FIRE AND INDUSTRIAL SAFETY

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Voronezh State University of Architecture and Civil Engineering

D.Sc. in Engineering, Prof. of Dept. of Heat and Gas Supply

and Oil and Gas Business S. N. Kuznetsov

Russia, Voronezh, tel. (473)271-53-21

e-mail: kuznetvrn@mail.ru

S. N. Kuznetsov

A MATHEMATICAL MODEL OF THE SPREAD OF FIRE GASES ON ESCAPE ROUTES

Statement of the problem. The existing methods used for the calculation of the concentration of fire gases on escape routes in the event of a fire do not allow one to account for the effect of anti-smoke ventilation with a great degree of precision. Mathematical modelling allows one to count with a sufficient degree of precision the concentration fields of smoke gases on escape routes.

Results and conclusions. For the description of aerodynamic and heat mass transfer processes the equations of continuity, momentum, heat and substance transfer were used. The mathematical model of the propagation of smoke gases in evacuation routes developed using MatLab system was implemented which allow one to account for the effect of anti-smoke ventilation with a great degree of precision.

Keywords: fire, smoke, mathematical modeling, escape routes.

Introduction

One of the major problems that arises in case of a fire is safe evacuation of people. The existing methods do not allow a highly accurate calculation of fire gas concentrations on escape routes.

The way fire gases spread on escape routes affected by anti-smoke ventilation has not been adequately studied up to this day. This paper presents a mathematical model of the spread of fire gases on escape routes.

1. Mathematical model

The mathematical model of the temperature field and fire smoke concentrations uses the equations based on the physical laws of the conservation of mass, momentum and energy averaged according to Reynolds [2—4, 6, 7, 9].

Let us examine the equations of the mathematical model of the temperature fields and fire gas concentrations. A continuity equation can be written as

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u_i)}{\partial x_i} = 0, \quad (1)$$

where t is the time, sec; ρ is the density of fire gases, kg/m^3; x_i is the i-th spatial coordinate, m; u_i is a flow rate component. In order to isolate the system, the ratios linking the strain tensors with the rates of fire gases are used. Newton's law that links the strain tensor and the deformation rate tensor of a viscous gas can be written as

$$\sigma_{ji} = -p\delta_{ij} + \mu\left(\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - \frac{2}{3}\delta_{ij}\frac{\partial u_k}{\partial x_k}\right), \quad (3)$$

where $i, j, k = 1, 2, 3$; p is the pressure, Pa; μ is the coefficient of dynamic viscosity of fire gases, kg/m·sec; δ_{ij} is Kronecker function ($\delta_{ij} = 1$if $i = j$ and $\delta_{ij} = 0$ if $i \neq j$); μ is dynamic viscosity of fire gases, kg/(m·sec).

Incorporating the equations (2) and (3), we have a non-stationary equation of momentum transfer:

$$\frac{\partial (\rho \overline{u}_i)}{\partial t} + \frac{\partial}{\partial x_j}\left(\rho \overline{u}_i \overline{u}_j\right) = -\frac{\partial \overline{p}}{\partial x_i} + \frac{\partial}{\partial x_j}\left(\mu\left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i} - \frac{2}{3}\delta_{ij}\frac{\partial \overline{u}_i}{\partial x_i}\right)\right) - \delta_{i3}\rho g + \frac{\partial}{\partial x_j}\left(-\rho \overline{u_i'u_j'}\right), \quad (4)$$

$$-\rho \overline{u_i'u_j'} = \mu_t\left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i}\right) - \frac{2}{3}\left(\rho k + \mu_t \frac{\partial \overline{u}_i}{\partial x_i}\right)\delta_{ij}, \quad (5)$$

where k is a kinetic energy of turbulence, m^2/sec^2; g is acceleration of gravity, m/sec^2; μ_t is a turbulent dynamic viscosity, kg/m·sec.In order to isolate the system of equations, non-stationary equations of the k-ε-turbulence model were used:

$$\frac{\partial(\rho k)}{\partial t}+\frac{\partial}{\partial x_i}(\rho k u_i)=\frac{\partial}{\partial x_j}\left(\left(\mu+\frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right)+G^k+G^b-\rho\varepsilon, \quad (6)$$

$$\frac{\partial(\rho\varepsilon)}{\partial t}+\frac{\partial(\rho\varepsilon u_i)}{\partial x_i}=\frac{\partial}{\partial x_j}\left(\left(\mu+\frac{\mu_t}{\sigma_\varepsilon}\right)\frac{\partial\varepsilon}{\partial x_j}\right)+\rho C_1 S_{ij}\varepsilon-\rho C_2\frac{\varepsilon^2}{k+\sqrt{\mu\varepsilon}}+C_{1\varepsilon}\frac{\varepsilon}{k}C_{3\varepsilon}G^b, \quad (7)$$

where ε is a dissipation rate of turbulence kinetic energy, m^2/sec^3.

$$G^k=2\mu_t\left(\sum_i\left(\frac{\partial u_i}{\partial x_i}\right)^2\right)+\mu_t\left(\sum_{i>j}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)^2\right), \quad (8)$$

$$G^b=\mu_t g\frac{1}{\rho}\frac{\partial\rho}{\partial x_3}. \quad (9)$$

A turbulent dynamic viscosity is given by the ratio

$$\mu_t=\rho C_\mu\frac{k^2}{\varepsilon}. \quad (10)$$

For constants of the turbulence model the following values are accepted:

$$C_{1\varepsilon}=1,42;\ C_{3\varepsilon}=1,38;\ C_2=1,8;$$
$$C_\mu=0,09;\ \sigma_k=1,0;\ \sigma_\varepsilon=1,1.$$

A non-stationary heat equation is

$$\frac{\partial(\rho h)}{\partial t}+\frac{\partial(u_i\rho h)}{\partial x_i}=\frac{\partial}{\partial x_i}\left(\frac{\lambda_e}{c_p}\frac{\partial h}{\partial x_i}\right)+q, \quad (11)$$

where h is a specific enthalpy of fire gases, J/kg; q is a specific heat of ignition, $Watt/m^3$.

A specific enthalpy of fire gases is given by the ratio

$$h=\sum_{i=1}^{N}h_i(T)c_i, \quad (12)$$

where c_i is a mass relative concentration of the i-th component of fire gases.

The coefficient of effective thermal conductivity of fire gases is determined by the

ratio

$$\lambda_e = \lambda + \frac{c_p \mu_t}{\sigma_t}, \quad (13)$$

where σ_t is a constant of the turbulence model.

The coefficient of thermal coefficient of fire gases is given by the ratio

$$\lambda = \sum_{i=1}^{N} \lambda_i (T) c_i, \quad (14)$$

where λ_i (T) is the coefficient of heat conductivity of the i-th component of fire gases, Watt/(m·K).

A specific isobaric heat capacity of fire gases is given by

$$c_p = \sum_{i=1}^{N} c_{pi} (T) c_i . \quad (15)$$

A dynamic viscosity of fire gases is given by

$$\mu = \sum_{i=1}^{N} \mu_i c_i , \quad (16)$$

where μ_i is a dynamic viscosity of the i-th component of fire gases, kg/(m·sec).

The equation of state of fire gases is

$$\rho RT = pM , \quad (17)$$

where R is the universal gas constant, J·(mole·K)$^{-1}$.

The molar mass of fire gases is given by the ratio

$$M = \frac{1}{\sum_{i=1}^{N} \frac{c_i}{M_i}}, \quad (18)$$

where M_i is a molar mass of the i-th component of fire gases, kg/mole.

A non-stationary equation of fire gas transfer is

$$\frac{\partial(\rho c_i)}{\partial t} + \frac{\partial(u_i \rho c_i)}{\partial x_i} = \frac{\partial}{\partial x_i}\left(D_e \frac{\partial \rho c_i}{\partial x_i}\right) + g_\partial . \quad (19)$$

where c_i is the concentration of fire gases; g_∂ is a specific fire gas emission of the ignition, kg/(m^3·sec); D_e is an effective coefficient of fire gas diffusion, m^2/sec:

$$D_e = D + \frac{\mu_t}{\rho \sigma_c}, \quad (20)$$

where D is a coefficient of fire gas diffusion, m^2/sec; σ_c is a constant of the turbulence model.

The initial conditions determine the distribution of the rates, pressure, air temperatures and concentration of fire gases at the initial moment of time and depend on the problem at hand. The air on escape routes either stays inert at the initial moment of time or is in motion affected by a ventilation system.

Solid boundary conditions. Enveloping structures comply with the condition of tightness (a zero normal speed component) and adhesion (zero speed components), the temperature or heat flows are specified and fire gases are gas-tight as required [1, 5, 8, 14, 16, 18].

Free boundary conditions. In open apertures and input-output openings of ventilation systems the boundary conditions are determined by the pressure or air speed, input air temperature and fire gas concentration in the input air [10—13, 15, 17].

2. Implementation of the mathematical model

The equations of the developed mathematical model of the temperature fields and concentration of fire gases were solved using the semi-implicit method for the equations linking the pressures *SIMPLER* that provide better stability and compliance of the iterative equations compared to the other methods.

For discretization of differential equations the model of the control volume was used. A discrete equation of the model is an algebraic equation that links the dependent variable in the group of the connected nods and formulates the same physical dependencies that the initial differential equation does. The dependent variable largely influences only the adjacent region. As there are more nods, the solution of the discrete presentation of the equations of the

model strives for the accurate solution of the differential equation of the model. Before the discretization, the region was split by the chess grid. Discrete analogues of differential equations of the model express law of the conservation of mass, momentum and energy for each control volume.

MatLab package was chosen as the development environment of the programs that implement the mathematical model in conjunction with the programming language *C*++.

While programming in the *MatLab* environment an interpreted language is used which means that each command is translated into the machine language commands as the program is running. On one hand, this enables to run the program both in the test and continuous run mode, but the time of the transformation of the commands makes the data processing a lot slower. In order to solve this problem, the *MatLab* environment offers *mcc* utility to transform *m*-files into *C* or C++. This enables the connection with the libraries of mathematical functions as well functions used for graphics. Besides making the execution faster, the use of *mcc* allows m-programs to be used on the systems where *MatLab* is not installed. The *mcc* utility transforms the applications implemented in *MatLab* into individual applications. Processing takes less time since the execution codes do not need to be rewritten from one language to another.

The result of the use of this tool depending on the settings can be ready-to-use execution files which are console applications as well as initial *C* or *C*++ files which are the functionality interface implemented in m-code. As object modules are being arranged, the applications statistically connects with the *MatLab* libraries such as the library of mathematical functions and plotting. When the applications with *MatLab* initially not installed on them are installed, a package of *MatLab Run-Time Libraries* need to be installed. All these libraries are part of a self-packaging archive file *mglinstaller.exe*.

Autonomous applications of either *C/C*++ dynamic-link libraries are created using off-site compilators such as *Microsoft C Compiler* as part of *Microsoft Visual C/C*++.

Conclusions

1. The existing methods used to calculate fire gas concentrations on fire escape routes does not allow for the effect of the anti-smoke ventilation. The problem of creating a mathematical model of the spread of fire gases on escape routes, which enables an accurate calcula-

tion of concentration fields of fire gases on escape routes. In order to describe aerodynamic processes and thermal and mass exchange the equations of continuity, momentum conservation, thermal and matter transfer were used.

2. The developed mathematical model of the spread of fire gases on escape routes was implemented as a *MatLab* computer software. The program allows for a highly accurate calculation of concentration fields of fire gases and the effect of the anti-smoke ventilation.

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