-to-ceiling window») has a height of 1.622 and width of 4.97.

5. Construction of the streamlines

According to the told above, it is sufficient to construct the lines, which are the images of horizontal sections of the rectangle with the dimensions of 1.622 x 4.97 under the action of display Φ_2^{-1} transferring it into a rectangle with a «window».

In turn, after stretching of the turned-out figure one more display is applied to these lines which transfer a figure into a rectangle «with a partition and a window».

The first of these two maps is constructed by analogy with [5]. Scheme «streamlines», built using the package MAPLE, is shown in (Fig. 8).

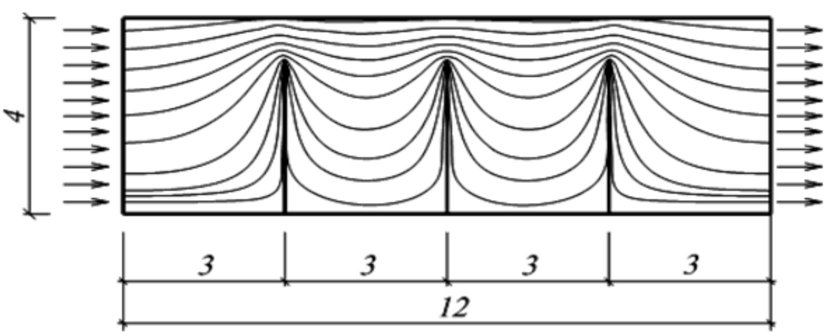

Fig. 8. Scheme of "streamlines" built with the software package MAPLE

The degree of conformity of the resulting scheme to real processes of ventilation is represented to authors quite acceptable. Possible experiments allowing to check such conformity, are in a development stage now.

Conclusions

1. The results of applying the method of conformal mapping to modeling streamlines of air flow in rooms with partitions can be regarded as close to real.
2. Computer calculations which have been made by means of this method, use only one tabulated (for example, in the software package MAPLE) function and therefore are, in fact, analytical.
3. Application of the offered way of calculation of air streams is possible at the organization of forcing-out ventilation in connection with typical for it low speeds of air streams and vortex-free movement of the environment.

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