HEAT AND GAS SUPPLY, VENTILATION, AIR CONDITIONING, GAS SUPPLY AND ILLUMINATION

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MATHEMATICAL MODEL OF CONVECTION HEAT TRANSFER WHEN CHARGING A HEAT ACCUMULATOR OF A HEAT SUPPLY SYSTEM

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Statement of the problem. Currently in order to improve the reliability of heat supply technologies of small energy sources are being implemented, e.g., small heating plant based on internal combustion engines. The thermal energy in such installations is mainly produced by utilizing the waste heat of outflow smoke gases having a high temperature and thus heating the water in the tank. It is important to search for the most adequate mathematical models allowing one to determine the time of heating of a coolant in tank-accumulators.
Results and conclusions. Mathematical models of unsteady processes of charging a storage tank used in a heating system are obtained. By means of identifying the models based on the minimization of the functional Gauss implemented in the algorithms, the most appropriate mathematical model was established. A simplified mathematical model of the charging process of a storage tank was obtained allowing one to determine the water temperature with an error of less than 8 %. Based on mathematical modeling, analytical dependences for determining the temperature of a heat carrier at the inlet and outlet of the storage tank are identified.

Keywords: heating, storage tank, heat transfer.

Introduction

One of Russia's top priorities is to improve its central heating system but their performance has to be improved by means of utilizing non-conventional energy sources [18]. One of the promising technologies to employ in energy supply are thermal power stations that cater for heat and energy demands.

These stations have the capacity from several kilowatts to dozens of megawatts [5] making their application range extremely wide. There is currently data available on how they are being implemented [6], but there are no reliable mathematical models to allow the temperatures of heat-carriers to be calculated over the operation time. An important issue is controlling the capacity of these stations in order to maintain the temperature modes within thermal networks [4, 7—9]. It becomes necessary to regard a mathematical model of charging a heat accumulator in the context of the operation in heat supply systems. Installing this kind of a device allows the daily hot water consumption to be more even.

1. Mathematical model of water heating using smoke gas

Water is heated by means of contact of smoke gas with the wall of the pipes of a heat exchanger operating in the cross mode (Fig. 1). Due to a difference in the temperatures and thus water density natural convection occurs. While circulating continuously, the water gradually heats up to a necessary temperature (95 °C). The convection rate u is not known beforehand.

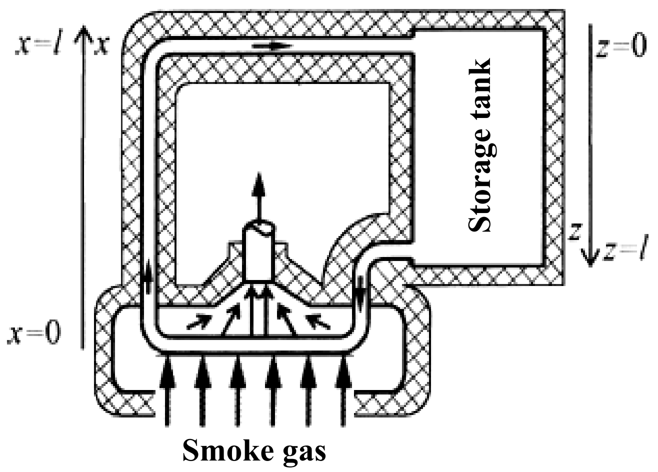

Fig. 1. Scheme of charging of the heat accumulator

Using the methods of engineering hydraulics [1] considering the movement mode (the Reynolds number) a maximum possible rate of a natural movement of water in a heat transfer pipe was determined. It is $u_{\max} = 0.424$ m/sec.

Let us write a mathematical model of heating water in the pipe whose side surface is thermally insulated and in the lower original part the heat is supplied from the smoke gas. At some point in time the rate of the outlet water flow in the pipe u can be assumed to be constant. The equation of heat conductivity is as follows

$$\frac{\partial T}{\partial \tau} + u\frac{\partial T}{\partial x} = a\frac{\partial^2 T}{\partial x^2} \qquad \left(0 < x < l,\ \tau > 0\right), \tag{1}$$

where T is the temperature, ºC; τ is the time, c; a is the coefficient of heat conductivity, m^2/sec; u is the rate of water, m/sec.

It is assumed that due to a small diameter of the pipe the temperature along the entire section is distributed evenly.

Let us introduce the following initial and boundary conditions for solving the equation (1). Counting the temperatures from T_0, the initial condition is as follows

$$T(x,0)=0 \qquad (0<x<l). \tag{2}$$

At the top end of the pipe $x = l$ the heat is transferred into the environment according to the Newton-Richman law (in the tank) with the coefficient of heat conductivity α:

$$\lambda \frac{\partial T}{\partial x}\bigg|_{x=l} = -\alpha \cdot T\bigg|_{x=l} \qquad (0 < \tau < 22 \text{ min}), \tag{3}$$

where α is the coefficient of heat conductivity, J/(m^2·sec); λ is the coefficient of heat conductivity of water J/(m·sec).

At the lower end of the pipe the water gets the heat from the smoke gas.

Due to an uncertainty the following options for the boundary conditions [19] for $x = 0$ and $x = l$ were considered:

1. $\lambda \frac{\partial T}{\partial x} = \alpha\left(T - T_{00}\right)$ for $x = 0$;

$\frac{\partial T}{\partial x} = 0$ for $x = l$;

2. $T = T_{00}$ for $x = 0$;

$\lambda \frac{\partial T}{\partial x} = -\alpha T$ for $x = l$;

3. $\lambda \frac{\partial T}{\partial x} = \alpha\left(T - T_{00}\right)$ for $x = 0$;

$\lambda \frac{\partial T}{\partial x} = -\alpha T$ for $x = l$,

where T_{00} is an unknown temperature of the wall of the pipe for $x = 0$.

The analytical solution of the equation (1) using the boundary and initial conditions (2) was obtained by means of Fourier's method (dividing the variables):

$$T(l,\tau) = T_{00}\left[1 - 2Nu\sum_{n=1}^{\infty} e^{-a\frac{\mu_n^2}{l^2}\tau} \frac{\cos\frac{\mu_n}{l}(l-u\tau) + \frac{Nu}{\mu_n}\sin\frac{\mu_n}{l}(l-u\tau)}{\mu_n^2 + Nu^2 + Nu}\right], \tag{4}$$

where μ_n are positive roots of the resulting characteristic equation such as

$$tg\mu_n = Nu / \mu_n, \tag{5}$$

where $Nu = \frac{\alpha l}{\lambda}$ is the Nusselt number.

As seen from Formula (4), the calculation value $T(l, \tau)$ depends on three parameters: T_{00}, Nu and u.

2. Identification of mathematical models

The resulting model should be identified, i.e. such numerical values of its parameters should be determined so that the models would be in good agreement with the empirical data [15, 16]. The structural scheme of the calculation of the identification of the model is given in Fig. 2.

Block 1. Introducing the original data: the initial temperature T_0; the coordinate x where the temperatures were measured at different points in time; the empirical time range $\tau(j)$ and a corresponding empirical temperature range $T_{on}(j)$; the values of the parameters of the model: the convection rate u, the Nusselt number Nu, the temperature T_{00}.

Block 2. The solution for the specified Nusselt number Nu of the characteristic equation (5) using the algorithm (9)—(10).

Block 3. Calculating the temperature $T(l, \tau(j))$ for each point in time $\tau(j)$ using the calculation scheme (4).

Block 4. Calculating the efficiency criterion, i.e. the Gaussian function that corresponds with the values of the parameters of the model u, Nu, T_{00}.

Block 5. Minimizing the Gaussian function by combining the methods of coordinate and shortest descend that includes planning the calculational experiment; going back to Block 1 to change the numerical value of the parameters u, Nu, T_{00}; switching to Block 6 as soon as the minimum value is achieved [12, 14].

Block 6. Comparing the optimal value of the Gaussian function with a dispersion of the reproducibility using the Fisher statistical criterion [20]. Introducing the mathematical model, i.e. identifying numerical values of the parameters u, Nu, T_{00}, in case of efficiency. Switching to Block 7.

Block 7. Printing the optimal values u, Nu, T_{00} and calculation values of the temperatures $T(x, \tau(j))$. Abandoning the mathematical model in case it is not efficient.

In order to identify the roots μ_n we use Newton's iteration [10]:

$$\mu_n^{нов} = \mu_n^{cm} - \frac{g\left(\mu_n^{cm}\right)}{g'\left(\mu_n^{cm}\right)}, \tag{6}$$

where using g and g' the following functions are denoted

$$g = \mu_n^{cm} tg \mu_n^{cm} - Nu \tag{7}$$

and its derivative

$$g' = tg\mu_n^{cm} + \frac{\mu_n^{cm}}{\cos^2 \mu_n^{cm}}. \tag{8}$$

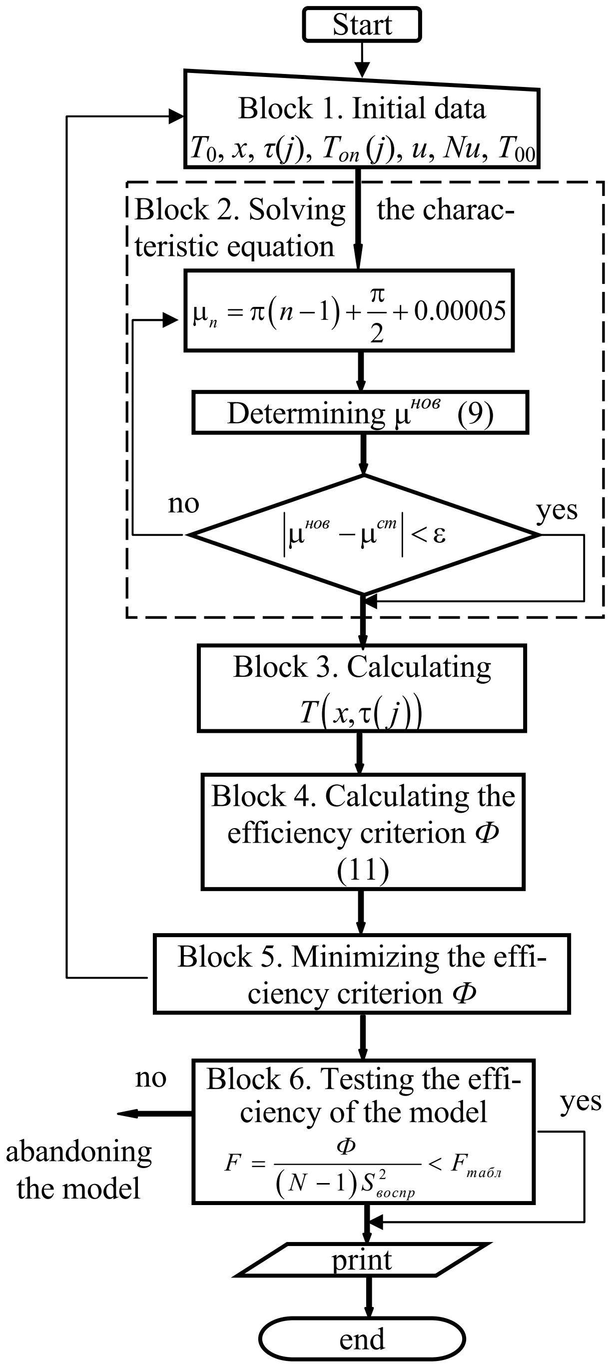

Fig. 2. Structural scheme of the identification of the mathematical model

Hence the iteration looks as follows

$$\mu_n^{нов} = \left(\mu - \frac{0.5\mu \sin 2\mu - Nu\cos^2 \mu}{0.5\sin 2\mu + \mu} \right)_{\mu=\mu^{cm}}. \tag{9}$$

The initial μ_n is specified using the range

$$\pi(n-1) < \mu_n < \pi(n-1) + \frac{\pi}{2}.$$

The calculation ends as soon as the previously specified accuracy (error) ε is achieved, e.g., when the following equality holds true

$$\left| \mu^{нов} - \mu^{cm} \right| < 0.000001 = \varepsilon. \tag{10}$$

The criterion for the efficiency of the model was the Gaussian function that is the following for the first option

$$\Phi = \sum_{j=1}^{N} \left[T_{on}(j) - T_{00} V(j) \right]^2, \tag{11}$$

where the expression in the square brackets of the right part of the formula (4) is denoted using $V(j)$ for the experimental value $\tau = \tau(j)$; N is the number of the experimental measurements $\tau(j)$ and $T_{on}(j)$.

Differentiating the function (11) using T_{00} and equating the derivative to zero we get

$$T_{00} = \frac{\sum_{j=1}^{N} T_{on}(j) V(j)}{\sum_{j=1}^{N} V(j) V(j)}. \tag{12}$$

The identification using the scheme (see Fig. 2) was performed for all the options of the model using the algorithmic language. The results are presented below.

Option 1. The characteristic equation is (5), the solution is (4). The optimal values are T_{00} = 12.64861, Nu = 0.9997329, u_1 = 0, u_2 = 0.3059973, u_3 = 0.1100073. The Gaussian function is Φ = 737.8488.

Option 2. The characteristic equation is

$$\mu_n ctg \mu_n = -Nu.$$

The solution is

$$T(x,\tau) = T_{00} \left[1 - 2\sum_{n=1}^{\infty} \frac{\mu_n^2 + Nu^2}{\left(\mu_n^2 + Nu^2 + Nu\right)\mu_n} \left(1 - \cos \mu_n\right) \sin \frac{\mu_n}{l} \left(x - u\tau\right) e^{-a\frac{\mu_n^2}{l^2}\tau} \right] \quad \text{for} \quad u \neq 0,$$

$$T\left(x,\tau\right)=T_{00}\left[1-\frac{Nu}{1+Nu}\cdot\frac{x}{l}-2\sum_{n=1}^{\infty}\frac{\mu_n^2+Nu^2}{\left(\mu_n^2+Nu^2+Nu\right)\mu_n}\sin\frac{\mu_n}{l}xe^{-a\frac{\mu_n^2}{l^2}\tau}\right] \quad \text{for} \quad u=0.$$

The optimal values are $T_{00}=-8.994679$, $Nu=0.500151$, $u_1=0$, $u_2=0.30388$, $u_3=0.101292$. The Gaussian function is $\Phi=3542.8$.

Option 3. The characteristic equation is

$$tg\mu_n=\frac{2Nu\mu_n^2}{\mu_n^2-Nu^2}.$$

The solution is

$$T\left(l,\tau\right)=T_{00}\left[1-2Nu\sum_{n=1}^{\infty}e^{-a\frac{\mu_n^2}{l^2}\tau}\left(1+\frac{\left(\mu_n^2+Nu^2\right)\cos\mu_n}{\mu_n^2-Nu^2}\right)\frac{\cos\frac{\mu_n}{l}\left(l-u\tau\right)+\frac{Nu}{\mu_n}\sin\frac{\mu_n}{l}\left(l-u\tau\right)}{\mu_n^2+Nu^2+2Nu}\right].$$

The optimal values are

$$T_{00}=16.00909,\; Nu=1.04837,\; u_1=0,$$
$$u_2=0.309943,\; u_3=0.0980236.$$

The Gaussian function is Φ = 1092.762.

Comparing three investigated options of the models we can see that the efficiency criterion (the sum of the roots of the deviations of the values of $T(x, l)$ from the calculated ones) has the smallest value for the first variant: Φ = 737.8488. The values of the unknown parameters of the mathematical model should be as according to the results for this particular variant, i.e.

$$T_{00}=12.69782,\; Nu=0.9997329,\; u_1=0,\; 0\le t\le 7 \text{ min},$$
$$u_2=0.3059973,\; 7<t\le 22 \text{ min},\; u_3=0.1100073,\; 22<t\le 92 \text{ min}.$$

Therefore the model of heating the water in a heat-transferring pipe is obtained and identified.

3. Calculating the temperature field in a storage tank

In order to calculate the temperature of the water in a storage tank depending on the time and coordinate, we solve the equation of heat conductivity [11, 13]:

$$\frac{\partial T_б}{\partial t}+u_б\frac{\partial T_б}{\partial z}=a\frac{\partial^2 T_б}{\partial z^2} \tag{13}$$

for a semi-restricted environment ($z \ge 0$) for the boundary condition

$$T_б\left(z,t\right)\Big|_{z=0}=f\left(t\right), \tag{14}$$

where $f(t)$ is the temperature of the water in the pipe at the storage tank inlet ($z = 0$) calculated based on the initial temperature T_0.

$$T_б\left(z,0\right)=T_0. \tag{15}$$

Further on the temperature of the water in a storage tank will be calculated based on T_0 so that at the initial point in time in the entire area

$$T_б(z,0)=0. \tag{16}$$

For the function $f(t)$ the dependence is previously obtained (4).

$$\begin{cases} T_{00}=12.6486; \quad Nu=0.9997; \\ u=u_1=0 \quad for \;\; 0\le t\le t_2=7 \text{ min}; \\ u=u_2=0.306 \text{ m/sec} \quad for \;\; t_2\le t\le t_3=22 \text{ min}; \\ u=u_3=0.11 \text{ m/sec} \quad for \;\; t_3\le t\le t_4=92 \text{ min}. \end{cases}$$

In the equation (13) the rate of the water in the tank is denoted using $u_б$:

$$u_б = k_б \cdot u \tag{17}$$

for a continuous flow

$$k_б=k_S=\frac{S_{mp}}{S_б}, \tag{18}$$

where S_{mp}, $S_б$ is the area of the heat-transferring pipe and tank respectively, m^2.

Using the influence function [2] the solution of the equation (13) for the boundary condition (14) and the initial condition (16) is as follows

$$T_б(z,t)=\frac{1}{2\sqrt{\pi a}}\int_{t_0}^{t}\frac{z-u_б(t-\tau)}{(t-\tau)^{3/2}}e^{-\frac{[z-u_б(t-\tau)]^2}{4a(t-\tau)}}\cdot f(\tau)d\tau. \tag{19}$$

The integration (19) was performed numerically using the rectangle rule. The calculation of the temperature of the water in the tank according to the formulae (19), (4) is implemented using the algorithmic language. The statistical evaluation based on the Fisher criterion shows that the equation (19) is good at describing the dependence the temperature of the water at the storage tank outlet on the time. Note that the function $f(t)$ as well as the experimental values of the temperature of the water in the pipe at the tank inlet is approximated using the linear dependence

$$f(t)=A+Bt, \tag{20}$$

where A = 75.968, B = 0.1391, for t > 22 min.

Then for solving the task (13) for the conditions (14), (15) we use the analytical dependence obtained [3, 17] by means of the operational method (the Laplace transformation) that is as follows:

$$\begin{aligned} T_б(z,t)&=\frac{1}{2}A\left\{\Phi^*\left[\frac{z-u_б t}{2\sqrt{at}}\right]+e^{\frac{u_б z}{a}}\Phi^*\left[\frac{z+u_б t}{2\sqrt{at}}\right]\right\}+ \\ &+\frac{B}{2u_б}\left\{(z+u_б t)e^{\frac{u_б z}{a}}\Phi^*\left[\frac{z+u_б t}{2\sqrt{at}}\right]-(z-u_б t)\Phi^*\left[\frac{z-u_б t}{2\sqrt{at}}\right]\right\}+t. \end{aligned} \tag{21}$$

In (21) an extra error function is denoted using $\Phi^{*}(x)$:

$$\Phi^{*}(x) = 1 - \Phi(x), \tag{22}$$

$$\Phi(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-\eta^2} d\eta \,. \tag{23}$$

The error function (the probability integral) (23) can be presented as an evenly convergent series:

$$\Phi(x) = \frac{2}{\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)\cdot n!}. \tag{24}$$

The experiment of calculating the error function shows that for $x < 1$ in order to obtain the results with the error of less than 0.000001 in the expansion (24), only 3 members have to be sustained, i.e. to approximate this function using the following polynomial:

$$\Phi(x) = \frac{2}{\sqrt{\pi}} \left[x - \frac{x^3}{3} + \frac{x^5}{10} \right]. \tag{25}$$

The results of the calculations of the temperature using the formulae (21) and (19) are no more than 8 % different from each other.

Fig. 3 shows the results of theoretical and experimental studies of charging the heat accumulator.

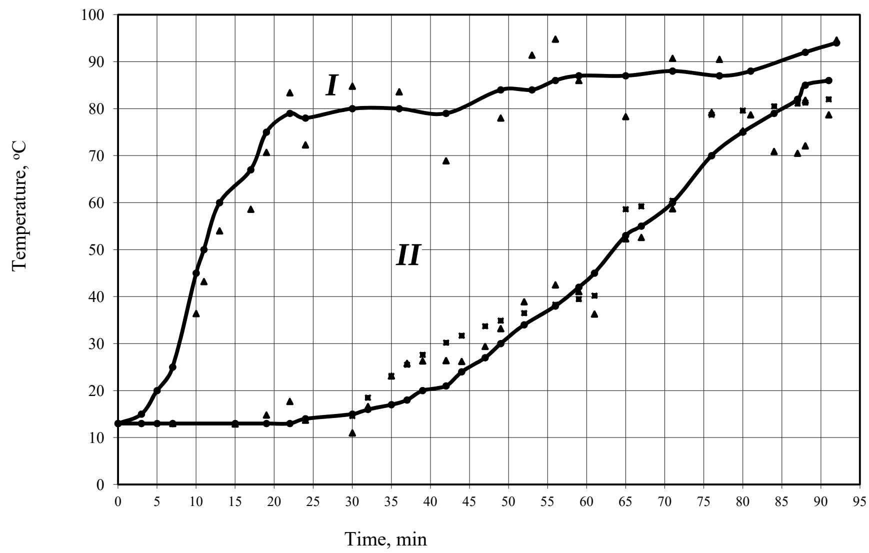

Fig. 3. Change in the temperature of the water in the storage tank:

I at the tank inlet; *II* at the tank outlet;

▲ is a theoretical one; ■ is an approximated one

Conclusions

1. The mathematical models to describe the convection heat exchange in the storage tank are developed. The optimal parameters when the model becomes sufficient for the experiment are obtained using identification.
2. The best model for heating water in a heat-transferring pipe using a Gaussian function as a criterion.
3. By means of mathematical modelling, analytical dependences to determine the temperature at the storage tank inlet and outlet are obtained.
4. A simplified mathematical model for charging the heat accumulator is also obtained.

References

1. Al'tshul'A. D., Zhivotovskiy L. S., Ivanov L. P. *Gidravlika i aerodinamika* [Hydraulics and aerodynamics]. Moscow, Stroyizdat Publ., 1987. 414 p.

2. Budak B. M., Samarskiy A. A., Tikhonov A. N. *Sbornik zadach po matematicheskoy fizike* [Collection of problems on mathematical physics]. Moscow, Nauka Publ., 1972. 688 p.

3. Karslou G., Eger D. *Teploprovodnost' tverdykh tel* [The thermal conductivity of solids]. Moscow, Nauka Publ., 1964. 488 p.

4. Kitaev D. N. Vliyanie sovremennykh otopitel'nykh priborov na regulirovanie teplovykh setey [The impact of modern heating devices for regulation of heat networks]. *Nauchnyy zhurnal. Inzhenernye sistemy i sooruzheniya*, 2014, vol. 2, no. 4 (17), pp. 49—55.

5. Kitaev D. N., Kurnosov A. T. Issledovanie znacheniy kpd mini-TETs. [The study values the efficiency of CHP plants]. *Vestnik Voronezhskogo gosudarstvennogo tekhnicheskogo universiteta*, 2008, vol. 4, no. 12, pp. 71—73.

6. Kitaev D. N., Zolotarev A. V., Shestykh N. S. Perspektivnye skhemy ispol'zovaniya kogeneratsionnykh ustanovok v sistemakh teplosnabzheniya [The future plan of the use of cogeneration in heat supply systems]. *Nauchnyy zhurnal. Inzhenernye sistemy i sooruzheniya*, 2012, no. 2 (7), pp. 26—29.

7. Kitaev D. N. Pogreshnost' rascheta temperaturnogo grafika teplovoy seti pri ispol'zovanii pokazateley otopitel'nykh priborov [The error of calculation of the temperature schedule of the heat network during the use of the indicators of heating appliances]. *Promyshlennaya energetika*, 2013, no. 7, pp. 37—37.

8. Kitaev D. N. Raschet temperatury naruzhnogo vozdukha v tochke izloma temperaturnogo grafika [Calculation of air temperature in the inflection point temperature chart]. *Novosti teplosnabzheniya*, 2012, no. 10 (146), pp. 46—48.

9. Kitaev D. N. Sovremennye otopitel'nye pribory i ikh pokazateli [The modern heating devices and their performance]. *Santekhnika, otoplenie, konditsionirovanie, energosberezhenie*, 2014, no. 1, pp. 48—49.

10. Lapchik M. P., Ragulina M. I., Khenner E. K. *Chislennye metody* [Numerical methods]. Moscow, Akademiya Publ., 2004, 384 p.

11. Mel'kumov V. N., Kuznetsov S. N. Vzaimodeystvie ventilyatsionnykh vozdushnykh potokov s konvektivnymi potokami ot istochnikov teploty [The interaction of ventilation air flows with convection flows from heat sources]. *Izvestiya vuzov. Stroitel'stvo*, 2009, no. 1, pp. 63—69.

12. Mel'kumov V. N., Kuznetsov I. S., Kobelev V. N. Vybor matematicheskoy modeli trass teplovykh setey [The choice of the mathematical model tracks thermal networks]. *Nauchnyy vestnik Voronezhskogo GASU. Stroitel'stvo i arkhitektura*, 2011, no. 2, pp. 31—36.

13. Mel'kumov V. N., Kuznetsov I. S. Dinamika formirovaniya vozdushnykh potokov i poley temperatury v pomeshchenii [Dynamics of formation of air streams and temperatures fields in premise]. *Nauchnyy vestnik Voronezhskogo GASU. Stroitel'stvo i arkhitektura*, 2008, no. 4, pp. 172—178.

14. Mel'kumov V. N., Kuznetsov I. S., Kobelev V. N. Zadacha poiska optimal'noy struktury teplovykh setey [The task of finding the optimal structure of heat networks]. *Nauchnyy vestnik Voronezhskogo GASU. Stroitel'stvo i arkhitektura*, 2011, no. 2, pp. 37—42.

15. Mel'kumov V. N., Loboda A. V., Chuykin S. V. Matematicheskoe modelirovanie vozdushnykh potokov v pomeshcheniyakh bol'shikh ob"emov [Mathematical modelling of air flows in premises of large volumes]. *Nauchnyy vestnik Voronezhskogo GASU. Stroitel'stvo i arkhitektura*, 2014, no. 2 (34), pp. 11—18.

16. Mel'kumov V. N., Kuznetsov I. S. Matematicheskoe modelirovanie poley kontsentratsiy vrednykh veshchestv pri proizvodstve stroitel'nykh materialov [Mathematical modeling of fields of concentrations of harmful substances in the manufacture of building materials]. *Nauchnyy vestnik Voronezhskogo GASU. Stroitel'stvo i arkhitektura*, 2013, № 1 (29), pp. 99—107.

17. Mel'kumov V. N., Kuznetsov S. N., Gulak V. V. Modelirovanie zadymlennosti pomeshcheniy slozhnoy konfiguratsii v nachal'noy stadii pozhara [Modeling of smoke content of premises of complex configuration at initial stage of fire]. *Nauchnyy vestnik Voronezhskogo GASU. Stroitel'stvo i arkhitektura*, 2010, no. 3, pp. 131—138.

18. Mel'kumov V. N., Chuykin S. V., Papshitskiy A. M., Sklyarov K. A. Modelirovanie struktury inzhenernykh setey pri territorial'nom planirovanii goroda [Modeling of the structure of utility networks the regional planning city]. *Nauchnyy vestnik Voronezhskogo GASU. Stroitel'stvo i arkhitektura*, 2015, no. 2 (38), pp. 41—48.

19. Muchnik G. F., Rubashov I. B. *Metody teorii teploobmena. Teploprovodnost'* [Methods of the theory of heat transfer. The thermal conductivity]. Moscow, Vysshaya shkola Publ., 1970. 288 p.

20. Sheffe G. *Dispersionnyy analiz* [Analysis of variance]. Moscow, Nauka. Gl. red. fiz.-mat. lit. Publ., 1980. 512 p.

21. Hadia N. M. A., Ryabtsev S. V., Seredin P. V., Domashevskaya E. P. Effect of the temperatures on structural and optical properties of tin oxide (SnO_x) powder. Physica B: Condensed Matter, 2010, vol. 405, iss. 1, pp. 313–317. doi: 10.1016/j.physb.2009.08.082.