BUILDING MECHANICS

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Yu. P. Nazarov[1], E. V. Poznyak[2]

THEORY OF A QUASI-STATIC ANALYSIS OF SPORT GRANDSTANDS UNDER LOADS FROM CONCERTED ACTIONS OF SPECTATORS

Central Research Institute of Building Structures Named after V. A. Kucherenko
(Federal State Unitary Enterprise Scientific Research Centre «Construction»)
Russia, Moscow, e-mail: nazarov-dom@mail.ru
[1]D. Sc. in Engineering, Prof., Member of the Russian National Committee on Theoretical and Applied Mechanics, Council of the Russian Academy of Architecture and Building Sciences, Head Research Fellow
National Research University «Moscow Power Engineering Institute»
Russia, Moscow, e-mail: PozniakYV@mpei.ru
[2]PhD in Engineering, Assoc. Prof. of the Dept. of Dynamics and Machinery Performance Named after V. I. Bolotin

Statement of the problem. This paper presents deterministic and stochastic methods for a quasi-static structural calculation of the influence of sporting stands on a semi-sinusoid impulse pulse load of audiences moving concertedly.

Results. Deterministic and probabilistic solutions are obtained for specific concerted load, i. e. by walking and jumping up. The study found a connection between the deterministic and probabilistic approaches. Both of the solutions are tested by calculating in a time domain. A method of the assessment of vibrations is described, which is perceived by an audience in accordance to maximum displacements and accelerations.

Conclusions. This study has shown such features of pulse loads as simultaneous excitation of several forms of oscillations and high dynamic factors. In case of high vibrations it is necessary to evaluate a dynamic comfort level. The results of the study can be useful for updating existing guidelines on loads and impacts.

Keywords: quasi-static analysis, impulse load, sporting stands, human effect on a structure, amplification factors, estimation of a dynamic comfort level.

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Introduction

Modern architectural forms of sport and public cultural constructions, which are suspended or console structures, call for calculations for a new type of dynamic impacts — human-structure interaction loads.

These impacts are due to synchronous movement of people and can be characterized as low-frequency periodic surface loads. If eigenfrequencies of stands are close to that of impulses generated by spectators, this might lead to an increased vibration level, degradation of operational criteria and sometimes local damage [9—10, 13, 14].

In order to investigate the impact from spectator movement, there has been an extensive series of experiments and theoretical studies performed abroad [13, 14, 16—21] where the approximation of a dynamic load on a stand was obtained as a sequence of half-sinusoidal impulses:

$$F(t)=\begin{cases} K_p G \sin\left(\dfrac{\pi t}{t_p}\right), & 0 \le t \le t_p, \\ 0, & t_p \le t \le T_p, \end{cases} \quad (1)$$

where $K_p = F_{\max} / G$ is an impact factor; $F_{\max}$ is a peak load; G is the weight of a static spectator load on a stand; t_p is a contact period; T_p is an impulse period. The ratio of a contact period t_p to a load time T_p is a contact ratio $\alpha = t_p / T_p$, the impulse frequency is $f_p = 1 / T_p$. It was experimentally shown that the impulse frequency may vary from 1 to 4 Hz [13, 14, 21].

The approximation (1) is the foundation for guidelines of the UK, Canada, Germany and is illustrated in the Eurocode EN 1991-1-1 [12, 15, 4]. In the guideline BS 6399-1:1996 there is a correspondence of different types of contact ratios and those of spectator actions [5]: the value $\alpha = 2/3$ corresponds to pedestrian movement and low-rhythmic aerobics, the value $\alpha = ½$ to rhythmic movement and high-rhythmic the value $\alpha = 1/3$ to regular jumps and the value $\alpha = ¼$ to high jumps. The decomposition of the function $F(t)$ into a Fourier series [4] is known:

$$F(t)=G\left[1+\sum_{n=1}^{\infty} a_n \cos\frac{2\pi n}{T_p}t+\sum_{n=1}^{\infty} b_n \sin\frac{2\pi n}{T_p}t\right]=G\left[1+\sum_{n=1}^{\infty} r_n \sin\left(\frac{2\pi n}{T_p}t+\varphi_n\right)\right], \quad (2)$$

where a_n, b_n are the Fourier coefficients at $2n\alpha = 1$, $a_n = 0$, $b_n = \pi/2$, in other cases

$$a_n = 0.5\left[\frac{\cos(2n\alpha-1)\pi-1}{2n\alpha-1}-\frac{\cos(2n\alpha+1)\pi-1}{2n\alpha+1}\right],$$

$$b_n = 0.5\left[\frac{\sin(2n\alpha-1)\pi-1}{2n\alpha-1}-\frac{\sin(2n\alpha+1)\pi-1}{2n\alpha+1}\right],$$

$$r_n=\sqrt{a_n^2+b_n^2}, \quad \varphi_n = \operatorname{arctg}(a_n/b_n).$$

The analysis of the series (2) shows that the impulse load (1) is successfully modeled by the first six harmonics. In Fig. 1 there are graphs of the function *F*, kN/m^2, of the time, sec, obtained by summing the first six harmonics in (2) for $\alpha = 2/3$ (pedestrian movement) and $\alpha = 1/4$ (high jumps) with the frequency 2 Hz, a static crowd load is the maximum actual weight 2.80 kN/m^2.

A dynamic response of a structure to an impact (1) can be determined in a few ways. The main one is the quasi-static method as it is most convenient and known to engineering designers for its common use in seismic resistance. The quasi-static method is convenient as exterior impulse force is applied to a structure as a static load, internal effort does not depend on time, which makes construction calculations easier to perform. Dynamic effects in the quasi-static method are considered using dynamic factors. The quasi-static method can be regarded from the point of view of determined or probabilistic loading [19—21, 12]. In the first case all the parameters of a calculation model and loading are considered strictly prescribed and dynamic factors are determined using the methods of the oscillation theory. In the second case loading is considered as an implementation of a random process and dynamic factors depend on its spectral density. A test calculation will be direct integration of movement equations in a temporary range with the resulting dependencies of the parameters of stress-strain on time. Below are three approaches to solving a dynamic task (a quasi-static determined, quasi-static probabilistic and test in a temporary area) and they are compared. Besides determining a dynamic response, a comfort level is evaluated. Based on the frequencies and amplitudes of vibro-displacements and vibro-accelerations, 6 gradations of spectators' perception ranging from "vibrations are not felt" to "vibrations are not pleasant during a short-term impact".

a) b)

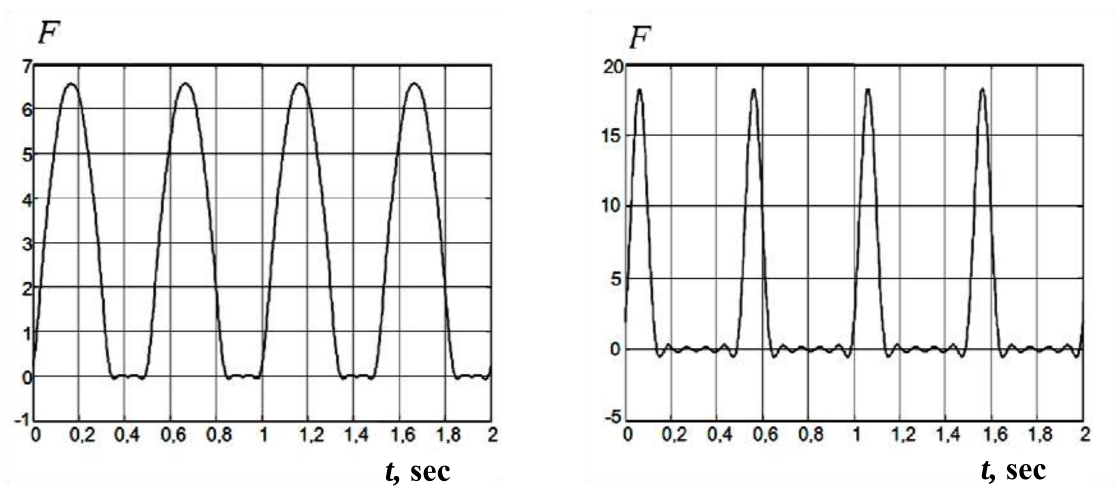


Fig. 1

1. Movement equations in the main coordinates

The movement equation of a dissipative system with N degrees of freedom takes the following form

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{B}\dot{\mathbf{q}} + \mathbf{C}\mathbf{q} = \mathbf{P}\,, \tag{3}$$

where q is a N-dimensional vector of generalized movements; M, B, C are matrices of inertia, damping and solidness of the dimension $N\times N$; P is a N-dimensional vector of generalized external impulse forces. The vector P consists of periodic forces P_i that can be arranged into a Fourier series using the formula (2):

$$P_i(t) = P_i^G\left[1 + \sum_{n=1}^{N_F} r_n \sin\left(\frac{2\pi n}{T_p}n + \varphi_n\right)\right], \tag{4}$$

where P_i^G is a static crowd load along the i-the generalized coordinate; N_F is a number of retained members of the Fourier series. As a vector

$$\mathbf{P} = \mathbf{P}^G + \mathbf{P}^G\sum_{n=1}^{N_F} r_n \sin\left(\frac{2\pi n}{T_p}n + \varphi_n\right).$$

Let us denote: $\mathbf{V}$ is a matrix of eigenvectors of the system (3) of the $N\times N$ dimension; $\boldsymbol{\Omega}^2 = \mathrm{diag}\left[\Omega_k^2\right]$ is a diagonal quadratic matrix of eigenfrequencies; Ω_k^2 is the square of the k-th eigenfrequency; M_{mod} is a diagonal modal matrix of the masses:

$$\mathbf{M}_{\mathrm{mod}} = \mathbf{V}^T\mathbf{M}\mathbf{V} = \mathrm{diag}\left[M_{\mathrm{mod},k}\right];$$

$M_{\mathrm{mod},k}$ is a modal mass along the k-th eigenform; $2\boldsymbol{\varepsilon} = \mathbf{M}_{\mathrm{mod}}^{-1}\mathbf{V}^T\mathbf{B}\mathbf{V}$ is a modal damping matrix. Damping is assumed to be small thus the damping matrix can be considered diagonal $\boldsymbol{\varepsilon} = \mathrm{diag}\left[\varepsilon_k\right]$, ε_k is a modal damping coefficient along the k-th eigenform. Using the transformation $\mathbf{q} = \mathbf{V}\mathbf{u}$ we obtain a system of independent movement equations in the space of the main coordinates:

$$\ddot{\mathbf{u}} + 2\boldsymbol{\varepsilon}\dot{\mathbf{u}} + \Omega^2\mathbf{u} = \mathbf{Q}\,, \tag{5}$$

where $\mathbf{u}$ is a vector of the main coordinates; Q is a vector of external impulse forces reduced to the main coordinates:

$$\mathbf{Q} = \mathbf{M}_{\mathrm{mod}}^{-1}\mathbf{V}^T\mathbf{P}\,.$$

In the component-wise form the equation (5) is as follows

$$\ddot{u}_k + 2\varepsilon_k\dot{u}_k + \Omega_k^2 u_k = Q_k\,, \tag{6}$$

where $$Q_k = \frac{1}{M_{\text{mod},k}} \mathbf{v}_k^T \mathbf{P};$$

$\mathbf{v}_k^T$ is a vector of the *k*-th eigenform. The solution in the initial base for the *j*-th generalized coordinate is

$$q_j = \mathbf{v}_j^T \mathbf{u}.$$

2. Quasi-static determined solution

The solution of the equation (5) in an established mode of forced vibrations can be obtained by means of the superposition method considering the decomposition (4) and summing a dynamic response along each harmonic component of a load (the detailed development is provided in [5]):

$$u_k = \frac{1}{\Omega_k^2 M_{\text{mod},k}} \mathbf{v}_k^T \mathbf{P}^G \left(1 + \sum_{n=1}^{N_F} \frac{r_n \sin\left(n\theta t + \psi_n\right)}{\sqrt{\left(1 - \frac{n^2\theta^2}{\Omega_k^2}\right)^2 + \left(\frac{2\varepsilon_k n\theta}{\Omega_k^2}\right)^2}} \right), \tag{7}$$

where ψ_n is a response phase to the *n*-th harmonic component of a load:

$$\psi_n = \varphi_n - \text{arctg}\left(\frac{2\varepsilon_k n\theta}{\Omega_k^2} \Big/ \left(1 - \frac{n^2\theta^2}{\Omega_k^2}\right)\right);$$

θ is an angular frequency of an impulse load:

$$\theta = \frac{2\pi}{T_p} = 2\pi f_p.$$

The summand (7) that is independent of time corresponds to the static movement u_{st}. A maximum dynamic movement along the *k*-th generalized coordinate $u_{\max,k}$ will be found by assuming the sinuses (7) to equal one. Then the modal dynamic factor corresponding to the *k*-th form of oscillations is

$$\beta\left(\Omega_k\right) = \frac{u_{\max,k}}{u_{st}} = 1 + \sum_{n=1}^{N_F} \frac{r_n}{\sqrt{\left(1 - \frac{n^2\theta^2}{\Omega_k^2}\right)^2 + \left(\frac{2\varepsilon_k n\theta}{\Omega_k^2}\right)^2}}.$$

This formula can be written using a modal damping coefficient ξ_k ($\varepsilon_k = \Omega_k \xi_k$) and the frequency expressed in Hz ($f_p = \theta/2\pi$, $f_k = \Omega_k/2\pi$):

$$\beta\left(f_k\right)=1+\sum_{n=1}^{N_F}\frac{r_n}{\sqrt{\left(1-\frac{n^2 f_p^2}{f_k^2}\right)^2+\left(\frac{2\xi_k n f_p}{f_k}\right)^2}}. \quad (8)$$

Fig. 2 shows the graphs of the dynamic factors in the axes of eigenfrequencies $\beta(f_k)$ at the impulse frequency f_p=2 Hz: a) for pedestrian movement $\alpha = 2/3$ and b) for high jumps $\alpha = 1/4$; both graphs are designed for 5 % damping (the grey line) and 2,5 % damping (the black line).

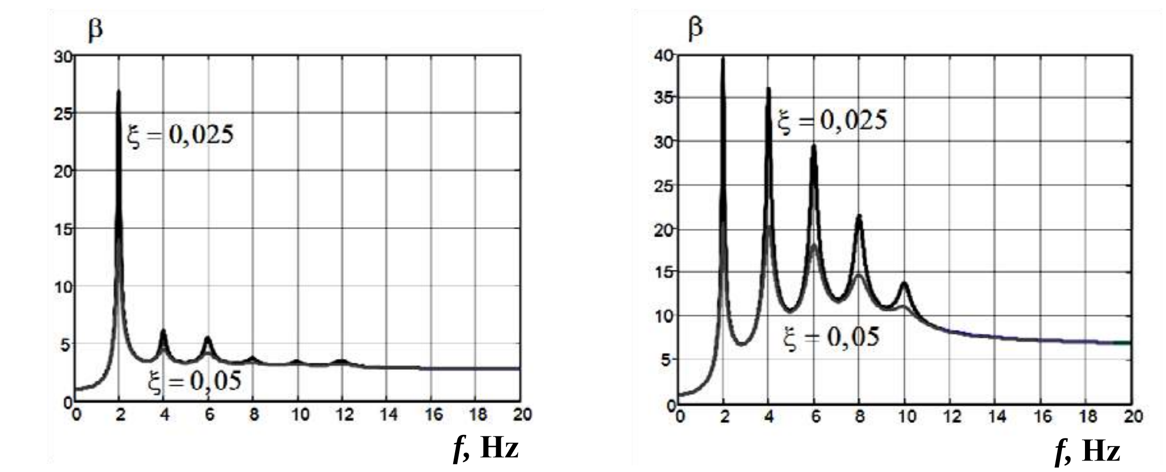


Fig. 2

3. Quasi-static probabilistic solution

A random process corresponding to an impulse load is narrow, i.e. it is determined for a discrete set of frequencies $n\theta$ using the formula (2). The spectral density of the impulse load is functions close to the δ-function along the coordinates corresponding with the impact frequencies $n\theta$, $n = 1, \ldots, N_F$. Fig. 3 shows the graphs of spectral densities S, $(kN/m^2)^2$/Hz corresponding with the impulse loads in Fig. 1 at the impulse frequency $f_p = 2$ Hz: a) for pedestrian movement $\alpha = 2/3$, b) for high jumps $\alpha = 1/4$.

In a temporary interval of active loading by a single impact a random process of spectator movement on a stand can be considered stationary, i.e. the average value and standard do not depend on time. The average value of a random process considering the ratios

$$K_p = \pi/2\alpha, \ \alpha = t_p/T_p$$

is

$$m_F = \frac{1}{T_p}\int_0^{T_p} F(t)\,dt = \frac{K_p G}{T_p}\int_0^{T_p}\sin\left(\frac{\pi t}{t_p}\right)dt = \frac{K_p G}{T_p}\cdot 2\frac{t_p}{\pi} = G. \quad (9)$$

Therefore a load from the coordinated crowd movement can be considered as a stationary random process with an average value of that of a static load G.

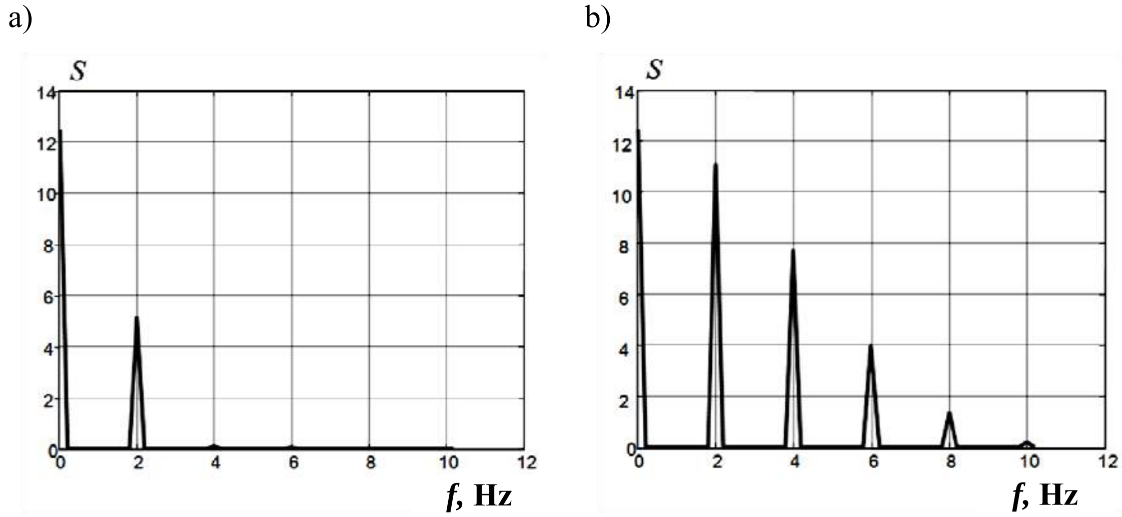


Fig. 3

The movement of the system (6) can be presented in the following way:

$$\ddot{u} + 2\varepsilon\dot{u} + \Omega^2 u = Q, \tag{10}$$

where $Q(t)$ is a stationary process at the input that corresponds with an external impact on the system; $u(t)$ is a stationary process at the output that corresponds with a response (movement) of a system.

If a spectral density of a stationary process at the input $S_Q(\theta)$ is known, a spectral density of a process at the output $S_u(\theta)$ for a linear stationary system (10) is given by the formula [1, 2]

$$S_u(\theta) = \frac{S_Q(\theta)}{\left(\Omega^2 - \theta^2\right)^2 + 4\varepsilon^2\theta^2}. \tag{11}$$

The concept of the dynamic factor is introduced similarly to determined established oscillations for harmonic loading. This factor shows by how many times a static load is to be increased so that dynamic effects are considered in the quasi-static approach.

Let us separate the constant multiplier G from the right part of the equation (10), which is an average value of a random load Q, then a generalized random force is

$$Q(t) = G\overline{Q}(t).$$

Based on linearity of the system (10),

$$u(t) = G\overline{u}(t),$$

where the random process $\bar{u}$ is movement from the force $\bar{Q}$. Then due to stationarity of the processes at the input and output the average square σ_u^2 (or the dispersion D_u) is

$$\sigma_u^2 = D_u = G^2 D_{\bar{u}}. \quad (12)$$

The dynamic factor β for random loads is determined as a ratio of the dynamic movement u to the static movement u_G (i.e. the movement from the static load G):

$$\beta = \frac{\sigma_u}{u_G}. \quad (13)$$

The square of the standard σ_u^2 is the dispersion of the process at the output D_u and is connected to the spectral densities $S_u(\theta)$ and $S_Q(\theta)$. Considering (12):

$$\sigma_u^2 = D_u = G^2 D_{\bar{u}} = G^2 \int_0^\infty S_{\bar{u}}(\theta)\, d\theta = G^2 \int_0^\infty \frac{S_{\bar{Q}}(\theta)}{\left(\Omega^2 - \theta^2\right)^2 + 4\varepsilon^2\theta^2} d\theta. \quad (14)$$

The static movement u_G does not depend on time and is determined using the equation (10) at $Q = G$:

$$u_G = G/\Omega^2. \quad (15)$$

The final formula for the dynamic factors of the system with one degree of freedom is obtained using the equation (13) considering (14) and (15):

$$\beta(\Omega) = \left(\int_0^\infty \frac{\Omega^4 S_{\bar{Q}}(\theta)}{\left(\Omega^2 - \theta^2\right)^2 + 4\varepsilon^2\theta^2} d\theta \right)^{\frac{1}{2}}.$$

For multidimensional systems when a dynamic factor is different for oscillations along each k-th eigenfrequency, it is assumed to be a function of the eigenfrequencies Ω_k:

$$\beta(\Omega_k) = \left(\int_0^\infty \frac{\Omega_k^4 S_{\bar{Q}}(\theta)}{\left(\Omega_k^2 - \theta^2\right)^2 + 4\varepsilon_k^2\theta^2} d\theta \right)^{\frac{1}{2}}.$$

This formula can be written using the modal damping coefficient ξ_k ($\varepsilon_k = \Omega_k \xi_k$) and the frequencies expressed in Hz ($S_{\bar{Q}}(\theta) = S_{\bar{Q}}(f)/2\pi$, $\theta = 2\pi f$, $\Omega_k = 2\pi f_k$):

$$\beta(f_k) = \left(\int_0^\infty \frac{S_{\bar{Q}}(f)}{\left(1 - \frac{f_p^2}{f_k^2}\right)^2 + 4\xi_k^2 \frac{f_p^2}{f_k^2}} df \right)^{\frac{1}{2}}. \quad (16)$$

Fig. 4 shows the graphs of the dynamic factors β(f_k) designed using the formula (16) for the frequency of the impulses f_p = 2 Hz: a) for pedestrian movement α = 2/3 and b) for high jumps α = 1/4. The black line corresponds to 2.5 % damping, the grey line to 5 % damping.

4. Test calculation using a temporary range and result analysis

Based on the example of the system with one degree of freedom, we will consider how the quasi-static solutions described in Section 3 and 4 correlate with the test calculation in a temporary range. The formula (7) for the established movements of a one-mass system (10) is as follows:

$$u(t)=\frac{G}{\Omega^2}\left(1+\sum_{n=1}^{N_F}\frac{r_n\sin(n\theta t+\psi_n)}{\sqrt{\left(1-\frac{n^2\theta^2}{\Omega^2}\right)^2+\left(\frac{2\varepsilon\, n\theta}{\Omega^2}\right)^2}}\right). \tag{17}$$

The test calculation is performed for a resonance mode when the frequency of an impulse load coincides with the eigenfrequency of the system: θ = Ω = 12.5664 rad/sec (2 Hz). Fig. 5 presents movements, m, as a time function, sec, for pedestrian movement (a) and high jumps (b) in the range of 10 sec.

The load G is assumed to be one. Using the graphs, let us determine the probabilistic and determined dynamic factors and compare them with the factors obtained in Section 2 (see Fig. 2) and Section 3 (Fig. 4).

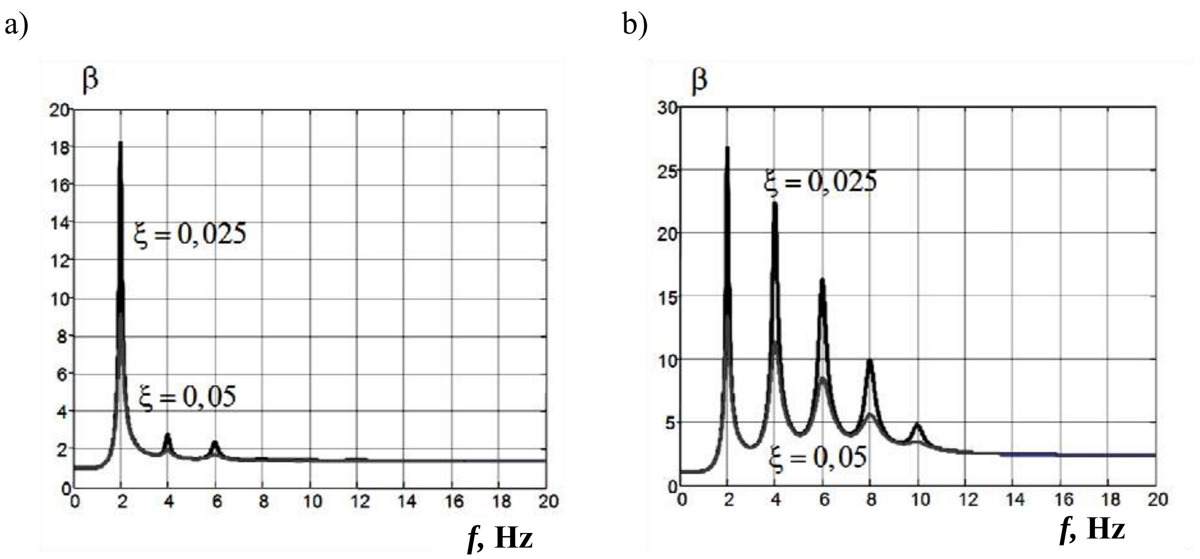


Fig. 4

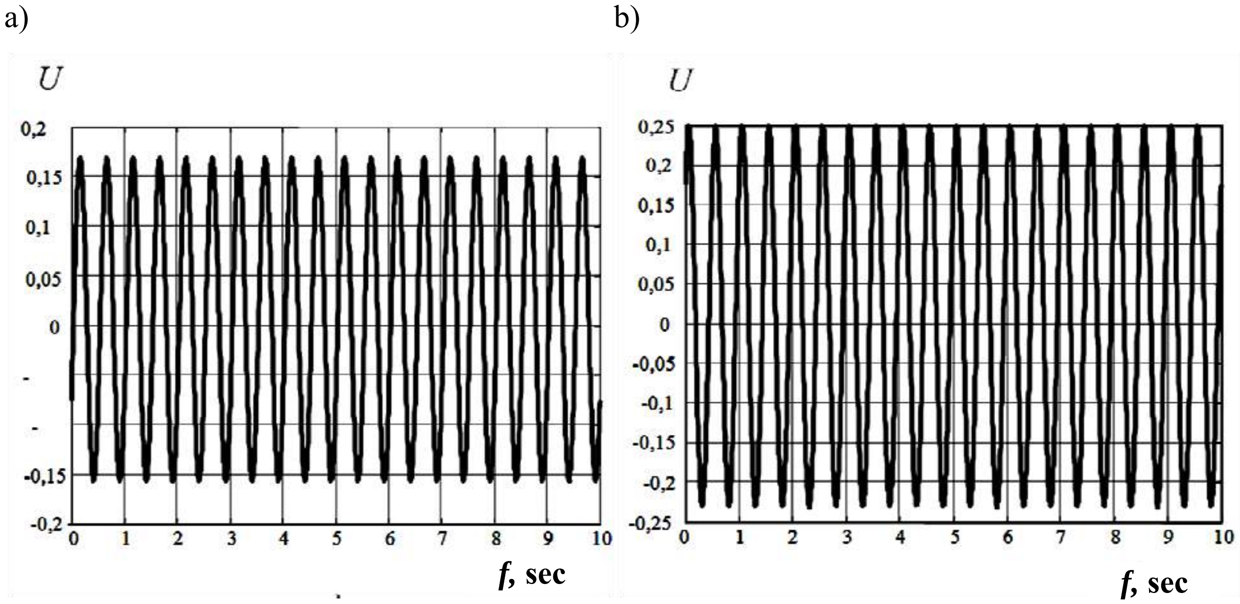


Fig. 5

For pedestrian movement: the amplitude of the oscillations $u_a = 0.1624$, average value $u_m = u_{st} = 0.0063$, standard $\sigma_u = 0.1152$; for the frequency 2 Hz the determined efficiency coefficient is $\beta_{det} = u_a / u_m = 25.78$, the probabilistic one $\beta_{stoch} = \sigma_u / u_m = 18.29$. The corresponding values in the diagrams (see Fig. 2a, 4a): $\beta_{det} = 26.79$ (the error is 3.8 %), $\beta_{stoch} = 18.24$ (the error is 0.3 %).

For high jumps: the amplitude of the oscillations $u_a = 0.2432$, average value $u_m = u_{st} = 0.0064$, standard $\sigma_u = 0.1689$; for the frequency 2 Hz the determined efficiency coefficient is $\beta_{det} = u_a / u_m = 38.00$, the probabilistic one $\beta_{stoch} = \sigma_u / u_m = 26.39$. The corresponding values in the diagrams (see Fig. 2b, 4b): $\beta_{det} = 39.44$ (the error is 3.7 %), $\beta_{stoch} = 26.69$ (the error is 1.1 %). The test resuts are indicative of the two above approaches being correct.

Note that as shown in [5], for the impulse spectral density $\beta_{stoch} / \beta_{det} = 1/\sqrt{2} = 0.707$. This is due to the fact that β_{stoch} is a standard of a random dynamic factor. The ratio $\beta_{stoch}/\beta_{det}$ will be dropping considering a random character of the load.

5. Evaluation of the spectators' perception of vibrations

A human body is a viscoelastic system with its own frequencies. Resonance frequencies of certain body parts are as follows: [3]: eyes — 12—27Hz, throat — 6—27 Hz, chest — 2—12 Hz, legs and arms — 2—8 Hz, head — 8—27 Hz, face and jaw — 4—27 Hz, lumbar spine — 4—14 Hz, stomach — 4—12 Hz. For the oscillation frequencies close to the resonance ones vibrations might have a range of negative impacts from unpleasant sensations to severe functional and physiological damage. The qualitative assessment of subjective perceptions caused by vibration is shown in Fig. 6 [3] as ranges of equal perception: a) depending on vibration movements and frequency, b) depending on vibration acceleration and frequency. Different levels of unpleasant sensations summed in Table correspond to each range of equal perception.

a) b)

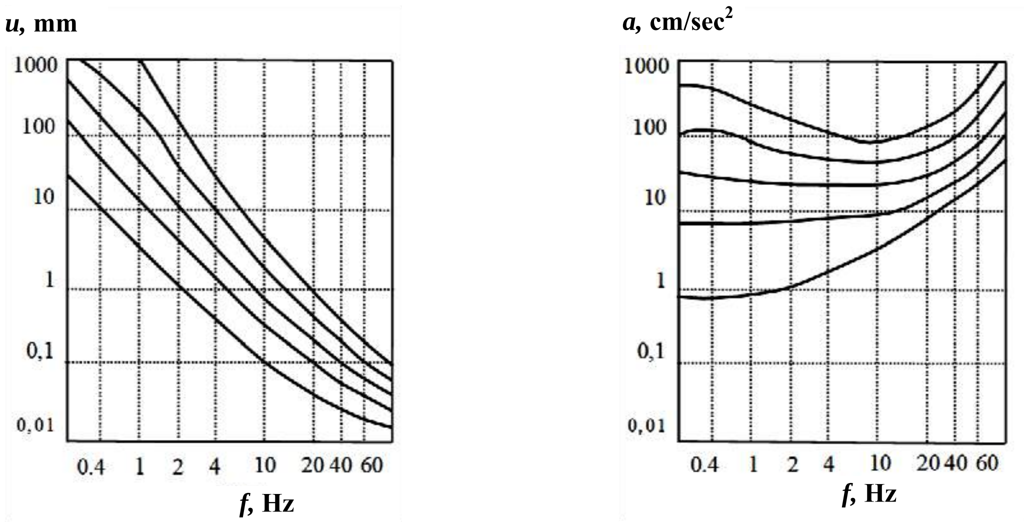


Fig. 6

Table 3

Range	Vibration	Range	Vibration
1	Imperceptible	4	Strongly perceived
2	Poorly perceptible	5	Unpleasant during long-term impact
3	Well perceived	6	Unpleasant during short-term impact

For significant vibration levels in the range of 4—10 Hz humans might feel discomfort and pain due to resonance in the “chest-stomach” system. The most comfortable condition for the vibrations with the frequencies from 1 to 4 Hz (vibrations are not perceived) is at the vibration accelerations of up to 1 cm/sec^2, vibration movements of up to 1 mm. This data is in agreement with the acceptable vibration acceleration documented in the health and safety guidelines [11]. Hence in order to evaluate the spectators’ perception using Fig. 6 and Table, it is necessary that the amplitudes of the movements and accelerations are known. The amplitudes of the movements and accelerations can be calculated according to the formula (7). Differentiating the movements (7) in time twice and assuming that the sinuses are one, we obtain the maximum modal acceleration:

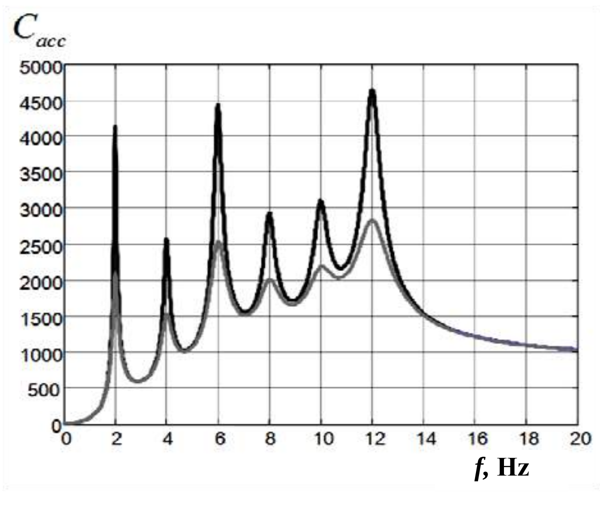
$$\max \ddot{u}_k = \frac{1}{\Omega_k^2 M_{\mathrm{mod},k}} \mathbf{v}_k^T \mathbf{P}^G \sum_{n=1}^{N_F} \frac{n^2\theta^2 r_n}{\sqrt{\left(1-\frac{n^2\theta^2}{\Omega_k^2}\right)^2 + \left(\frac{2\varepsilon_k n\theta}{\Omega_k^2}\right)^2}} . \quad (18)$$

Let us introduce the modal coefficient of reducing the static movements to the accelerations C_{acc} (the coefficient that the static movement should be multiplied by in order for the maximum acceleration to be obtained):

$$C_{acc} = \theta^2 \sum_{n=1}^{N_F} \frac{n^2 r_n}{\sqrt{\left(1-\frac{n^2\theta^2}{\Omega_k^2}\right)^2 + \left(\frac{2\varepsilon_k n\theta}{\Omega_k^2}\right)^2}} . \quad (19)$$

The coefficients C_{acc} are determined for each eigenfrequency (Fig. 6) and are thus modal as well as the dynamic factor. An example of a diagram of C_{acc} in Fig. 6 is given for the impulse frequency f_p =2 Hz: a) for pedestrian movement α = 2/3 and b) for high jumps α = 1/4. The black line corresponds to 2.5 % damping, the grey one to 5 % damping. Using the values of the dynamic movements and accelerations at a known impact frequency the spectators' perception level is determined according to Table and Fig. 6.

a)



b)

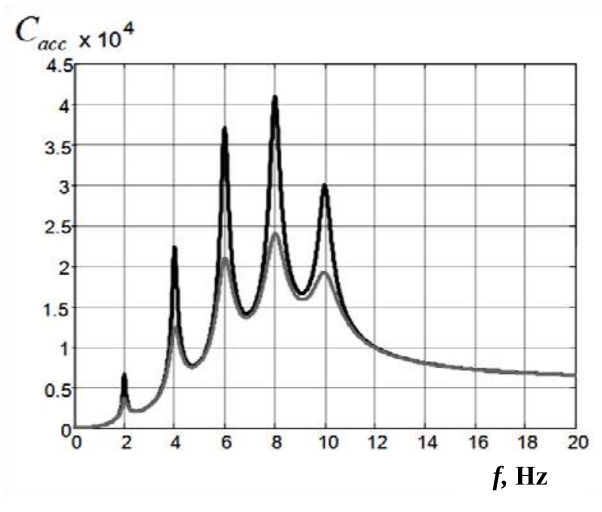


Fig. 7

Conclusions

The study revealed the following features of loading caused by coordinated crowd activity:

1. Impulse impact (1) has such a property that at the same impulse frequency several forms of

oscillations in a structure are excited. Hence spectators synchronously jump with the frequency of 2 Hz, resonances will be not only for eigenfrequencies of a structure of 2 Hz but also for 4, 6, 8 Hz if there are any (Fig. 2, 4). It is most dangerous when a spectrum of eigenfrequencies interrupts a range of possible frequencies of forced oscillations of 1—4 Hz. The dynamic factors reach their maximum. As noted in [5], if this is the case, a dynamic calculation is not advisable, it is thus necessary to take construction measures in order to avoid resonance. A dynamic response is on the rise if static movements are large (particularly for console structures);

2. Loading is narrow, i.e. the entire impulse energy is focused on certain frequencies *nf* and the spectral impact density is a sequence of impulse functions at these frequencies, $n = 1, 2, \ldots, N_F$. Therefore the dynamic factors turned out to be incredibly high: up to 14 at 5 % and up to 26 at 2.5 % damping in a resonance mode for pedestrian movement, up to 20 at 5 % and up to 39 at 2.5 % damping in a resonance mode for high jumps. Note that these dynamic factors correspond to absolutely synchronous human movement with the same phase, frequency and amplitude, which is obviously unlikely. It is reasonable to reduce the dynamic factors considering inconsistency of crowd movement. In foreign methods the parameters of a dynamic response are suggested to be multiplied by the non-synchronous coefficient, which is 0.67 [12]. However, this can be dealt with in a more accurate manner by introducing the random parameters of dynamic load such as amplitude, movement phase as well as spatial distribution. Spectral density will be smoother, the peak ordinates will drop and so will the dynamic factors;

3. In order to reduce the amplitudes of oscillations, it is reasonable to make use of the technologies enhancing the damping properties of structures;

4. Since a vibration level might be high and unpleasant for spectators, it is necessary that a calculation is supplemented by the evaluation of a level of perceived vibrations. Note that if the healthy and safety regulations [11] are adhered to, stands are to provided the “vibrations are not felt” level, which is barely impossible to implement for sporting structures.

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