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MATHEMATICAL MODELLING OF AIR STREAMS IN LARGE SPACES

Statement of the problem. The greatest contribution to solution the problem of mathematical modeling of air streams in large spaces was made using the methods which are based on the system of the equations of a hydraulic gas dynamics. These methods involve a great deal of computer calculations, and errors can affect the accuracy of the resulting lines of current, and its qualitative character on the whole. In this article the analytical way of the calculation of geometrical and numerical characteristics of air streams of large premises based on the conformal mapping method is considered.

Results. The mathematical model of air streams of displacement ventilation of spectator galleries based on the theory of conformal mapping is developed. The model is based on the use of symmetry of rectangular rooms and elliptic integral. The developed mathematical model can be applied to a mode of a current of the environment with a minimum vortex formation.

Conclusions. The implementation of a function of elliptic integral in packages of symbolical mathematics maintains a high level of accuracy of numerical studies of the suggested model that allows an increase in the reliability of the obtained results. The important characteristics of model is possibly accurate analytical solution of the problem of designing air velocity rates in spectator tribunes.

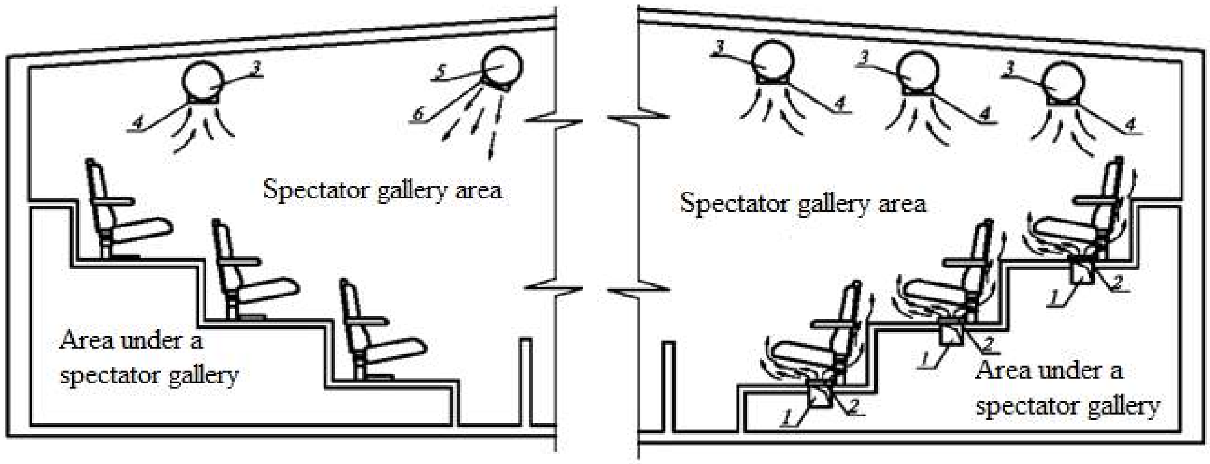
Keywords: displacement ventilation, velocity rates, current lines, conformal mapping, air conditioning, mathematical modeling.

Introduction

While designing ventilation and air conditioning systems for public spaces, most optimal systems of air distribution need to be found which can be organized according to the principles of mixing and displacement ventilation.

Typically, the air in a premises in mixing ventilation moves in inflow fluxes (due to ejection) at high speeds while making use of the inside air. The inflow and outflow air mix resulting in relatively even distribution of temperature and velocity fields in the entire premises. When displacement ventilation is used, the temperature and velocity fields are more evenly distributed, which positively affects the comfort of people in spaces with systems of ventilation and air conditioning. However, the existing calculation methods do not normally meet the modern accuracy requirements. Therefore, ways and methods of calculating systems of displacement ventilation based on analytical modelling of airflows in a premises need to be developed.

Previously, in [1, 2] a point was made that modelling of displacement ventilation air flow should make use of the conformal mapping method, which relies on the properties of a first order elliptical integral and a reverse function called an elliptic sinus. The present paper deals with a large space such as a spectator gallery, which seems to be most likely to utilize displacement ventilation. Fig. 1 shows the main schemes of heat transfer using mixing (Fig. 1a) and displacement (Fig. 1b) ventilation.



a)

b)

Fig. 1. Heat transfer: a) upward, b) downward:
1 are incoming air ducts; 2 are low-speed heat transfer devices; 3 is an exhaust duct;
4 are exhaust devices; 5 are incoming air ducts; 6 are ceiling heat transfer devices

In the first case in the organization of upward heat transfer of spectator galleries grouped incoming air ducts are placed in the upper part of the space. The air is removed through the air inlet over spectator galleries. In the second method, the incoming air of 20 m^3/h per person is supplied at the speed of 0.25 m/sec under the gallery seats with the temperature difference of 3 °C between the incoming air and operating area air. The air was removed from the upper area over the galleries. According to [1—3], the calculation of velocity fields by means of the theory of conformal mapping takes place in two stages: 1) designing current lines of airflows, 2) determining numerical values of velocity. Let us look at each in more detail.

1. Designing current lines of displacement ventilation airflows of spectator galleries

In the organization of heat transfer of spectator galleries according to the schematic in Fig. 1, let us consider the simplest model where the entire area of the space is divided so that each of them included an incoming and outlet air duct in the lower and upper parts respectively.

For a distinguished element of the area, a calculation figure takes the form as shown in Fig. 2. The distinguished element is a square *ABDF* with the sides parallel to the X and Y axis. Besides, there are lines *CJ* and *EF* on the sides *BD* and *DF* which are inlet and outlet ducts respectively. The inlet and outlet ducts are buried with each side of the square being a current line.

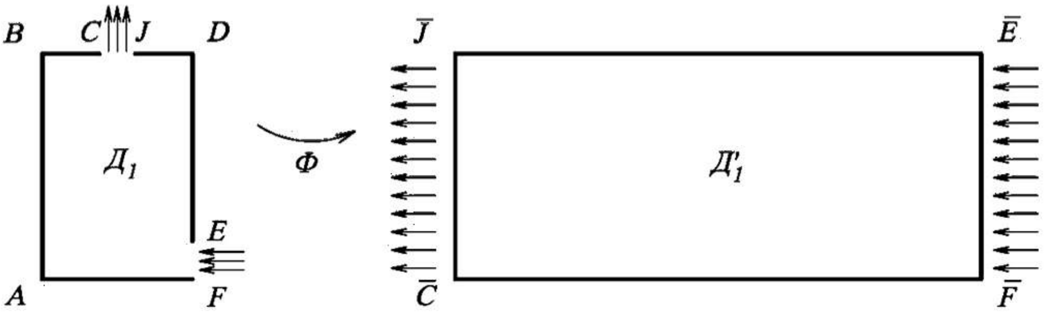


Fig. 2. Original figure of the first element of the distinguished area

In the case under consideration in order to design current lines and subsequently to determine velocities of airflows in the original randomly shaped figure (Fig. 2), it is necessary to draw a conformal map of the figure $Д_1(ABDF)$ onto a “more simple” $Д'_1\overline{F}\overline{E}\overline{J}\overline{C}$. The inlet and outlet ducts of the ventilation system needs to be placed in the figure $Д'_1$ so that they suggested a “natural” configuration of the current lines. The current lines on a real figure are designed using reverse conformal mapping. The main assumptions for using conformal mapping here are conditions of incompressibility of the air and its vortex-free motion.

At the first stage of designing this map, conformal maps of the original square $Д_1$ with all the distinguished points *ABDF* and *CJE* are drawn onto an additional upper half-plane and detected on the real line of their maps. Here, a half-plane with controlled points is mapped onto the square $Д'_1$, with the map of the inlet *EF* and outlet *CJ* ducts are the right $\overline{F}\overline{E}$ and left $\overline{J}\overline{C}$ sides of the square respectively (Fig. 2).

It is known that a half-plane can be mapped onto a square using an elliptic integral:

$$F(z,\alpha)=\int_0^z \frac{dt}{\sqrt{(1-t^2)(1-\alpha^2t^2)}}\,(0<\alpha<1). \tag{1}$$

The points -1/α; -1; 1; 1/α move to the tops of the square. For the first stage a map reverse to the elliptic integral is used. At the second stage the half—plane is mapped onto itself so that four of the distinguished control points take the “ideal” position, i.e. correspond to a set of points -1/α; -1; 1; 1/α with some parameter of the elliptic integral. At the third stage the half-plane with “regularly” positioned control points is mapped onto the square with “regular” inlet and outlet ducts.

There are a few additions to the three main stages. In order for the square to be map of the upper half-plane (Fig. 3), using some map *F*(*z;*α) by means of the elliptic integral, it is necessary that its vertical and horizontal sizes were specially made to meet the parameter α. The ratio of the vertical and horizontal sides *K''/K'* of the square changes monotonously and continuously from zero if the parameter α changes from zero to one (*K'* is made equal to half of the total width of the square).

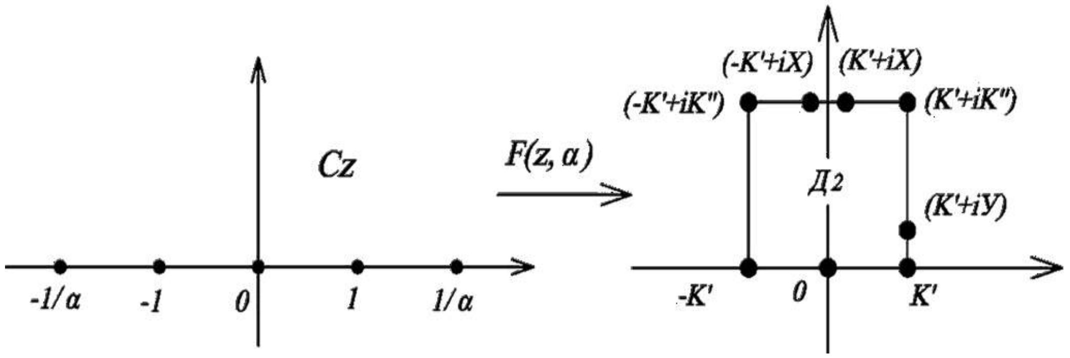


Fig. 3. Conformal map of the upper half-plane onto the “inside” of the square

This means that the sizes of a real part of the space should be expanded (compressed) to the “ideal” condition. For example, for the first part with the size $Д_1$ (5 m; 1.5 m) the ratio *K''/K'* is 6.67. This ratio is achieved at $\alpha = 1.15\cdot 10^{-4}$ and $K' = 1.57$, $K'' = 10.48$. As a result, before using the above conformal map, it is necessary to expand the original figure $Д_1$ with the coefficient 2.096 = 10.48/5 = 1.57/(1.5/2).

In a fixed parameter α the function (1) conformally maps the upper half-plane onto a square $Д_2$ symmetrical relative to the imaginary axis of a complex plane with the tops at the points K', $(K' + iK'')$, $(-K' + iK'')$, $-K'$ (see Fig. 2). The control points *F, E, J, C* of the original figure move to K', $(K' + iУ)$, $(K' + iX)$, $(-K' + iX)$ respectively.

Conformal map *Φ* of the original figure onto a “regular” square can be a superposition of three maps taking the form

$$\Phi = F(w,\beta) \quad w \quad F \quad (z,\alpha), \tag{2}$$

where $F(w, \beta)$ is a map designed using an elliptic integral with some parameter β; *w* is some linear fractional mapping onto itself on the upper half-plane; $\zeta = F^{-1}(z, \alpha)$ is an elliptic sinus with a known parameter α.

In a reverse mapping $F^{-1}(z; \alpha)$ the control points (*FEJC*) of the square $Д_1$ move into four points of the real line Z_1, Z_2, Z_3, Z_4 (1;1.6; 17813; -17813). Linear fractional mapping *w* can be written as follows:

$$w = \frac{C\xi + F}{\xi + J}, \tag{3}$$

where *C*, *F*, *J* are some real coefficients.

Geometrical requirements for the mapping *w* are that the four points Z_1, Z_2, Z_3 and Z_4 are to move to the four points as follows

$$\left(-\frac{1}{\beta}, -1, 1, \frac{1}{\beta}\right)$$

with a coefficient $\beta \in (0,1)$.

In this case $\beta \approx 0.983$ and the linear fractional mapping *w* is as follows

$$w = \frac{74,64\xi + 72,63}{\xi + 1,008}. \tag{4}$$

Using the superposition (2) previously supplemented with expansion, we get at $\alpha = 1.15\cdot10^{-4}$ and $\beta = 0.983$ a mapping of the figure $Д_1$ in the counter image *Ф* onto the square $Д_3$ in the map (Fig. 4) with the size (6.24; 1.58).

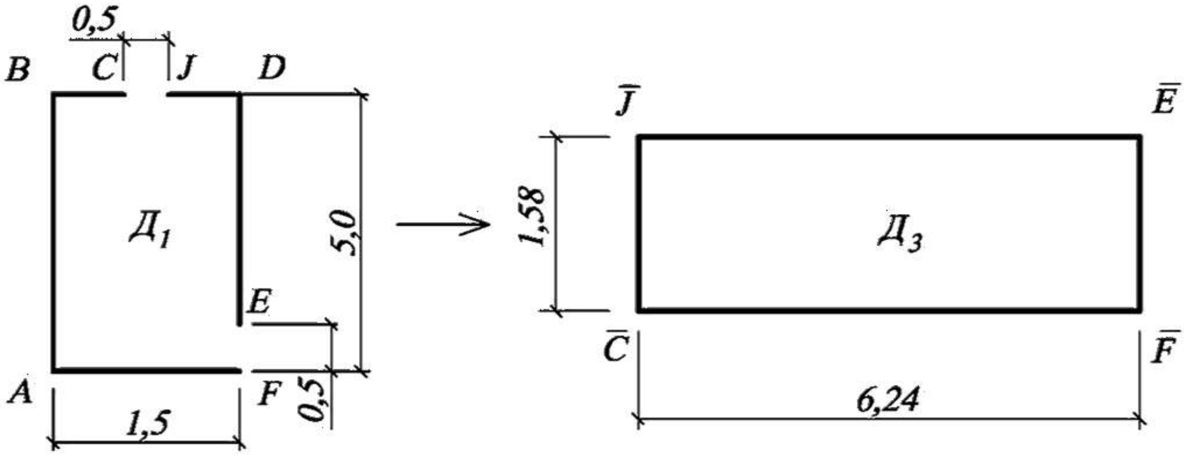


Fig. 4. Conformal mapping $Д_1$ onto the square $Д_3$

Further on, using a software package *Maple,* current lines are designed. At the final stage of designing the current lines in the original space, individual current lines are joined together. The final schematic of the development of airflows in this case is in Fig. 5.

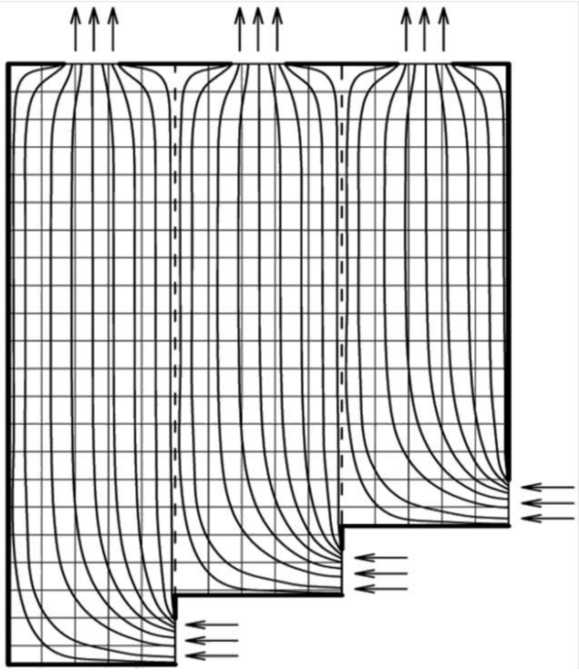


Fig. 5. Schematic of the “current lines” of spectator galleries after some lines were joined together

Note. Division of a large space into parts as was suggested above is in some sense artificial. Therefore viability of the model (e.g., where the lines are joined together) needs to be checked using other tools.

2. Determining numerical values of the velocities of air

A complex potential derivative is a complex vector connected with a velocity vector. If there is a known map of a complex figure onto a half-plane, the current line targentials (determining the velocity of the flow) are calculated using simple differentiation. I.e. if complex potential is known, velocity projections are given by the formula

$$\frac{dw}{dz} = v_x + iv_y, \tag{5}$$

where v_x is a velocity projection onto the axis *OX*; v_y is a velocity projection onto the axis *OY*; w is a complex flow potential.

The scalar value of airflow velocity is given by the equation

$$v = \left| v_x + iv_y \right| = \sqrt{v_x^2 + v_y^2}. \tag{6}$$

Since when current lines are designed that determine the velocity of the flow in spectator galleries the above four-step mapping is used, the air velocity is the product of the derivatives of each transformation step

$$v = A_1 \cdot A_2 \cdot A_3 \cdot A_4, \tag{7}$$

where A_1, A_2, A_3, A_4 are the derivatives of the function at each transformation step.

The general mapping is designed in the reverse order as a superposition of simple mappings. A subsequent calculation of a ventilation flow needs derivatives at each calculation point at each transformation step. As a result, the mathematical model defining the air velocity at any point of spaces of spectator galleries takes the form

$$v = \frac{2,264 \cdot 10^{-2}}{\sqrt{\dfrac{\left(t_1^2 - 1\right)\left(0,9676 \cdot t_1^2 - 1\right)}{\left(0,01 \cdot t_1 + 0,01\right)^4 \left(t_2^2 - 1\right)\left(0,49 \cdot t_2^2 - 3,906 \cdot 10^7\right)}}}, \tag{8}$$

where the variables t_1 and t_2 correspond with the position of the point at the first and second transformation steps respectively.

Software packages such as *IcePak, Coolit, Flotherm* applied for numerical modelling of velocity fields for this type of problems are effective in solving hydrogasdynamics tasks. But the above packages are technically challenging to use for designing since they are rather expensive. Therefore the use of methods of mathematical modelling based on the theory of conformal mapping seems to be the simplest alternative to the above.

Conclusions

1. There are mathematical models based on solving a system of equations with partial derivatives that involve a great deal of computer calculations. Errors made through the course o such calculations (since they are not stable or insufficiently developed in terms of convergence) can affect the accuracy of the final current lines and their quality as well.

2. Using the developed mathematical model based on the theory of conformal mapping, an analytical solution for designing velocity fields of displacement ventilation air of spectator galleries. The model relies on the symmetry of square spaces and the properties of elliptic integration. This function employed in symbolic mathematics packages maintains high accuracy in numerical studies of the model, which proves the results to be reliable.

3. The developed mathematical model is applied for low-vortex areas. In case of turbulence, the results need to be extra checked and compared with the experimental or natural experiment data.

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