

BUILDING STRUCTURES, BUILDINGS AND CONSTRUCTIONS

UDC 624.046

Yu. V. Krasnoshchekov¹, S. A. Makeev², L. V. Krasotina³

APPLICATION OF A CHART OF FLEXIBLE FILAMENT FOR THE CALCULATION OF CEILING AT CRASH OF COLUMN OF FRAMEWORK

Siberian Automobile and Road University

Russia, Omsk, tel.: (3812)23-64-71, e-mail: makeev608079@mail.ru

¹*D. Sc. in Engineering, Prof. of the Dept. of Building Structures*

²*D. Sc. in Engineering, Prof. of the Dept. of Building Structures*

³*PhD. in Engineering, Assoc. Prof. of the Dept. of Building Structures*

Statement of the problem. Development of models of the analytical description of behavior of designs during progressive destruction of buildings is a relevant task. At the same time it is necessary to consider a set of factors including effects of dynamic behavior of designs. The task is set of model building of behavior of a framework of the multi-storey building during a momentary failure of the central column of a random floor with the development of options of design strengthening of elements of overlappings reducing risks of an avalanche failure.

Results. The mathematical behavior model of a concrete framework of the multi-storey building during a momentary failure of the central column of a random floor taking into account dynamic effects is developed. Two options of strengthening of combined reinforced concrete overlappings for the purpose of prevention of the progressing collapse are considered. The calculated scheme of flexible thread is applied to static and dynamic calculation of the reinforced overlapping. An increase in tensile strains during a momentary failure of a column is recommended to be considered by introducing a coefficient of impact. The way of calculating a coefficient of impact taking into account inelastic deformations of fittings is developed and approved. Examples of calculation of the building for progressive failure are given.

Conclusions. During a failure columns in buildings with a framework strengthening of overlappings and providing their carrying capacity on stretching according to the scheme of flexible thread is required. A momentary failure of a column is followed by fluctuations of structural elements and an increase in efforts in them. It is suggested that an increase in the tensile strains is determined by introducing a coefficient of impact. The way of calculating a coefficient of impact taking into account inelastic deformations of fittings is developed and approved.

Keywords: emergency, failure of a column, progressive failure, flexible thread, dynamic effect.

Introduction. The main condition for designing structures is safety. Lately there has been a lot of discussion about longevity, i.e. reliability under emergency loads. There has been no agreement as to the definition of the term for construction objects [19].

According to Prof. V. D. Reizer, longevity is structures' capacity to retain its performance in emergencies with no avalanche-like (cascade) development of excitements and failures [10]. The only thing to add is that this is a systematic property [4, 15, 19]. Based on the definition, progressive failure is to be excluded in order to provide longevity.

The causes of progressive (avalanche-like) collapse of construction objects are local failures of construction elements in emergencies which are explosive and seismic dynamic impacts as well as loads resulting from fires, karst failures, non-sanctioned replanning, etc.

A building or a structure should be designed in a way that a failure of any individual element had no impact on the overall performance of an object and its primary parts over the period while emergency measures are being taken (e.g., evacuation in the event of a fire). Safe and consistent functioning of these elements secure a construction object even if it can no longer be used unless major repairs are performed [14]. This problem is to be immediately addressed as current construction practices make no consideration for progressive collapse [14, 17]. The idea of designing structures with emergency impacts in mind is relatively new [6, 8, 16—20]. Progressive collapse calculations are performed for buildings and structures of the KS-3 and KS-2 type that are able to house large numbers of people. The suggested set of buildings and structures of the type is presented in Appendix B of the GOST 27751-2014 "Reliability of Construction Structures and Foundations". In particular, they are buildings (residential, offices, administrative, public, etc.) that are five-storeyed or more. For these buildings standard construction systems of a braced framing from assembled ferroconcrete elements, e.g., a frame of the series 1.020-1/87, are used.

A failure of one of the column is one of the possible outcomes that is considered while designing frame buildings. A major focus is on calculating enclosure elements whose flights are to be considerably increased. In buildings with a braced framing providing spatial rigidity, beam elements of overlappings almost lose their capacity to bend. Therefore a calculation scheme of an overlapping over a removed column is considered as a membrane or a flexible thread (string).

In [3] there is an example of using the above model to study longevity of buildings with a braced framing whose overlappings are reinforced with rope bars. The results of an approximate calculation of efforts in the bar and movements are rather contradictory. A dynamic effect caused by a sudden removal of the column from the calculation was not considered.

A possible use of construction schemes of a braced framing for buildings and structures of the type KS-2 housing large numbers of people requires a special study of longevity. The objec-

tive of the ongoing study is to make the calculation method for overlappings using a flexible thread more accurate considering a dynamic effect under an emergency impact caused by a failure of a column.

1. Model of longevity. Implementation of the principle of efficiency using the systemic approach involves a set of models of constructive systems [4]. There are two models of longevity, i.e. a deterministic and probabilistic one.

A deterministic (semi-probabilistic) model implemented using the method of limiting states involves the analysis of the stress-strain of a constructive system to evaluate the strength in case of a failure of one or more load-bearing elements (modeling a possible collapse) [7, 13]. In [7] there are the results of modeling a spatial overlapping of a sports facility. The analysis of the stress-strain of an overlapping with removed elements was performed for normative values of constant and long-term components of time loads considering a dynamic effect. A similar model was set forth for the third group of limiting states (according to longevity) by V. D. Reiser [11].

The criteria for probabilistic models are indices for reliability (failure-free). E.g., this might be an index of reliability of the method of two moments that are given by the formula

$$\beta = (\bar{R} - \bar{F}) / \sqrt{s_R^2 + s_F^2}, \quad (1)$$

where \bar{R} and \bar{F} are mathematical expectations of the bearing capacity and load; s_R^2 and s_F^2 are the dispersions of the bearing capacity and load.

V. D. Reiser suggests that the longevity index is used to evaluate it

$$I = \frac{\beta_{INT}}{\beta_{INT} - \beta_D}, \quad (2)$$

where β_{INT} , β_D are indices of longevity of a non-damaged and damaged structure.

The use of probabilistic models requires standardizing the indices of reliability and longevity (identifying the limit values for a variety of situations). We believe that the above values of the indices are good at characterizing the difference between reliability and longevity.

Generally the calculation of longevity involves that of the strength of a building and structure against progressive collapse considering plastic deformations under limited loads. In [10] it is suggested that the longevity calculation is performed in two stages. At the first stage a calculation is performed in the operational stage proceeding a local failure. A calculation with removed elements is performed at the second stage considering physical and geometric nonli-

nearity on the impact of a load and effort identified at the first stage with an increase by the coefficient considering a dynamic effect of a local failure. According to the authors, this calculation is computer modeling of adjusting a structure to a new calculation situation.

While investigating longevity of braced buildings with a frame scheme providing spatial rigidity, a failure of one of the lower-storey columns is generally considered. As a result of large movements, a structure can adapt to a new situation with a possible change in the calculation scheme with that of the overlapping over the removed column considered as a membrane due to large movements.

In [11] it is shown that in frame buildings with beamless ferroconcrete overlappings when a certain size of a grid of columns is exceeded, it is of most importance to calculate against progressive collapse considering plastic deformations under limited loads. Only special combinations of loads including constant and long-term loads with the coefficients of combination and reliability that equal one as well as the most hazardous schemes of a local failure are considered. The values of movements (bendings) and width of the opening of cracks in structures are not specified and the strength should be provided for a minimum rigidity of construction elements and nod connections corresponding with the maximum acceptable deformations of concrete and reinforcement. The criteria of the bearing capacity are the same as for regular calculations using limited states.

Presently a calculation using the method of limited states provides reliability of buildings and structures. In the last edition of the GOST 27751-2014 apart from the first and second groups of limited states, there are those that emerge under particular impacts and circumstances that if exceeded, cause emergencies to occur.

Particular impacts are grouped into standardized (e.g., seismic) and emergency ones that occur, e.g., if a load-bearing element of a construction system fails. Particular impacts are included into particular combinations of loads where short-term loads do not have to be considered.

Particular loads are considered to cause emergencies. Therefore while calculating them, an emergency calculation situation should be taken into account that corresponds with exceptional operational conditions of a structure that might lead to social, environmental and economic losses.

The calculation values of particular loads are specified in the guidelines and designing takss considering possible losses in case of a failure.

Various strategies are employed to increase longevity [2, 16—20].

2. Determining the tensile load and movement of a rope bar. For a statistical calculation of a bar the solution of stretching a string [12]. A string fixed at two points with the flight $2l$ can have an initial stretching N_0 . If the force F is applied in the middle of the flight, it acquires extra stretching N and is moved by f under the force (Fig. 1).

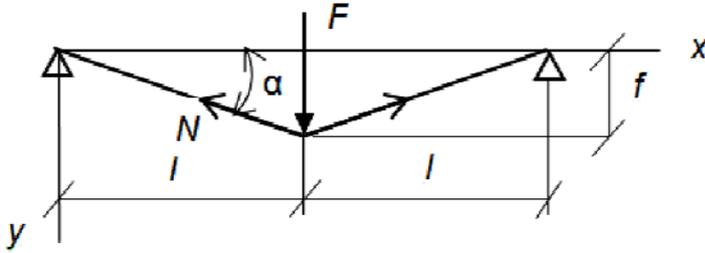


Fig. 1. Calculation scheme of a flexible thread

Designing a sum of the projections of the forces onto the axis γ , we obtain $\alpha = fl$ considering $\sin \alpha \approx \text{tg}$

$$F - \frac{2Nf}{\sqrt{f^2 + l^2}} = 0. \quad (3)$$

An increase in the length of each branch of the string with the elasticity modulus E and the area of the section A is

$$\Delta l = \sqrt{f^2 + l^2} - l = l(N - N_0) / EA. \quad (4)$$

Using the condition of the equality of the operation of the internal force on the movement f and external forces on the movement Δl the following equation is obtained

$$Ff = l(N^2 - N_0^2) / EA. \quad (5)$$

Solving a combination of the equation (3) and (5), we obtain a vertical movement (arch) of the string:

$$f = l(N^2 - N_0^2) / EAF. \quad (6)$$

An effort of stretching the string is determined using a cubic equation

$$4X^3 - (8N_0^2 - F^2)X^2 + 2N_0^2(N_0^2 + F^2)X - F^2(N_0^4 + F^2E^2A^2) = 0, \quad (7)$$

where $X = N^2$.

Based on equation (7), an expression for a conditional elasticity modulus of a flexible thread corresponding with the limited value $N_u = R_{sn}A$ was obtained:

$$E_u = \sqrt{4N_u^6 - (8N_0^2 - F^2)N_u^4 + 2N_0^2(2N_0^2 + F^2)N_u^2 - F^2N_0^4} / F^2A. \quad (8)$$

According to the obtained value E_u using the formula (8), a movement of a flexible thread f under the force F can be made more accurate.

Let us use the results of the calculation of the rope bar from [5] as an example. With the area of the section $A = 7.05 \text{ cm}^2$ and standardized resistance of the rope $R_{sn} = 1300 \text{ MPa}$ the limited effort in the rope is $N_u = 916.5 \text{ kN}$. Considering $N_0 = 0$ and $F = 222 \text{ kN}$ using the formula (8) $E_u = 44600 \text{ MPa}$ was obtained. This corresponds with the coefficient of plasticity (the ratio of the complete bend to the elastic one)

$$K_{pl} = 180000 / 44600 \approx 4 < \bar{K}_{pl}$$

(with the elasticity modulus $E = 180000 \text{ MPa}$). A limited value of the elasticity coefficient is given by [1]:

$$\bar{K}_{pl} = \bar{\varepsilon}_{s2} E / (R_{sd} + 0.002E) \approx 4.7, \tag{9}$$

where $\bar{\varepsilon}_{s2} = 0.05$ is a maximum acceptable an even relative increase in the length; R_{sd} is the resistance of the rope under a dynamic load: $R_{sd} = 1.2 \cdot 1300 = 1560 \text{ MPa}$.

According to the formula (6) at $E = E_u$ a deflection of stretching $f = 72 \text{ cm}$ was determined.

Note that the use of the initial stretching N_0 of the rope without an increase in the area of the section has almost no influence on the value of the deflection of stretching due to a reduction in the conditional elasticity modulus. E.g., at $N_0 = 300 \text{ kN}$ $E_u = 40000 \text{ MPa}$ was obtained as well as $K_{pl} = 4.5$ and $f = 72 \text{ cm}$. An increase in the initial stretching up to $N_0 = 500 \text{ kN}$ was not acceptable as $E_u = 31800 \text{ MPa}$ and $K_{pl} = 5.7 > 4.7$.

3. A dynamic calculation of the flexible thread. A dynamic effect during a sudden failure of the elements of construction schemes is essential due to the complexity of the task [9, 10].

For a dynamic calculation of the flexible thread a model that is described in [12] is used. A calculation scheme of stretching is considered as a thread with one end fixed and the other one stretched by a conditional counterbalance through a block (a vertical part of the thread is considered unstretchable) (Fig. 2).

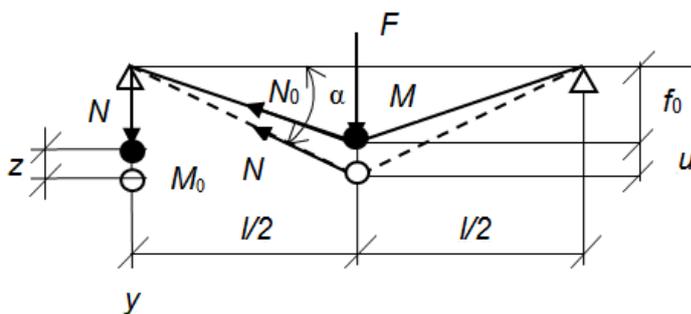


Fig. 2. Calculation scheme of the thread under a dynamic impact

It is assumed that as a result of a static load F the corresponding mass M is moved at the distance f_0 and there is an effort N_0 in the stretching. Due to a momentary application of the mass M there is an oscillation of the thread with the movements u and the mass of the counterbalance M_0 that generates the stretching N during the oscillations of the thread is increased by z . In order to solve the task, differential equations of the movement of a material point are used as

$$My'' = \Sigma Y_i, \quad (10)$$

where y'' is the acceleration of the point towards the axis γ ; Y_i are the projections of the forces applied to the point onto the axis γ .

The masses M_0 and M are obtained by dividing the forces N and F into the free fall acceleration g .

Due to a small angle α let us assume that $\sin \alpha \approx \operatorname{tg} \alpha = 2f/l$ and the equations (10) are

$$\begin{aligned} N_0 + M_0 z'' &= N; \\ (F - Mf'')l/4 &= N, \end{aligned} \quad (11)$$

where $f = f_0 + u$ is the deflection caused by the oscillations.

The dependence of the variables z and f is determined using the expression

$$z = 2 \left(\sqrt{(l/2)^2 + f^2} - \sqrt{(l/2)^2 + f_0^2} \right) - Z/c, \quad (12)$$

where $Z = N - N_0$ is a changeable component of stretching of the thread during the oscillations; $c = EA/l$ is a single force of the elastic extension; Z/c is a changeable component of the deformation of the thread.

In the formula (12) the expression in the brackets is transformed by adding the summands f^4/l^2 and f_0^4/l^2 to the radical sums. The acceptable equality of extra summands determined the approximation of the formula (12) as it is more conveniently transformed into

$$z = 2(f^2 - f_0^2)/l - Z/c. \quad (13)$$

Considering that in the expression f the component f_0 is a constant value, we obtain $f'' = u''$.

Then the second equation (11) can be written as follows

$$Fl/4 - Mu''l/4 = (Z + N_0)(f_0 + u) = Zf_0 + N_0f_0 + Zu + N_0u.$$

Considering that $Fl/4 = N_0f_0$ (Fig. 2) and neglecting the product Zu , we finally obtain

$$Z = -Mu''/4f_0 - N_0u/f_0. \quad (14)$$

The movement z is written as follows

$$z = 2(2f_0u - u^2)/l - Z/c \approx 4f_0u/l - Z/c = (Ml/4f_0c)u'' + (4f_0/l + N_0/f_0c)u. \quad (15)$$

Inserting (14) and (15) into the first equation (11), we get a differential equation

$$u^4 + b_1 u'' + b_2 u = 0, \quad (16)$$

where

$$b_1 = (N_0 + 4f_0^2 c / l) 4 / Ml + c / M_0;$$

$$b_2 = 4N_0 c / MM_0 l.$$

Using the solution of the equation (16) in [13], the expressions for the characteristics of the oscillations were obtained:

— the frequencies:

$$\omega_{1,2} = \sqrt{b_1 / 2 \pm \sqrt{b_1^2 / 4 - b_2}}; \quad (17)$$

— the amplitude:

$$A_1 = k (\omega_2^2 - 4N_0 / Ml) / (\omega_2^2 - \omega_1^2); \quad (18)$$

— the calculation parameters:

$$h_1 = Ml / 4f_0 c; \quad h_2 = 4f_0 / l + N_0 / f_0 c; \quad (19)$$

— the maximum acceleration:

$$z''_{\max} = \pm 2\omega_1^2 A_1 (h_1 \omega_1^2 - h_2); \quad (20)$$

— the maximum stretching of the thread:

$$Z_{\max} = M_0 z''_{\max}. \quad (21)$$

Let us perform a dynamic calculation using the above initial data (with the flight $l = 12$ m and $k = f_0 = 0.72$ m).

The calculation masses are

$$M = 222 / 9.81 = 22.6 \text{ kN} \cdot \text{sec}^2 / \text{m};$$

$$M_0 = 916.5 / 9.81 = 93.4 \text{ kN} \cdot \text{sec}^2 / \text{m}.$$

A single force is

$$c = 44600 \cdot 7.05 / 12 \cdot 10 = 2620 \text{ kN/m}.$$

The coefficients of the equation (16) are

$$b_1 = 4(916.5 + 4 \cdot 0.72 \cdot 2620 / 12) / 22.6 \cdot 12 + 2620 / 93.4 = 50.84 \text{ sec}^2;$$

$$b_2 = 4 \cdot 916.5 \cdot 2620 / 22.6 \cdot 93.4 \cdot 12 = 379.12 \text{ sec}^{-4}.$$

The frequencies of the oscillations are

$$\omega_1 = \sqrt{50.84 / 2 - \sqrt{50.84^2 / 4 - 379.12}} = 3.01 \text{ sec}^{-1};$$

$$\omega_2 = \sqrt{50.84 / 2 + \sqrt{50.84^2 / 4 - 379.12}} = 6.46 \text{ sec}^{-1}.$$

The amplitude is:

$$A_1 = 0.72(6.46^2 - 4 \cdot 916.5 / 22.6 \cdot 12) / (6.46^2 - 3.01^2) = 0.62 \text{ m.}$$

The calculation parameters are

$$h_1 = 22.6 \cdot 12 / 4 \cdot 0.72 \cdot 2620 = 0.036 \text{ sec}^2;$$

$$h_2 = 4 \cdot 0.72 / 12 + 916.5 / 0.72 \cdot 2620 = 0.726 .$$

The maximum acceleration is

$$z''_{\max} = \pm 2 \cdot 3.01^2 \cdot 0.62(0.036 \cdot 3.01^2 - 0.726) = 4.49 \text{ m/sec}^2.$$

The maximum growth in the stretching of the thread during the oscillations is $Z_{\max} = 4.49 \cdot 93.4 = 420 \text{ kN}$. The dynamic factor is $k_d = 1 + 420/916.5 = 1.46$. This means that it is necessary to strengthen the rope bars almost by 1.5 times.

4. A variant of strengthening overlappings of buildings with a braced framing. In order to find a more effective solution of overlappings of buildings with a braced framing, a variant of strengthening using a reinforcement of the type A 500 is employed [5]. Fig. 3 shows a construction scheme of an assembled ferroconcrete overlapping with traditional elements. Reinforcement of cross-bars was calculated based on an operational load. In order to achieve the continuity of the cross-bars, the areas adjoining them with the column are strengthened with a reinforcement equivalent in the strength and rigidity of the longitudinal reinforcement of the cross-bars. The position of the strengthening reinforcement along the height of the cross-bars has no value as the main purpose of the strengthening is to provide the axial rigidity and strength of the belts of the cross-bars for the tensile operation according to the flexible thread scheme. In order to improve the efficiency of the strengthening, a similar reinforcement is provided along the axis of the slabs between the columns as well.

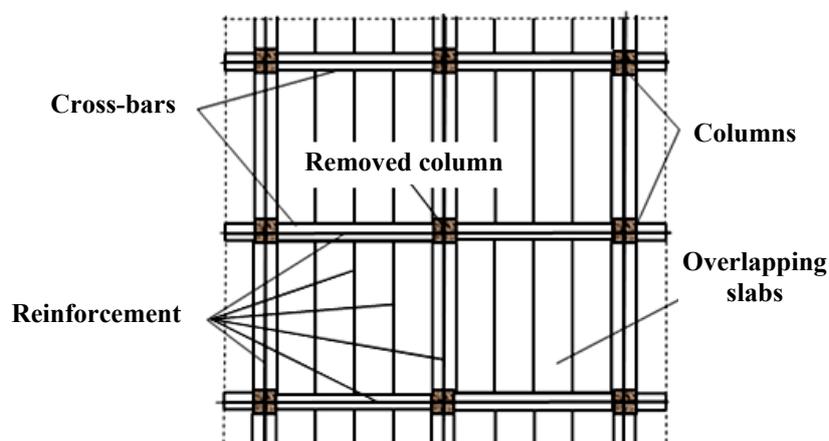


Fig. 3. Construction scheme of the reinforced overlapping of a building with a braced framing

As an example let us consider a structure of overlapping with a column grid 6×6 m loaded with an evenly distributed calculated load $q = 12 \text{ kN/m}^2$ (a standard value of a long-term part of the load $g = 7.5 \text{ kN/m}^2$). At the operational stage the cross-bars function according to the split beam scheme. A longitudinal operational reinforcement of the cross-bars $4\text{Ø}25$ A 500 has the area of the section $A = 19.625 \text{ cm}^2$. The elasticity modulus of the reinforcement $E = 200000 \text{ MPa}$, the standard resistance of the reinforcement is $R_{sn} = 500 \text{ MPa}$.

As the middle column is removed, there is an emergency with an increase in the flight of a structure up to $l = 12 \text{ m}$. Even for a long-term part of the load, the elements of overlappings lose their capacity to operate for bending. Thus the calculation scheme of the overlapping over the removed column is considered as a flexible thread (a string). The strengthening of the stretching of the thread $N_u = 500 \cdot 19.625 \cdot 1.1 = 1079.4 \text{ kN}$ is determined considering the dynamic factor $k_d = 1.1$. A thread with the length $l = 12 \text{ m}$ in each of the directions is loaded in the middle of the flight with the force $F = 7.5 \cdot 6 \cdot 6 / 2 = 135 \text{ kN}$.

Considering $N_0 = 0$ according to the formula (6) a value of a conditional elasticity modulus is $E_u = 70460 \text{ MPa}$. This corresponds with the plasticity coefficient

$$K_{pl} = 200000 / 704600 \approx 2.84 < \bar{K}_{pl} = 10 \text{ [5]}.$$

According to the formula (6) at $E = E_u$ a deflection of the stretching is determined

$$f_0 = 6 \cdot 1079.4^2 / 70460 \cdot 19.625 \cdot 135 \cdot 10 = 0.375 \text{ m}$$

(here according to Fig. 1 it is assumed that $l = 6 \text{ m}$). According to the obtained data, a dynamic calculation at $N_0 = 1079.4 \text{ kN}$, $f_0 = 0.375 \text{ m}$ is performed.

The calculation masses are

$$M = 135 / 9.81 = 13.8 \text{ kN} \cdot \text{sec}^2 / \text{m};$$

$$M_0 = 1079.4 / 9.81 = 110 \text{ kN} \cdot \text{sec}^2 / \text{m}.$$

A single force is

$$c = 70460 \cdot 19.625 / 12 \cdot 10 = 11520 \text{ kN/m}.$$

The coefficients of the equation (15) are

$$b_1 = 4(1079.4 + 4 \cdot 0.375 \cdot 11520 / 12) / 13.8 \cdot 12 + 11520 / 110 = 165.58 \text{ sec}^2;$$

$$b_2 = 4 \cdot 1079.4 \cdot 11520 / 13.8 \cdot 110 \cdot 12 = 2730.5 \text{ sec}^{-4}.$$

The frequencies of the oscillations are

$$\omega_1 = \sqrt{165.58 / 2 - \sqrt{165.58^2 / 4 - 2730.5}} = 4.31 \text{ sec}^{-1};$$

$$\omega_2 = \sqrt{165.58 / 2 + \sqrt{165.58^2 / 4 - 2730.5}} = 12.12 \text{ sec}^{-1}.$$

The amplitude is

$$A_1 = 0.375(12.12^2 - 4 \cdot 1079.4 / 13.8 \cdot 12) / (12.12^2 - 4.31^2) = 0.35 \text{ m.}$$

The calculation parameters are

$$h_1 = 13.8 \cdot 12 / 4 \cdot 0.375 \cdot 11520 = 0.0096 \text{ sec}^2;$$

$$h_2 = 4 \cdot 0.375 / 12 + 1079.4 / 0.375 \cdot 11520 = 0.375.$$

The maximum acceleration is

$$z''_{\max} = \pm 2 \cdot 4.31^2 \cdot 0.35(0.0096 \cdot 4.31^2 - 0.375) = 0.59 \text{ m/sec}^2.$$

A maximum growth of the stretching of the thread at the oscillations $Z_{\max} = 0.59 \cdot 110 = 65.3 \text{ kN}$. The dynamic factor is $k_d = 1 + 65.3/1079.4 = 1.06 < 1.1$. This means that extra reinforcement is not necessary.

Conclusions. Therefore if columns in buildings with a braced framing collapse, overlappings have to be reinforced and their tensile load-bearing capacity according to the flexible thread scheme has to be retained.

A momentary failure of a column is accompanied with an oscillation of construction elements and an increase of efforts in them. An increase in the tensile effort can be determined by introducing a dynamic coefficient.

A method of calculating a dynamic coefficient considering non-elastic deformations of reinforcement was developed and tested.

References

1. *Armirovanie elementov monolitnykh zhelezobetonnykh zdaniy: posobie po proektirovaniyu* [Reinforcement of elements of monolithic reinforced concrete buildings: a guide to design]. Moscow, FGUP «NITs Stroitel'stvo», 2007. 117 p.
2. De B'yadzi V. Povyshenie zhivuchesti sooruzheniya s pomoshch'yu uslozhneniya konstruktivnykh skhem [Improving the vitality of the structure by complicating the design schemes]. *Vestnik TGASU*, 2015, no. 4, pp. 92—100.
3. Krasnoshchekov Yu. V., Mel'nikova S. O., Ekimov A. A. Zhivuchest' mnogoetazhnogo zdaniya so svyazevym karkasom [Survivability of a multi-storey building with a connecting frame]. *Vestnik SibADI*, 2016, no. 2 (48), pp. 100—104.
4. Krasnoshchekov Yu. V. *Nauchnye osnovy issledovaniy vzaimodeistviya elementov zhelezobetonnykh konstruksii* [The scientific basis of research of interaction of elements of reinforced concrete structures]. Omsk, SibADI Publ., 1997. 276 p.
5. Krasnoshchekov Yu. V. Raschet karkasnogo zdaniya na progressiruyushchee obrushenie pri avariinom otkaze kolonny [Calculation of the frame building on the progressive collapse in case of an emergency failure of the column]. *Stroitel'naya mekhanika i raschet sooruzhenii*, 2017, no. 1, pp. 54—58.

6. Klyueva N. V., Bukhtiyarova A. S., Kolchunov S. I. Issledovanie zhivuchesti zhelezobetonnykh ramno-sterzhnevyykh prostranstvennykh konstruksii v zapredel'nykh sostoyaniyakh [Investigation of the survivability of reinforced concrete frame-rod spatial structures in out-of-limit States]. *Promyshlennoe i grazhdanskoe stroitel'stvo*, 2012, no. 2, pp. 55—59.
7. Kudishin Yu. I., Drobot D. Yu. K voprosu o zhivuchesti stroitel'nykh konstruksii [On the question of the vitality of building structures]. *Stroitel'naya mekhanika i raschet sooruzhenii*, 2008, no. 2, pp. 36—43.
8. Nazarov Yu. P., Gorodetskii A. S., Simbirkin V. N. K probleme obespecheniya zhivuchesti stroitel'nykh konstruksii pri avariinykh vozdeistviyakh [To the problem of ensuring the survivability of building structures in case of emergency]. *Stroitel'naya mekhanika i raschet sooruzhenii*, 2009, no. 4, pp. 5—9.
9. Perel'muter A. V. *Izbrannye problemy nadezhnosti i bezopasnosti stroitel'nykh konstruksii* [Selected problems of reliability and safety of building structures]. Moscow, ASV Publ., 2007. 256 p.
10. Raizer V. D. K probleme zhivuchesti zdaniy i sooruzhenii [To the problem of survivability of buildings and structures]. *Stroitel'naya mekhanika i raschet sooruzhenii*, 2012, no. 5, pp. 77—78.
11. Raizer V. D. *Teoriya nadezhnosti sooruzhenii* [The theory of reliability of structures]. Moscow, ASV Publ., 2010. 384 p.
12. Rekach V. G. *Rukovodstvo k resheniyu zadach prikladnoi teorii uprugosti* [Guide to solving problems of applied theory of elasticity]. Moscow, Vysshaya shkola Publ., 1973. 384 p.
13. Sventikov A. A. Otsenka progressiruyushchego razrusheniya prostranstvennykh visyachikh sterzhnevyykh pokrytii [Estimation of progressive destruction of spatial hanging rod coverings]. *Stroitel'naya mekhanika i raschet sooruzhenii*, 2010, no. 5, pp. 34—38.
14. Tikhonov I. N., Kozelkov M. M. Raschet i konstruirovaniye zhelezobetonnykh monolitnykh perekrytii zdaniy s uchetom zashchity ot progressiruyushchego obrusheniya [Calculation and construction of reinforced concrete monolithic floors of buildings taking into account protection against progressive collapse]. *Beton i zhelezobeton*, 2009, no. 3, pp. 2—8.
15. Shiyanov S. M., Shepelina P. V., Kurantsov V. V., Kormilitsin A. I. O zhivuchesti nesushchikh konstruksii slozhnykh tekhnicheskikh sistem [Survivability of load-bearing structures of complex technical systems]. *Dvoynye tekhnologii*, 2013, no. № 1 (67), pp. 17—19.
16. Aakash A. *Assessment, Design and Mitigation of Multiple Hazards*. USA, Michigan Technological University, 2011. 78 p.
17. Asprone D., Chiaia B., Biagi V. De, Manfredi G., Parisi F. Implementation of progressive damage in finite element codes for the assessment of robustness. *ECCOMAS Congress 2016 — VII European Congress on Computational Methods in Applied Sciences and Engineering*, At Crete Island, Greece. Crete, 2016, pp. 1—14.
18. De Biagi V. Enhancing structural robustness by complexity maximization. *Vestnik TSUAB. English version appendix*, 2015, no. 1—5, pp. 26—36.
19. De Biagi V., Chiaia B. Complexity and robustness of frame structures. *International Journal of Solids and Structures*, 2013, vol. 50, no. 22, pp. 3723—3741.
20. Ventura A., Chiaia B., Biagi V. De Robustness Assessment of RC Framed Structures against Progressive Collapse. *IOP Conference Series: Materials Science and Engineering*, 2017, vol. 245, no 3, pp. 1—10.