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NON-LINEAR CALCULATION OF MULTILAYER AIRFIELD PAVEMENTS ACCORDING TO THE THEORY OF PLASTIC CURRENT

Statement of the problem. Low fracture strength of asphalt, promoting formation of the reflected cracks in a strengthening layer interferes with strengthening of precast slab pavings PAG by asphalt layers of strengthening. Constructive-technological decisions aimed at decreasing the reflected crack formation in practice do not produce a due result because of the absence of a strict mathematical model of the operation of non-rigid layers of strengthening. Thus, for the purpose of the definition of the parameters of the design of strengthening operation of the mathematical model based on the research of change of an intense-deformed condition of a multilayer airfield pavement, exposed to impact of a static load from aircrafts is required.

Results. Elastic and plastic model of the deformation of the multilayer airfield pavement is obtained based on mathematical assumptions of the theory of a current. Strength and plasticity criterion are proved accordingly for materials of the artificial airfield pavement, a natural earth foundation. The method for settlements of elastic and plastic model is suggested using the method of final elements.

Conclusions. The obtained data can be used in the course of designing of layers of reinforcement of rigid airfield pavement from plates PAG.

Keywords: elastic plastic model, yield surface, ultimate strength (plasticity), finite element method, airfield pavement.

Introduction

Current airfield network of the state aviation is largely comprised of airfield pavements of ferroconcrete smooth airfield paving that needs extensive maintenance and repairs following a long-term operation. The major way of maintaining precast pavements is replacing the damaged by new ones. However, the industrial capacity of the only plant manufacturing smooth airfield paving in Leningrad region does not rise to the challenge of catering for the growing needs of the state aviation.

Besides, the paving prices considering transportation costs in the Far East and Siberia are constantly on the rise. Alternatively, assembled airfield pavings can be put back into use by increasing the number of asphalt concrete reinforcement layers.

Asphalt concrete reinforcement layers are generally known to be prone to wear and tear, evenness, high resistance to water, salts and reagents used to combat icing. One of the major advantages of asphalt concrete pavements is recycling, mechanization and construction times as well as quick operation starts. All in all, this accounts for relatively low maintenance costs.

Besides that, low crack resistance of asphalt concrete causing reflective cracks in the reinforcement layer prevents reinforcement of precast pavements of smooth airfield pavings. The causes of reflected cracks in asphalt concrete reinforcement layers are fragility of asphalt concrete at low temperatures, reduction in the length of cement concrete pavings as temperatures get lower, mutual vertical displacements of the edges in the joints and cracks of cement concrete pavement under aircraft loads.

A construction technology solution to deal with reflective tracks in asphalt concrete reinforcement layers is 7-16 cm crack-intercepting sublayers from large-grained highly porous asphalt concrete mix (20-30% pores of the total volume). Mechanical energy developed at the crack site caused by temperature stress and aircraft loads is known to significantly dissipate instead of focusing at the reinforcement base. However, in practice, the joints of the existing asphalt concrete paving are reflected on the asphalt concrete reinforcement layer. The reason for that is that there is no comprehensive mathematical model of the operation of non-rigid reinforcement layers. The developed construction technology solutions are empirical. A random change in the intensity of the exterior factors inadvertently causes reflective cracks. Therefore, in order to identify the parameters of a reinforcement structure, a mathematical model should be developed based on the study of changes in the stress-strain state of a multi-layer airfield pavement exposed to static aircraft loads considering physical nonlinearity of the materials.

1. Elastic-plastic model of deformation of a multi-layer airfield pavement from the standpoint of the flow theory

The currently used calculation methods can only be applied at the new construction stage when an airfield pavement is in the elastic stage. The basic assumptions were the following [1]:

– aircraft wheel load is accepted to be static. A dynamic force is accounted for by introducing the dynamic coefficient determined using an airfield paving and aircraft wheel pressure;

– a mathematical model of paving is specified as Kirchhhoff-Lyav plate on the elastic foundation with centrally positioned aircraft wheel load. A non-symmetrical position of loads and ways of joining the plates is accounted for by the inclusion of the correction coefficient;

– change in temperature stresses, concentration of strains, growth of concrete strength and aging of organic binding materials in time, seasonal oscillations of subgrade strength, etc. are accounted for by the inclusion of the operating coefficient.

Therefore, at the heart of the calculations of airfield paving is a joint operation of the paving and subbase. The strength is determined by aircraft wheel load resistance of subgrade. The most common calculation models of the subgrade are

– model of the subgrade coefficient of Fuss-Winkler;

– model of the elastic semispace;

– model of linearly deformed semispace.

In the first two models the subgrade is specified as an absolutely elastic material not experiencing any plastic deformations. The third model undergoes residual deformations, however the actual pressure curve is made a scheme of by replacing some parts of it on certain areas by a straight line. The common disadvantage of the models is negligence of the non-linear behavior of the subgrade as plastic deformations progress.

A multi-layer airfield pavement is generally a non-linear deformed structure where the dependence of a load and heaving transmitting it is curvilinear. The stress-strain state of these structures is described by more complex elastic plastic models relying on the notion of yield point, i.e. the elastic state boundaries determined by a level of strains achieved under repeated loads following prior unloading.

Let us examine an area of multi-layer airfield pavement identified in Fig.1 that has been in use for some time $T = t_i$, thus causing plastic deformations to progress and the elastic boundary to emerge. Let us consider a unit volume dV which is general for any point of the construction layers of an airfield pavement restrained by the deflection bowl. Under an aircraft wheel load P the complete deformations $d\{\varepsilon\}$ progress in the unit volume.

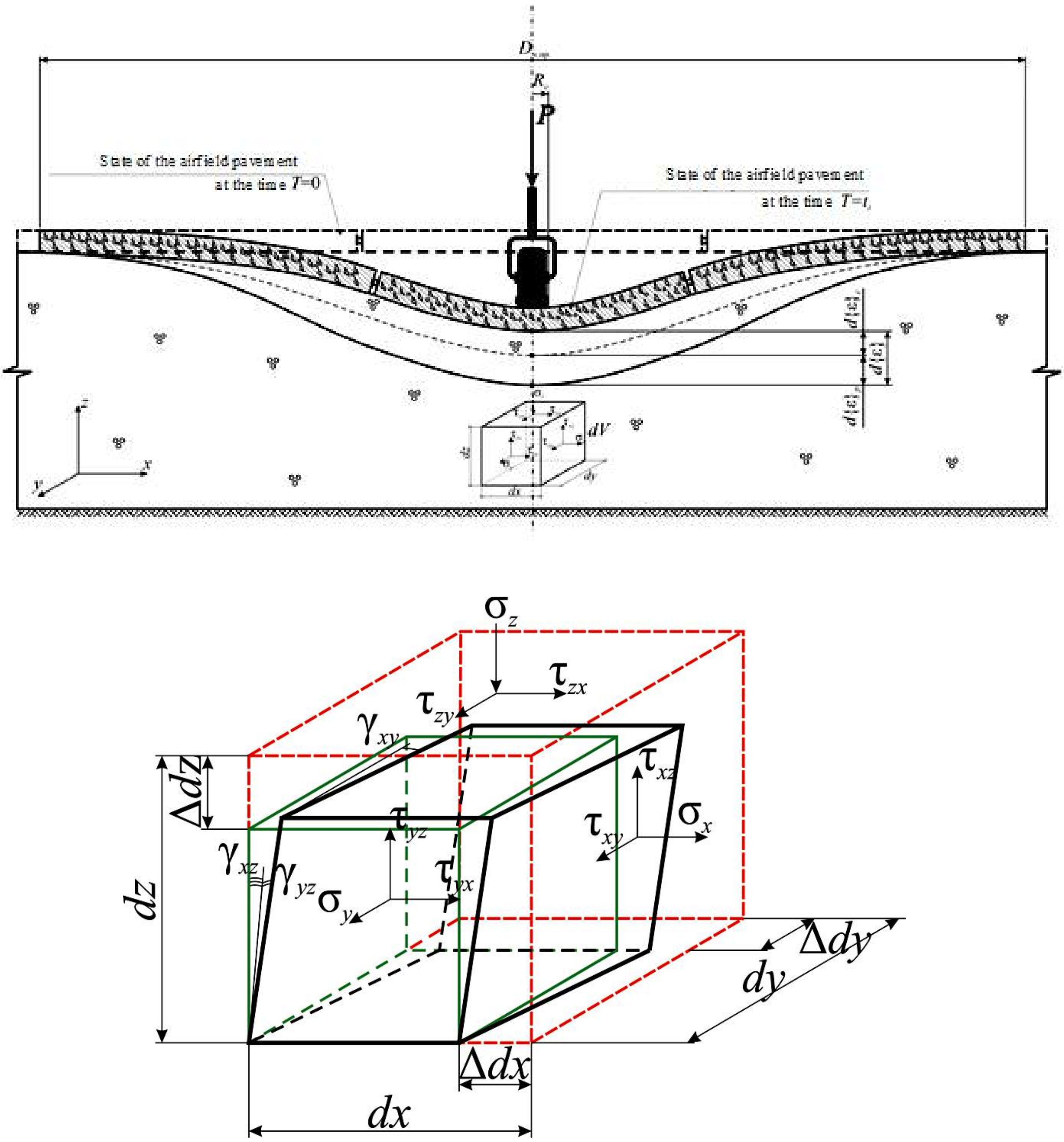

Fig. 1. Schematic of a multi-layer airfield pavement under an aircraft wheel load: $D_{ч.пр.}$ is the diameter of the deflection bowl; R_e is the radius of the circle as large as the area of wheel track image; $d\{\varepsilon\}_e$ is an increase in the elastic deformations; $d\{\varepsilon\}_p$ is an increase in the plastic deformations

The total deformation increase $d\{\varepsilon\} = \{d\varepsilon_x\ d\varepsilon_y\ d\varepsilon_z\ d\gamma_{xy}\ d\gamma_{yz}\ d\gamma_{xz}\}$ is

$$d\{\varepsilon\} = d\{\varepsilon\}_e + d\{\varepsilon\}_p, \quad (1)$$

where $d\{\varepsilon\}_e$ is an increase in the elastic deformations; $d\{\varepsilon\}_p$ is an increase in the plastic deformations.

Under consecutive and subsequent loads there will be elastic deformations in the pavement if the stresses are not over the achieved ones $\{\sigma\}_A$ and accompanied by increasing residual deformations $d\{\varepsilon\}_p$ if they are. The stresses occurring in the airfield construction layers achieved under consecutive loads following prior unloading is the elastic boundary, i.e. the yield point.

The elastic deformation increase is given by stress increases using the general Hooke's law [2]:

$$d\{\varepsilon\}_e = [D]^{-1} d\{\sigma\}, \tag{2}$$

where $[D]$ is the original elasticity matrix of the material; $d\{\sigma\} = \{d\sigma_x\ d\sigma_y\ d\sigma_z\ d\tau_{xy}\ d\tau_{yz}\ d\tau_{xz}\}$ is a stress increase vector.

Increases in plastic deformations are determined by the ratio [2]:

$$d\{\varepsilon\}_p = \lambda \frac{\partial F}{\partial\{\sigma\}}, \tag{3}$$

where λ is the plastic coefficient; $F = F(\{\sigma\},k)$ is the plastic potential (scalar function of the principal stresses and strengthening parameter k describing the loading history).

Plastic deformations are progressing along the surface normal determined by the plastic potential F within the range of principal stresses. The multiplier $\partial F/\partial\{\sigma\}$ characterizes the direction and the scalar parameter λ an increase in the plastic deformations.

Inserting the expressions (2) and (3) into (1), we get the law for increasing complete deformations in airfield pavements under an aircraft wheel load:

$$d\{\varepsilon\} = [D]^{-1} d\{\sigma\} + \lambda \frac{\partial F}{\partial\{\sigma\}}. \tag{4}$$

Unlike the previously used dependencies for the deformations of an airfield pavement based on the theory of elasticity, the obtained law accounts for plastic deformations and as they progress, the resistance of the construction layers to the current loads changes.

Within the range of principal stresses, the yield point forms the yield surface given by

$$F\left(\{\sigma\}, k\right) = 0. \tag{5}$$

Plastic deformations occur when a direction trajectory travels along the yield surface, the inside the surface is elastic and the stress trajectory outside the surface is not acceptable. The shape of the surface is largely determined by the nature of a material and is rather complex.

In order to solve the equation (4), the equation of the yield surface needs to be grounded which can be expressed using the principal stresses or stress tensor components with the corresponding substitutions. The equations reflected on a number of strength criteria for fragile binding friction materials in a complex stress state and plasticity criteria for non-binding materials.

2. Selection and justification of the strength and plasticity criteria

A yield criterion of artificial paving is William-Warne criterion [3] used to predict deterioration or failure of binding friction materials in a complex stress state. This criterion suggests that a material is ideally elastic-plastic (with no strengthening), a specific surface is convex, continuously differentiated and adapted to check data in a low compression range.

A section of a specific surface according to William-Warne criterion as shown in Fig.2 in a deviatory plane has a shape of a curved triangle for which there are only two radii r_{at} and a_{rc} corresponding with the Lode angle θ = 0 and θ = π/3. A specific surface F is given by the expression

$$F = \sqrt{J_2} + \sqrt{\frac{5}{2}} r(\theta) \left(\frac{1}{3\chi} I_1 - f_c \right) = 0, \tag{6}$$

where

$$\chi = \frac{f_b f_t}{f_b f_c - f_t f_c};$$

J_2 is the second invariant of a deviatory part of the stress tensor; r (θ) is the function describing an elliptic segment at $0 \le \theta \le \pi/3$; f_{ace} is a specific one-axial tensile strength; f_t is a specific one-axial tensile strength; f_{ib} is a specific two-axial compressive strength; I_1 is the first invariant of the stress tensor.

The following physical and mathematical characteristics are necessary for the calculation: the elasticity modulus E, Poisson coefficient μ, specific one-axial tensile strength f_t, specific one-axial compressive strength f_{ace}, specific two-axial compressive strength f_{ib}.

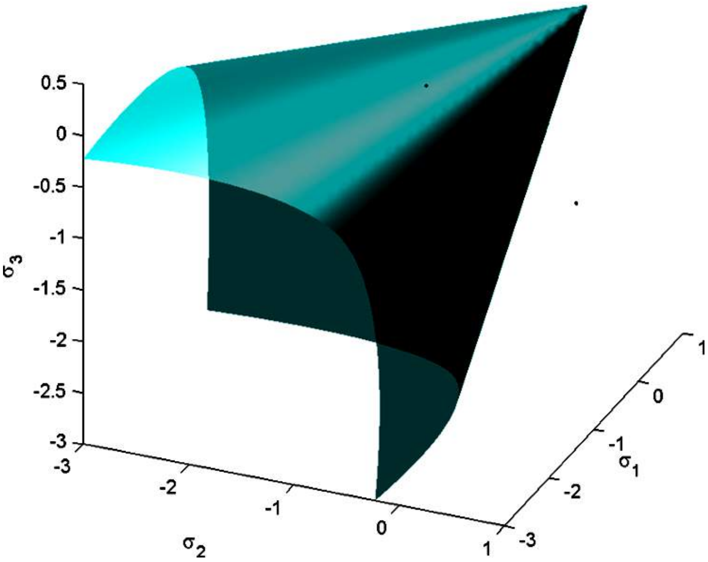

Fig. 2. Specific surface in the principal stresses according to the *Willam-Warnke* yield criterion

Unlike artificial paving materials, the subgrade material is porous and experiences plastic deformations in a deviatoric as well as hydrostatic component of the stress state. The elastic deformation area as shown in Fig. 3 in the principal stresses is restrained by the yield surface F_1 in relation to a deviatoric component and F_2 in relation to the hydrostatic one.

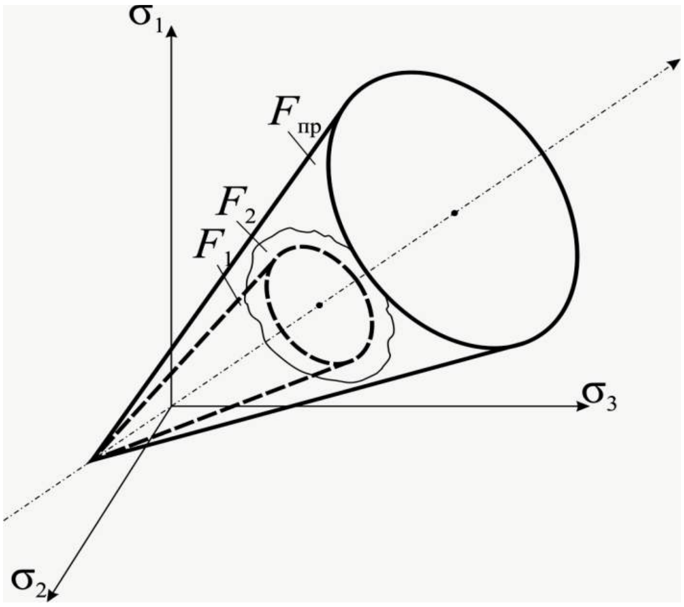

Fig. 3. Yield surfaces *Drucker-Prager*: F_{np} is a specific surface

The yield criterion *Drucker-Prager* [4] is best at describing a physical non-linearity of the subgrade base. The shape of the surface is given by the equation

$$F = \sqrt{J_2} + \frac{2}{\sqrt{3}} \frac{\sin\varphi}{(3 \pm \sin\varphi)} I_1 - \frac{2\sqrt{3} c \cos\varphi}{(3 \pm \sin\varphi)}, \quad (7)$$

where J_2 is the second invariant of a deviatoric part of the stress tensor; I_1 is the first invariant of the stress tensor; c is the specific adhesion coefficient; φ is the internal friction angle.

The following physical and mechanical characteristics of subgrades are necessary for the calculations: a deformation modulus E, Poisson coefficient μ, a specific adhesion coefficient c and the internal friction angle φ.

3. Calculation of the elastic-plastic model using the finite element method

An analytical calculation of multi-kayer airfield pavements considering actual boundary conditions and physical non-linearity of materials of construction layers is daunting due to elaborate systems of differential equations. Therefore it is performed using approximated analytical solutions based on the theory of elasticity. The use of numerical methods allows for a more accurate solution of the elastic-plastic model of airfield pavements under complex boundary conditions.

The most common numerical method is the finite element method for obtaining solutions using software implementing it, concentrate a finite element net in the expected area of high gradients of the investigated parameters, specify any boundary conditions, physical and mechanical properties of materials, loading parameters, etc.

The viability of the model is largely determined by a form of a finite element by which the geometry of the calculation model is approximated.

Finite elements of ferroconcrete structures are normally based on applied technical theories allowing a transition from a three-dimensional to a two-dimensional task, which makes the calculation easier. The main dependencies of finite elements were obtained based on the mechanics of composites. This is only consistent with the results for the structure where a width-height ratio is not significant. Structures where the width is a lot larger than the height should be calculated using three-dimensional finite elements.

In the paper [5] a special volumetric finite element as shown in Fig.4, which is a modification of a standard finite element in the shape of a parallelepiped and based on the general principles and ratios of three-dimensional mechanics of a deformed solid body, was examined in order to model multi-layer structures. A matrix of mechanical characteristics of this element is calculated layer by layer, which makes it suitable for use in physically non-linear materials when stresses change non-linearly according to the linear law of deformation.

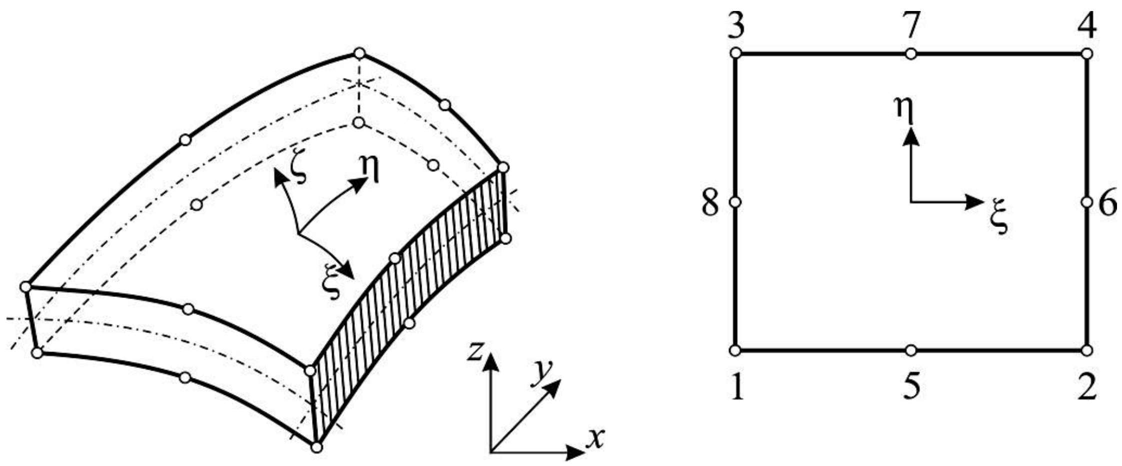

Fig. 4. Finite element in the shape of a curvilinear parallelepiped in a local normalized coordinate system (ξ, η, ζ)

The study of the stress state of multi-layer airfield pavements considering physical non-linearity of the properties of construction layer materials makes it necessary to solve non-linear algebraic equations.

A system of functions with displacement vector components

$$\{u\} = \{u\ \{x, y, z\}\ v\{x, y, z\}\ w\{x, y, z\}\},$$

stresses

$$\{\sigma\} = \{\sigma_x\{x, y, z\}\ \sigma_y\{x, y, z\}\ \sigma_z\{x, y, z\}\ \tau_{xy}\{x, y, z\}\ \tau_{yz}\{x, y, z\}\ \tau_{zx}\{x, y, z\}\}$$

and deformations

$$\{\varepsilon\} = \{\varepsilon_x\{x, y, z\}\ \varepsilon_y\{x, y, z\}\ \varepsilon_z\{x, y, z\}\ \gamma_{xy}\{x, y, z\}\ \gamma_{yz}\{x, y, z\}\ \gamma_{zx}\{x, y, z\}\}$$

are mathematically reduced to boundary task for a system of equations including

– balance equations:

$$[\Phi]^T \{\sigma\} = \{G_v\}, \quad (8)$$

– geometric equations (Cauchy):

$$\{\varepsilon\} = [\Phi]\{u\}, \tag{9}$$

– determining (physical) equations:

$$\{\sigma\} = [D]\{\varepsilon\}, \tag{10}$$

where $\{G_v\} = \{X\{x, y, z\}\ Y\{x, y, z\}\ Z\{x, y, z\}\}$ is a vector-function of volumetric forces; $[D]$ is a matrix of mechanical characteristics of a material sized 6×6; $[\Phi]$ is a matrix of differential operators:

$$[\Phi] = \begin{bmatrix} \partial/\partial x & 0 & 0 \\ 0 & \partial/\partial y & 0 \\ 0 & 0 & \partial/\partial z \\ \partial/\partial y & \partial/\partial x & 0 \\ 0 & \partial/\partial z & \partial/\partial y \\ \partial/\partial z & 0 & \partial/\partial x \end{bmatrix} \tag{11}$$

Solving the combination of (9), (10) in (8) in relation to unknown displacement, we get the following permittivity equation

$$[\Phi]^T [D][\Phi]\{u\} = \{G_v\}. \tag{12}$$

The ratios (10) are determined by a chosen model of the material and designed based on the general physical equations considering its major properties.

In order to solve a system of non-linear algebraic equations, it would be best to apply a step-wise method of increasing loads using the Newton-Raphson in each step.

As a result of solving the equation system by means of *COMSOL Multiphysics* the following are obtained:

– total deformations of a multi-layer airfield pavement (Fig. 5);

– normal stresses along the axis *x*, *y*, *z* (Fig. 6);

– tangent stresses in the planes *xy*, *xz*, *yz*;

– initiation areas of plastic deformations (Fig. 7);

– deviatoric part of the stress tensor along the axis *x*, *y*, *z*;

– stress state caused by the hydrostatic component;

– stresses in the reinforcement carcass of an airfield pavement (Fig. 8).

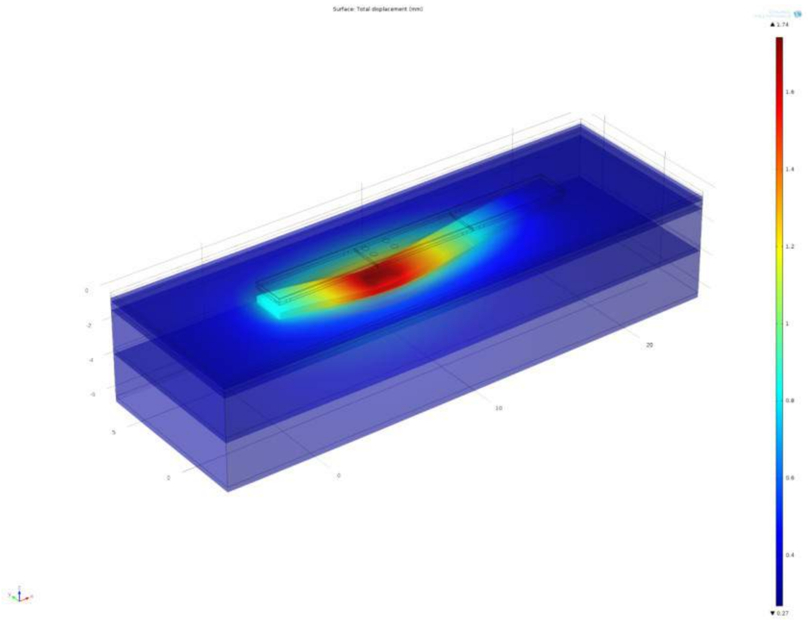

Fig. 5. Total deformations of a multi-layer airfield pavement

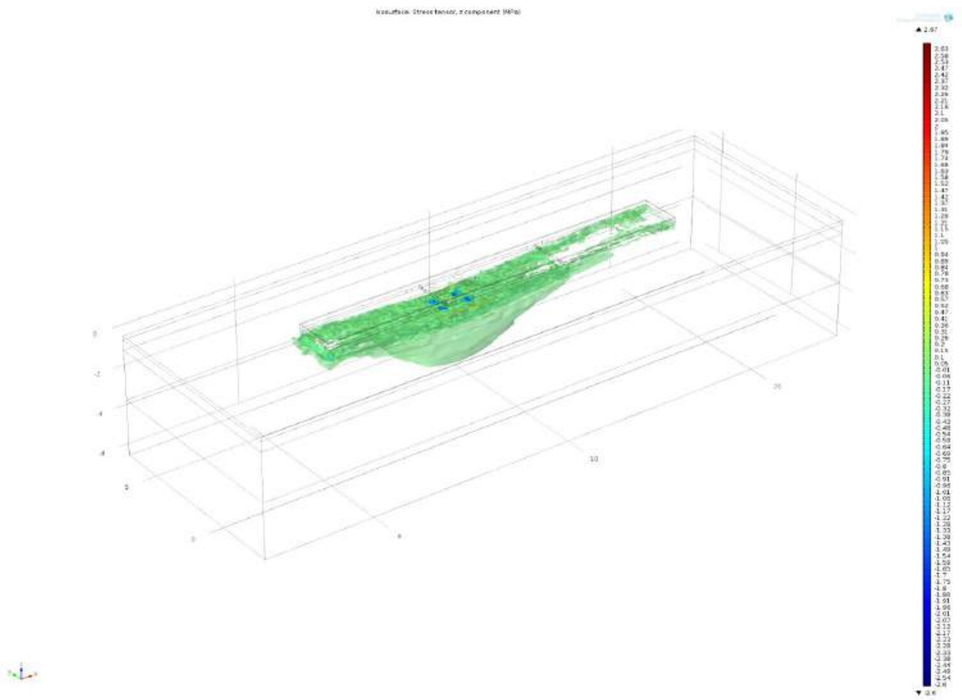

Fig. 6. Isofields of normal stresses σ_z

in the construction layer of a multi-layer airfield pavement

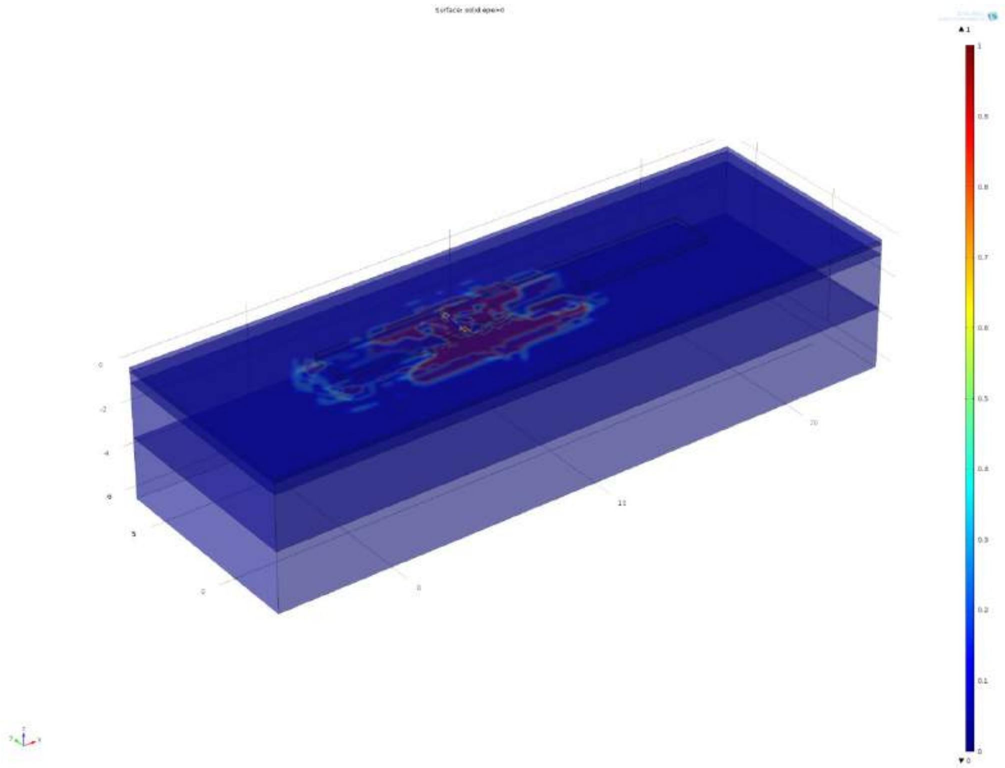

Fig. 7. Initiation areas of plastic deformations

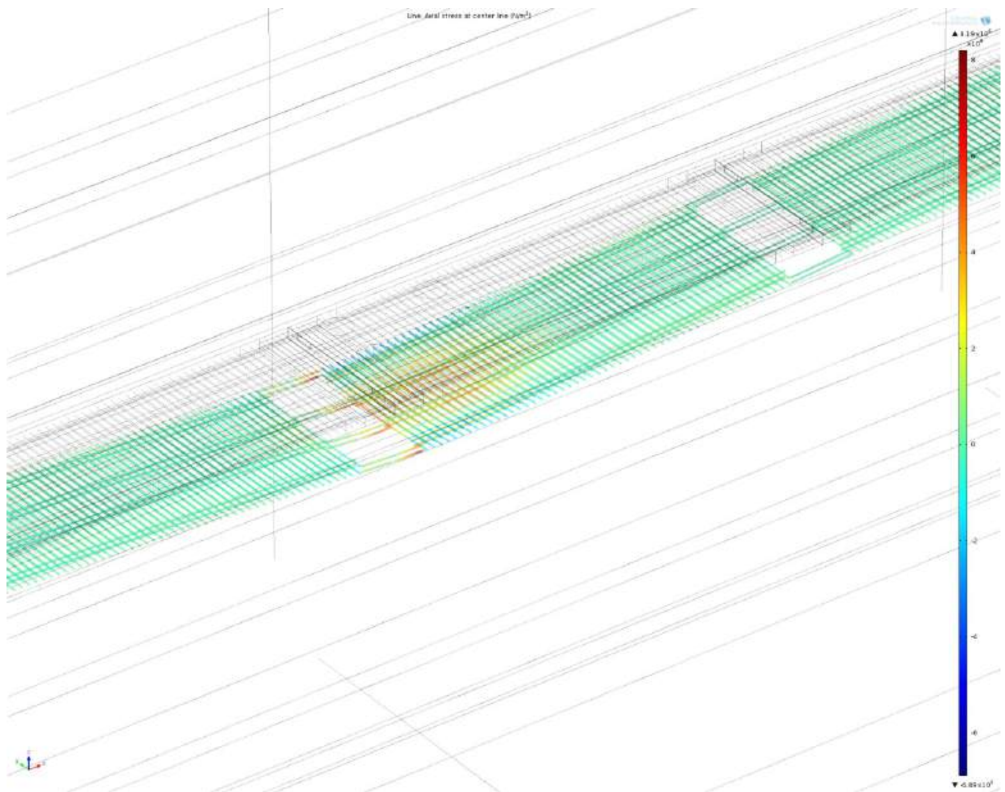

Fig. 8. Stresses in the reinforcement carcass
of previously stressed ferroconcrete airfield paving ПАГ–14V

Conclusions

1. The law was identified for determining deformations of airfield pavings which, unlike those previously used, accounts for plastic deformations and as they progress, the resistance of the construction layers to the current loads changes.

2. The yield and plasticity criteria have been used for materials of construction layers in order to describe the stress state of a multi-layer airfield paving. Plastic deformations occurring in fragile binding friction materials, which concrete is, are described by the yield criterion by *Willam-Warnke* and connected with a deviatoric part of the stress tensor. In non-binding materials such as a natural subgrade, plastic deformations are described by the yield criterion by *Drucker-Prager* and occur due to the deviatoric and hydrostatic component of the stress tensor.

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