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ELASTIC-PLASTIC TRANSVERSE BENDING OF A ROD DURING LIMITED PLASTIC DEFORMATION

Statement of the problem. The problem of calculating bending of a rod in the state of flat transverse elastoplastic bend during limited plastic deformation is discussed. The transverse section of a rod with two axes of symmetry has a form of a fifty-fifty beam with two shelves: external and internal. A complex form of cross section explains practically unsolved difficulties in terms of its analytical solution. It leads to the use of math software and programming and math support, particularly MathCAD. In the first case we take as external loading a point force applied in the middle of the flange; in the second case it is an evenly distributed loading acting along the whole rod.

Results. As a result the bearing ability of the rod during limited plasticity is determined as well as the boundary of plastic and elastic deformations; residual stress in the rod following its complete unloading; deflected rod axe and residual deflection in the rod following its complete unloading.

Conclusions. The calculation shows that the use of modern information technologies, particularly software and math support of PC allow one to deal with difficult and laborious problems, in terms of the design of the analytical solution of mechanics of a deformed solid body.

Keywords: plasticity, elastic-plastic bend, bearing ability of a rod, deformation, a section with two axes of symmetry.

Introduction

The calculation of bending structures considering plastic deformations reveals hidden strength potentials with the rigidity normally not being provided. This makes it necessary to use limited plastic deformations in calculations when yield occurs not at all the points of a cross section of the rod. If a cross section is different from a rectangular one and its width changes discretely along the height, an investigation of the stress-strain of the rod faces mathematical challenges associated with analytical expression for the height of the elastic core as a function of a longitudinal coordinate. If readily available mathematical PC packages, e.g. MathCAD, are used to calculate a rod bending under an elastic plastic load, these challenges can be eventually, though with some difficulties, addressed.

1. Theoretical foundations for the calculation of bending rods considering plastic deformations

A flat transverse bending of rods, cross section of which has two symmetry axes is the simplest problem of the theory of plasticity according to the ideal Prandtl curve. The major assumptions of the calculation of an elastic plastic bending of a rod with two symmetrical axes are detailed in special literature, e.g. in [1—5].

They all come down to the following. For a cross section with two symmetry axes, a neutral axis during elastic plastic deformation (axis x) coincides with the central axis (axis x_0). It is assumed that the upper fibers in the elastic plastic deformation area are compressed and the lower ones are stretched.

A destructive load in flat transverse elastic plastic bending of a rod is found according to the condition that a bending moment $M_x(z_0)$ in a dangerous section z_0 of a rod should not be larger than destructive moment $M_{разр}$:

$$M_x\left(z_0\right) \le M_{разр} . \tag{1}$$

A destructive moment for limited plastic deformation is given by the ratio

$$M_{разр} = \sigma_T \left\{ \left[S_{tx}^T + S_{sx}^T \right] + \frac{2I_x^Y}{h_0^Y} \right\}, \tag{2}$$

where S_{tx}^T is a static moment in relation to the neutral axis x of a part of a transverse (dangerous) section in the area of the stretched fibers where stresses are equal to the yield point σ_T (absolute magnitude):

$$S_{tx}^T = \int\limits_{-\frac{1}{2}h}^{-\frac{1}{2}h_0^Y} y \cdot b\left(y\right) dy ,$$

where $b(y)$ is the width of a cross section of a rod; S_{sx}^T is a static moment in relation to the neutral axis x of a part of a cross (dangerous) section in the area of compressed fibers where stresses are equal to the yield point σ_T (absolute magnitude):

$$S_{sx}^{T} = \int_{+\frac{1}{2}h_0^Y}^{+\frac{1}{2}h} y \cdot b(y)\, dy;$$

h_0^Y is the height of the elastic core in a dangerous section of the rod; I_x^Y is an inertia moment in relation to the neutral axis x of a part of a cross (dangerous) section corresponding with the elastic core:

$$I_x^Y = \int_{-\frac{1}{2}h_0^Y}^{+\frac{1}{2}h_0^Y} y^2 \cdot b(y)\, dy.$$

An increase in the carrying ability of the bent rod calculated according to the destructive moment is evaluated using the coefficient

$$\beta = \frac{M_{разр}}{M_{x_0}^{T}}, \tag{3}$$

where $M_{x_0}^{T}$ is a bending moment that causes yield in the fibers in the dangerous section:

$$M_{x_0}^{T} = \sigma_T \frac{2I_{x_0}}{h};$$

I_{x_0} is an inertia moment of the transverse section with respect to the central axis; h is the height of the cross section.

The coefficient β shows how many times loading increases by since the moment yield occurs in the fibers of the dangerous section till its carrying ability is completely gone.

In order to calculate the functions $h^Y(z)$ of the height of the elastic core, the equation of the internal moment is used

$$M_T(z) = \sigma_T \left\{ \left[S_{tx}^{T}(z) + S_{sx}^{T}(z) \right] + \frac{2I_x^Y(z)}{h_0^Y(z)} \right\} \tag{4}$$

bending moment created by the external moment $M_x(z)$.

Here

$$S_{tx}^{T}(z)=\int_{-\frac{1}{2}h}^{-\frac{1}{2}h^{Y}(z)} y\cdot b(y)\,dy;\quad S_{sx}^{T}(z)=\int_{+\frac{1}{2}h^{Y}(z)}^{+\frac{1}{2}h} y\cdot b(y)\,dy;\quad I_{x}^{T}(z)=\int_{-\frac{1}{2}h^{Y}(z)}^{+\frac{1}{2}h^{Y}(z)} y^{2}\cdot b(y)\,dy\,.$$

The boundary of the elastic and elastic plastic areas of the rod z^T is found using the condition

$$\sigma_T\,\frac{2I_{x_0}}{h}=M_x\left(z^T\right), \qquad (5)$$

where $M_x\,(z^T)$ is a bending moment created by the internal loading.

Normal stresses in the cross section of the rod experiencing elastic plastic bending in the area of plastic deformations, i.e when

$$y\le-\frac{1}{2}h^{Y}(z)\ \text{ or }\ y\ge\frac{1}{2}h^{Y}(z),$$

are equal to the yield point:

$$\sigma_z(z,y)=\sigma_T\,. \qquad (6)$$

In the area of plastic deformations for when $|y|<\frac{1}{2}h^{Y}(z)$, normal stresses are given by the ratio

$$\sigma_z(z,y)=\sigma_T\,\frac{2y}{h^{Y}(z)}\,. \qquad (7)$$

Tangential stresses in the plastic area of the cross section equal zero: $\tau_{zy}=0$.

Tangential stresses in the area of elastic plastic deformations at the points of the cross section experiencing plastic deformations are

$$\tau_{zy}^{s}(z,y)=\frac{\sigma_T}{b(y)}\left\{\frac{\partial A_{s}^{T}(z)}{\partial z}+\frac{2}{h^{Y}(z)}\frac{\partial S_{sx}^{Y,omc}(z,y)}{\partial z}-\frac{2S_{sx}^{Y,omc}(z,y)}{\left[h^{Y}(z)\right]^{2}}\frac{\partial h^{Y}(z)}{\partial z}\right\}. \qquad (8)$$

Tangential stresses in the area of elastic plastic deformations at the points of the cross section experiencing elastic tensile deformations are given by the formula

$$\tau_{zy}^{t}(z,y)=\frac{\sigma_T}{b(y)}\left\{\frac{\partial A_s^T(z)}{\partial z}+\frac{2}{h^Y(z)}\frac{\partial\left[S_{sx}^Y(z)-S_{tx}^{Y,omc}(z,y)\right]}{\partial z}-\frac{2\left[S_{sx}^Y(z)-S_{tx}^{Y,omc}(z,y)\right]}{\left[h^Y(z)\right]^2}\frac{\partial h^Y(z)}{\partial z}\right\}$$

, (9)

where $A_s^T(z)$ is the area of the compressed plastic part of the cross section; $S_{sx}^{Y,omc}(z,y)$ is a static moment of the cut off compressed elastic part of the cross section with respect to the neutral axis *x*; $S_{tx}^{Y,omc}(z,y)$ is a static moment of the cut off stretched elastic part of the cross section with respect to the neutral axis *x*; $S_{sx}^{Y}(z)$ is a static moment of the compressed elastic part of the cross section with respect to the neutral axis *x*, with

$$A_s^T(z)=\int_{+\frac{1}{2}h^Y(z)}^{+\frac{1}{2}h} b(y')dy'; \quad S_{sx}^{Y,omc}(z,y)=\int_{y}^{+\frac{1}{2}h^Y(z)} y'\cdot b(y')dy'; \quad 0\le y\le+\frac{1}{2}h^Y(z);$$

$$S_{sx}^{Y}(z,y)=\int_{0}^{+\frac{1}{2}h^Y(z)} y'\cdot b(y')dy'; \quad S_{tx}^{Y,omc}(z,y)=\int_{y}^{0} y'\cdot b(y')dy'; \quad -\frac{1}{2}h^Y(z)\le y\le 0.$$

If a rod under transverse elastic plastic bending is experiencing an evenly distributed load *q*, there are normal stresses $\sigma_y(z, y)$ on horizontal areas. In the plastic area of the compressed part of the cross section stresses on the horizontal areas are given by the ratio

$$\sigma_y(y)=\frac{q}{b(y)}. \tag{10}$$

Normal stresses on the horizontal areas within the elastic core are calculated by the formula:

$$\sigma_y(z,y)=\frac{q}{b(y)}-\frac{1}{b(y)}\int_{A_{omc}^Y(z,y)}\frac{d\tau_{zy}(z,y)}{dz}dA. \tag{11}$$

Stresses on the horizontal areas within the plastic area of the stretched part of the cross section are

$$\sigma_y(z,y)=\frac{q}{b(y)}-\frac{1}{b(y)}\int_{A^Y(z)}\frac{d\tau_{zy}(z,y)}{dz}dA. \tag{12}$$

In the formulas (10)—(12) $A_{отс}^{Y}(z,y)$ is the area of the cut off stretched part of the elastic core of the cross section; $A^{Y}(z)$ is the area of the elastic core of the cross section, with

$$A_{отс}^{Y}(z,y)=\int_{y}^{+\frac{1}{2}h^{Y}(z)} b(y')\,dy'; \; A^{Y}(z)=\int_{-\frac{1}{2}h^{Y}(z)}^{+\frac{1}{2}h^{Y}(z)} b(y')\,dy'.$$

The residual normal $\sigma_{z}^{ост}$, targential $\tau_{zy}^{ост}$ and normal stresses on the horizontal areas $\sigma_{y}^{ост}$ in the cross section of the rod in the plastic deformation area following its complete loading is determined using algebraic sum of the corresponding stresses $\sigma_{z}(z,y)$, $\tau_{zy}(z,y)$, $\sigma_{y}(z,y)$ at the plastic loading stage under elastic unloading $\sigma_{z}^{разгр}(z,y)$, $\tau_{zy}^{разгр}(z,y)$, $\sigma_{y}^{разгр}(z,y)$:

$$\sigma_{z}^{ост}(z,y)=\sigma_{z}(z,y)-\sigma_{z}^{разгр}(z,y); \tag{13}$$

$$\tau_{zy}^{ост}(z,y)=\tau_{zy}(z,y)-\tau_{zy}^{разгр}(z,y); \tag{14}$$

$$\sigma_{y}^{ост}(z,y)=\sigma_{y}(z,y)-\sigma_{y}^{разгр}(z,y); \tag{15}$$

Here

$$\sigma_{z}^{разгр}(z,y)=\frac{M_{x_0}(z)}{I_{x_0}}y; \quad \tau_{zy}^{разгр}(z,y)=\frac{Q_y(z)\cdot S_{x_0}^{отс}(y)}{I_{x_0}\cdot b(y)};$$

$$\sigma_{y}^{разгр}(z,y)=\frac{q}{b(y)}-\frac{1}{b(y)}\int_{A_{отс}(y)}\frac{d\tau_{zy}^{отс}}{dz}dA; \quad A_{отс}(y)=\int_{y}^{+\frac{1}{2}h} b(y')\,dy'.$$

A deflection curve of elastic plastic deformed rod is determined using joint integration of the differential equation of the axis of the deflection rod in the area of elastic deformations

$$\frac{d^2V(z)}{dz^2}=\frac{M_{x_0}(z)}{E\cdot I_{x_0}}, \tag{16}$$

and a differential equation of the deflected axis of the rod in the area of elastic plastic deformations:

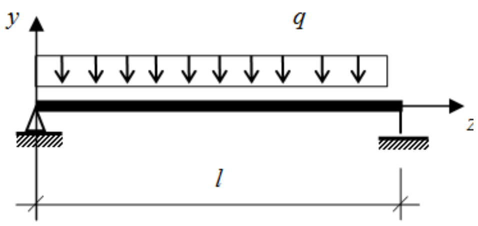


Fig. 1. Schematic of the rod

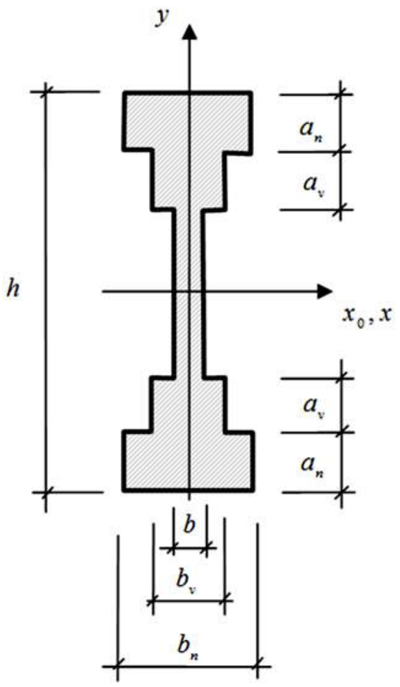


Fig. 2. Cross section of the rod

$$\frac{d^2V(z)}{dz^2} = \frac{2\sigma_T}{E \cdot h^Y(z)}, \tag{17}$$

where E is an elasticity modulus of the rod material.

Integration constants are determined using the condition of continuity and smoothness of the deflected axis of the rod at the adjoining ends of the elastic and elastic plastic areas.

The differential equation of the deflected axis of the rod in the area of elastic plastic deformations following its complete unloading is given by the ratio

$$\frac{d^2V_{ocm}(z)}{dz^2} = \frac{2\sigma_T}{E \cdot h^Y(z)} - \frac{M_{x_0}(z)}{E \cdot I_{x_0}}. \tag{18}$$

Residual bends in the elastic plastic deflected axis following its complete unloading are calculated by the integration of the sum of the differential equations (16) and (18) under the condition of continuity and smoothness of the deflected axis of the rod of the elastic and elastic plastic areas and conditions at the supports.

2. Example

Let us consider a steel two-support rod under a flat transverse elastic plastic bending (Fig. 1). The cross section of the rod is shaped like an equal double tee (Fig. 2). The carrying ability of the rod needs to be determined for limited plasticity as well as the boundary of the elastic and plastic deformations, residual stresses in the rod following its completed unloading, deflected axis of the rod and residual bends in the rod following its complete unloading. The rod material operates according to the ideal Prandtl curve.

The original data are as follows:

- length of the rod: $l = 6.0$ m;
- sizes of the cross section: $h = 0.40$ m; $b = 0.025$ m; $a_n = 2b$; $a_v = 2b$; $b_n = 3b$; $b_v = 2b$;

- height of the elastic core in the dangerous section: $h_0 = h - 2(a_n + a_v) = 0.2$ m;
- yield point: $\sigma_T = 240 \cdot 10^6$ Pa;
- modulus of elasticity: $E = 200 \cdot 10^9$ Pa;
- safety factor: $k = 1.2$.

Furthermore, the results of the solution of the problem in MathCAD are presented.

A bending moment when stresses in the fibers reach the yield point $M_T = 3.271 \cdot 10^5$ Nm. A destructive moment is $M_{разр} = 4.208 \cdot 10^5$ Nm.

Therefore $\beta = M_{разр} / M_T = 1.287$. A destructive load as a concentrated force acting on the rod halfway through the span is

$$F = \frac{4M_{разр}}{l} = 2{,}806 \cdot 10^5 \text{ N.}$$

The boundaries of the elastic and elastic plastic areas of the rod in this case are determined by the coordinates $z_{лев} = 2.332$ m, $z_{прав} = 3.668$ m based on the ratio (5).

A destructive load as an evenly distributed load acting on the entire rod is

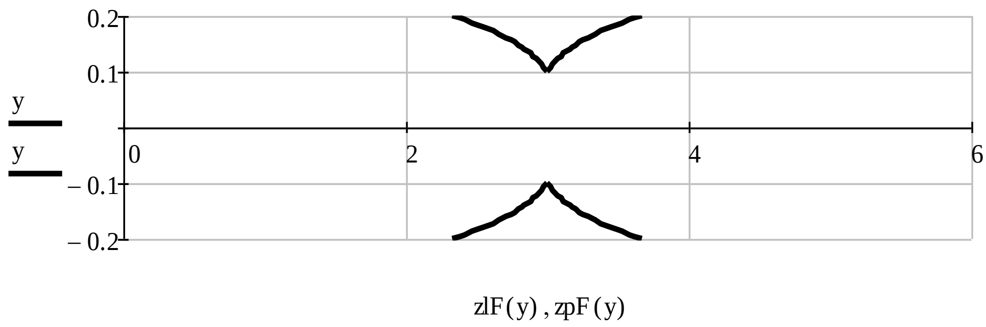
$$q = \frac{8M_{разр}}{l^2} = 9{,}352 \cdot 10^4 \text{ N/m.}$$

The boundaries of the elastic and elastic plastic areas of the rod are determined by the coordinates $z_{лев} = 1.584$ m, $z_{прав} = 4.416$ m.

In finding the boundary is identified between the areas of the elastic and plastic deformations, static moments of compressed and stretched plastic areas of the cross section as well as an inertia moment of the elastic core with respect to the neutral axis as a function of the height of the elastic core. Based on the geometry of the cross section, three cases were considered: a) the boundary of the elastic and plastic areas is within the wall of the cross section; b) the boundary of the elastic and plastic areas is within the internal wall of the cross section; c) the boundary of the elastic and plastic areas is within the external wall of the cross section. Then, specified by the height of the elastic core h^Y, based on the formula (4) the corresponding coordinate z of the boundary between the areas of elastic and plastic deformation.

The boundary between the areas of elastic and elastic plastic deformation along the rod is shown in Fig. 3.

a)



b)

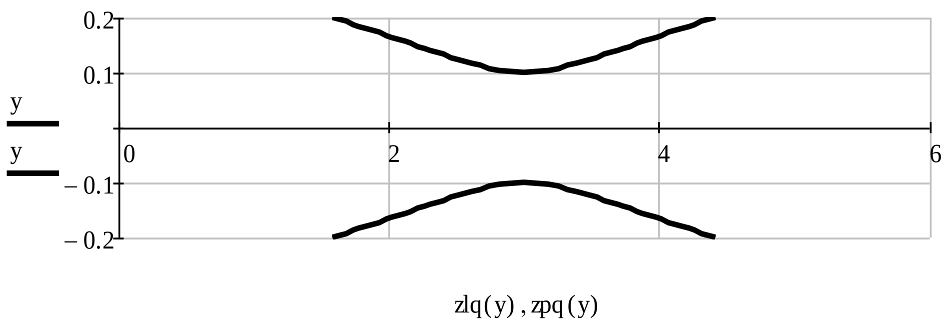


Fig. 3. Boundary between the areas of elastic and plastic deformation:

a) when the rod is loaded with a concentrated force;

b) when the rod is loaded with an evenly distributed load

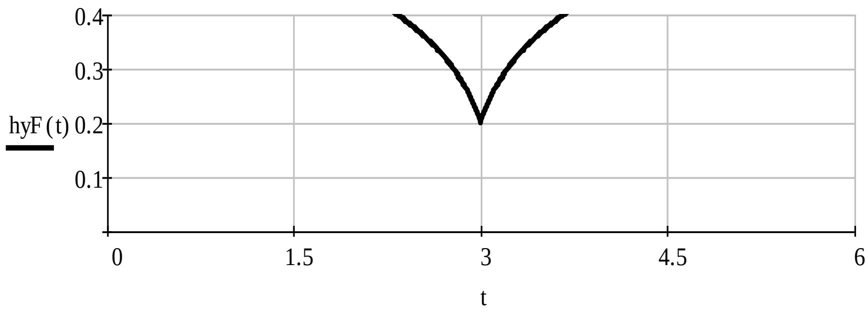
According to Fig. 3, the following can be concluded:

1. The shape of the cross section of the rod has an effect on the length of the area of plastic deformations;

2. Under a distributed external loading, the length of the area of plastic deformations is larger than the length of the area of plastic deformations under a concentrated force.

The analytical expression for a function $h^Y = h^Y(z)$ of the height of the elastic core was designed using a cubic spline interpolation. Fig. 4 shows the graphs $h^Y = h^Y(z)$, approximated by cubic splines.

a)



b)

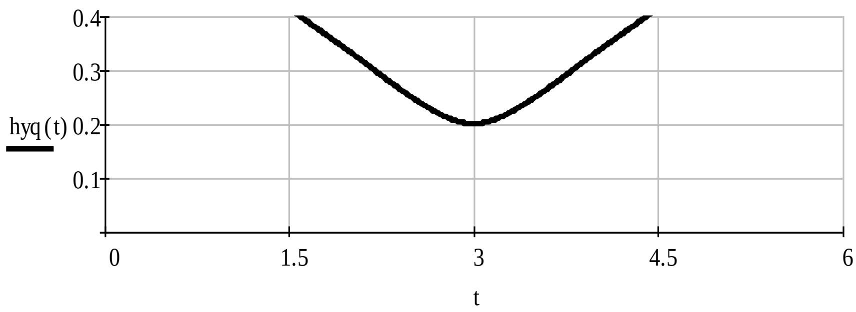


Fig. 4. Graph $h^Y = h^Y(z)$: a) when the rod is loaded with a concentrated force; b) when the rod is loaded with an evenly distributed load

Fig. 5 shows curves of normal stresses $\sigma_z(y)$ and curves of residual stresses $\sigma_z^{ocm}(y)$ in the dangerous cross section of the rod.

It should be noted that normal stresses in the dangerous section when the rod is under a concentrated force and normal stresses in the dangerous section when the rod is loaded with a distributed load coincide.

a)

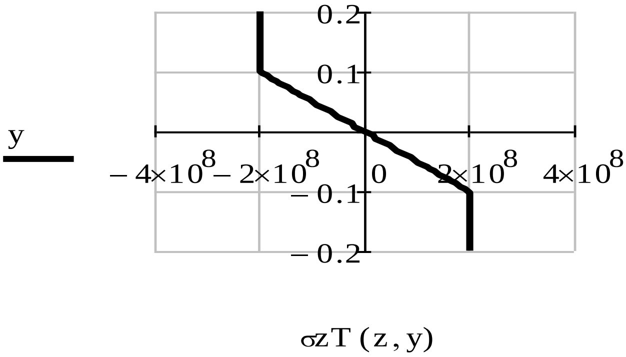


Fig. 5. Curves of normal stresses in a section z = 3.0 m of the rod: a) curve of normal stresses $\sigma_z(y)$; b) curve of residual normal stresses $\sigma_z^{ocm}(y)$

b)

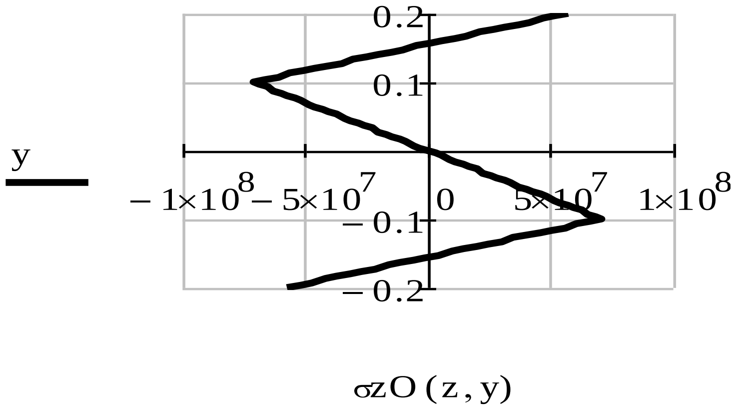


Fig. 5 (ending). Curves of normal stresses in a section z = 3.0 m of the rod: a) curve of normal stresses $\sigma_z(y)$; b) curve of residual normal stresses $\sigma_z^{ocm}(y)$

Tangential stresses in the area of elastic plastic deformations occur only in the area of the elastic core; in the area of plasticity tangential stresses is zero. When the rod is loaded with a concentrated load, as the section under consideration approaches the dangerous section, tangential stresses are on the rise, while the area of tangential stresses narrows down to the height of the elastic core. In the immediate vicinity of the dangerous section, tangential stresses start decreasing and become zero in the dangerous section itself.

Fig. 6a shows curves of tangential stresses in some cross sections when the rod is loaded with a concentrated force. When the rod is loaded with a distributed force, tangential stresses in the area of elastic plastic deformations decrease as the section under consideration approaches the dangerous section, while their area narrows down to the height of the elastic core.

Fig. 6b shows curves of tangential stresses in the cross sections when the rod is loaded with an evenly distributed load.

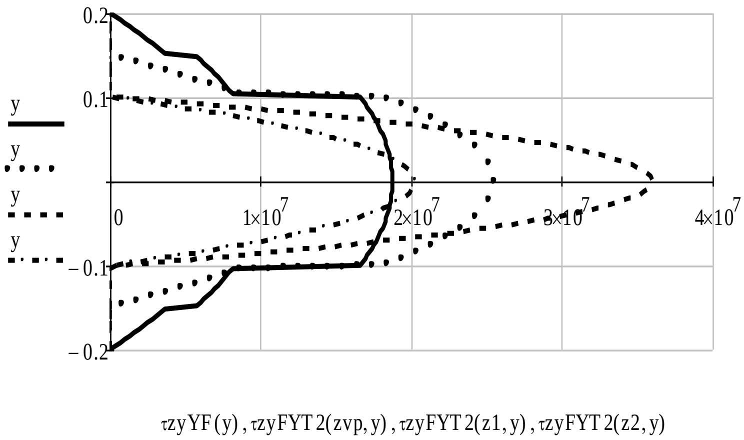
Curves of residual tangential stresses designed in cross sections where elastic core crosses the intersection of the external and internal shelves are in Fig. 7. A curve of normal stresses on horizontal areas in the area of elastic deformations (z = 1.584 m) is in Fig. 8a, in the area of elastic plastic deformations at (z = 2.95 m) is in Fig. 8b. A curve of residual normal stresses on horizontal areas in a section in the immediate vicinity of the dangerous section of the rod is shown in Fig. 9.

Curves of bends were designed by means of the immediate integration method for the differential equation of the bent rod as well as curves of residual bends of the rod following its complete unloading.

Three areas were identified on the rod: left and right elastic areas and middle one – elastic plastic ones. As the rod is loaded with a concentrated force, the left elastic area is an interval ($0 \le z \le z_{лев}$ = 2.332m), elastic plastic area is ($z_{лев}$ = 2.332 m $\le z \le z_{прав}$ = 3.668 m), the right elastic area is an interval ($z_{прав}$ = 3.668m$\le \le z \le l$ = 6.0 m).

As the rod is loaded with an evenly distributed load, the left elastic area is an interval ($0 \le z \le z_{лев}$ = 1.584 m), the elastic plastic area is ($z_{лев}$ = 1.584 м $\le z \le z_{прав}$ = 4.416 m), the right elastic area is an interval ($z_{прав}$ = 4.416 m $\le \le z \le l$ = 6.0 m).

a)



b)

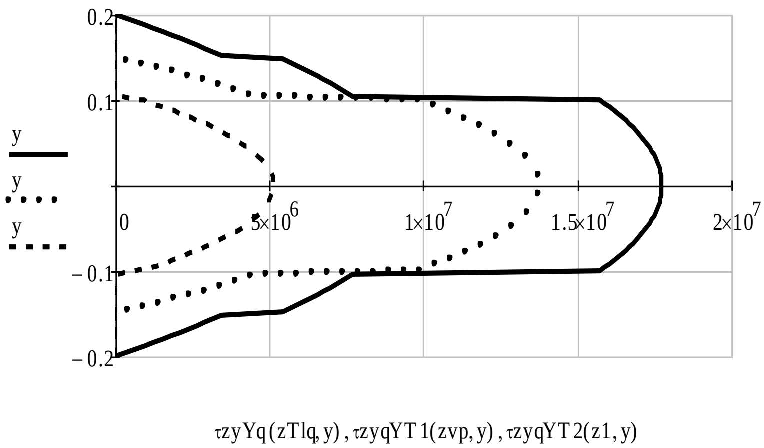


Fig. 6. Curves of tangential stresses in the sections:
a) when the rod is loaded with a concentrated force:
$z = 2{,}332$ m is a continuous line (area of elastic deformations);
$z = 2{,}782$ m is a dotted line (intersection of the internal and external shelves);
$z = 2{,}998$ m is a dot line;
$z = 2{,}990$ m is a chain line;
b) when the rod is loaded with an evenly distributed load: $z = 1{,}584$ m is a continuous line (area of elastic deformations);
$z = 2{,}192$ m is a dotted line (intersection of the internal and external shelves);
$z = 2{,}800$ m is a dot line

a)

b)

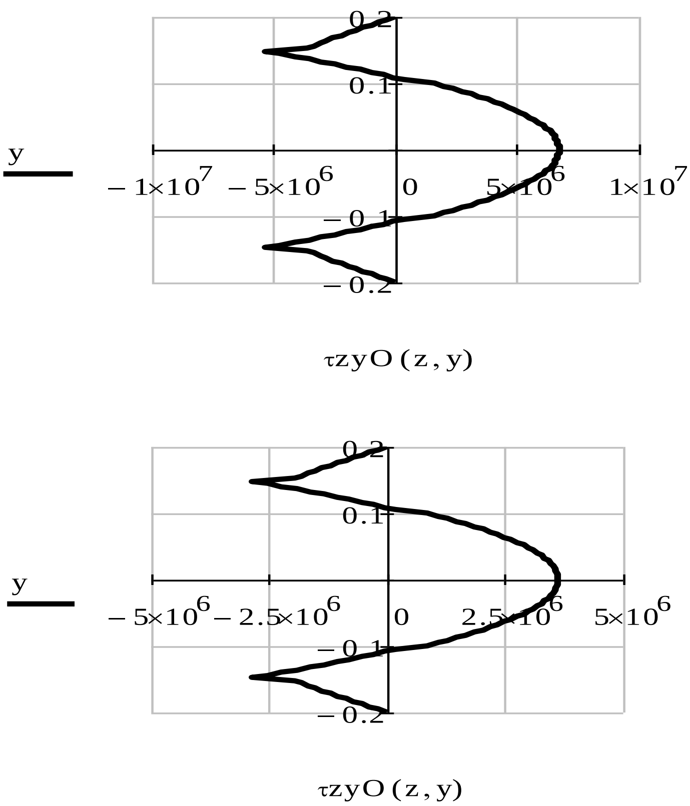


Fig. 7. Curve of residual tangential stresses:
a) in the section $z = 2{,}782$ m when the rod is loaded with a concentrated force;
b) in the section $z = 2{,}192$ m when the rod is loaded with an evenly distributed load

a)

b)

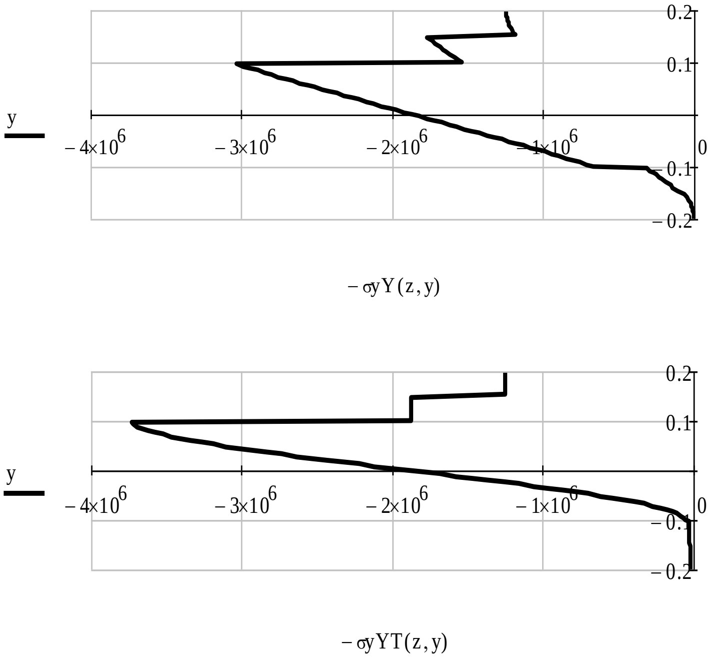


Fig. 8. Curve of normal stresses on horizontal areas:
a) in the section $z = 1{,}584$ m;
b) in the section $z = 2{,}950$ m

When the curves of bends were designed, a combination of differential equations (16) and (17) were integrated. When the curves of residual bends were designed, a combination of differential equations (16) and (18) was integrated.

Constant integrations were determined according to the conditions of continuity and smoothness of the deflected axis at the intersections of the areas.

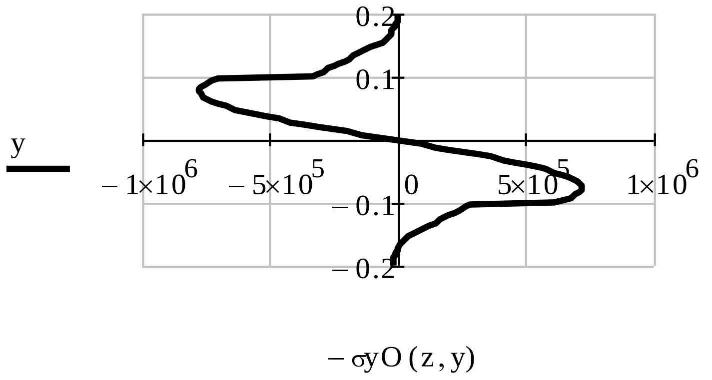


Fig. 9. Curve of residual normal stresses on horizontal areas in the section $z = 2{,}950$ m

Fig. 10 depicts the curves of bends when the rod is loaded with a concentrated load. The curves of bends when the rod is loaded with an evenly distributed load are in Fig. 11. Fig. 12 shows the curves of residual bends following a complete loading.

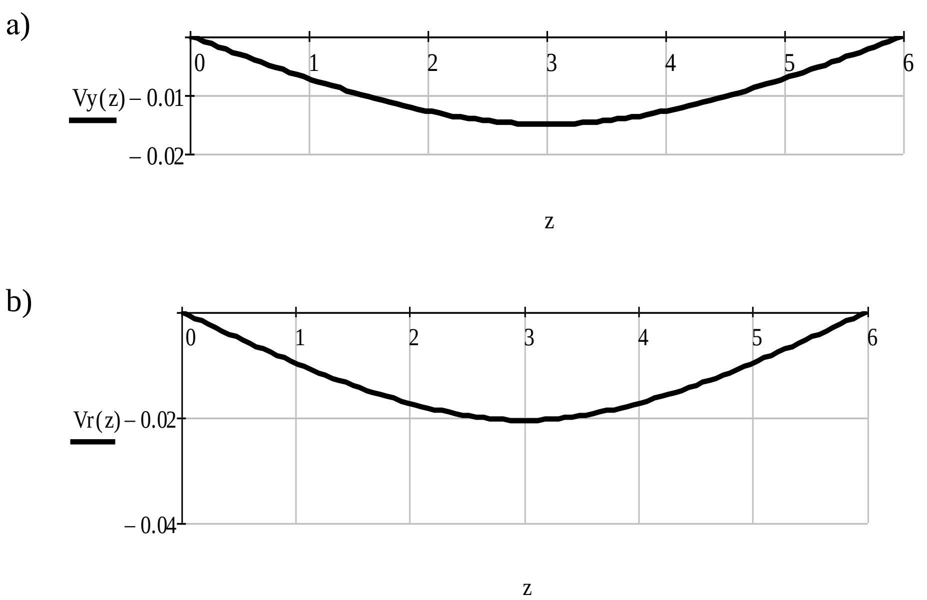
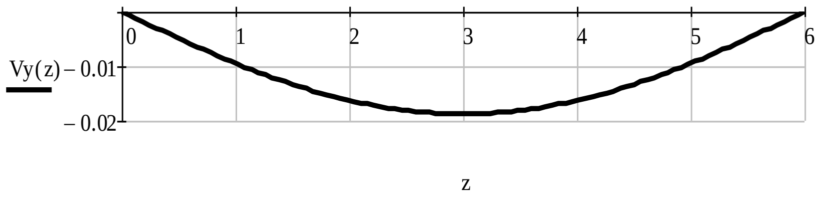


Fig. 10. Curves of bends when the rod is loaded with a concentrated force:
a) curve of bends corresponding with the stresses in the fibers which are equal to the yield point, the maximum bend V_{vax} = -0.015 m b) curve of bends corresponding with the plastic hinge, maximum bend V_{vax} = -0.021 m

a)



b)

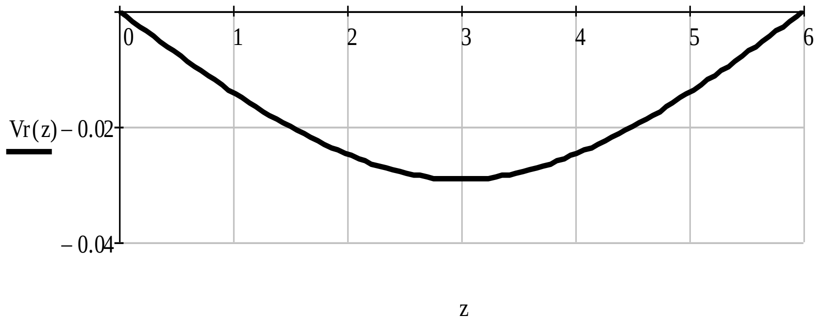


Fig. 11. Curves of bends when the rod is loaded with an evenly distributed load:
a) curve of residual bends when the rod is loaded with a concentrated load,
maximum bend V_{vax} = -0.0013 m; b) curve of residual bends when the rod is loaded with evenly distributed load, maximum load V_{vax} = -0.0051 m

Comparing Fig. 10 and 11, it is evident that when the rod operates at the elastic stage, its bends caused by a concentrated force and evenly distributed load are not considerably different. When the rod is under elastic plastic bending, its bend under an evenly distributed load is larger than a bend caused by a concentrated force. The analysis of Fig.12 concludes that residual bend of the rod loaded with an evenly distributed load is considerably higher than residual bend caused by a concentrated force.

a)

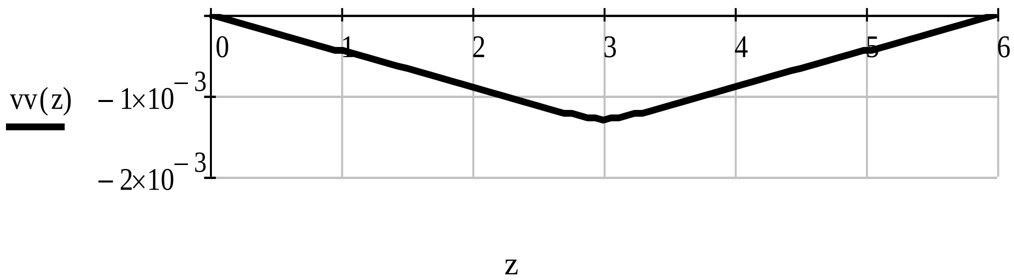


Fig. 12. Curves of residual bends: a) curves of residual bends when the rod is loaded with a concentrated force, maximum bend V_{vax} = -0.0013; b) curves of residual bends when the rod is loaded with a evenly distributed force, maximum bend V_{vax} = -0.0051 m

b)

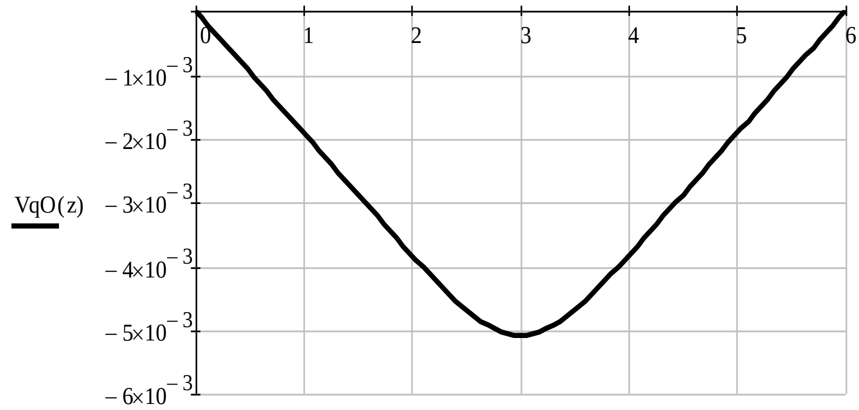


Fig. 12. Curves of residual bends: a) curves of residual bends when the rod is loaded with a concentrated force, maximum bend V_{vax} = -0.0013; b) curves of residual bends when the rod is loaded with a evenly distributed force, maximum bend V_{vax} = -0.0051 m

Conclusions

1. The results of a numerical solution show that the use of information technology, particularly mathematical PC software packages allow one to deal with analytically complex solutions.

2. The rod was first calculated experiencing elastic plastic bend, cross section of which is different from the rectangular one whose width changes discretely along the height of the section.

3. The results detailed in the article can be made use of while dealing with rod systems considering plastic deformations.

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