UDC 624.046

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CRACK-RESISTANCE AND STRENGTH OF A CONTACT JOINT OF A REINFORCED CONCRETE COMPOSITE WALL BEAM WITH CORROSION DAMAGES UNDER LOADING

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Statement of the problem. The main problem with calculating ferroconcrete composite structures has to do with determining a shear modulus in the contact area of the elements of these structures. Considering a long-term mode character of their loading and likely corrosion damage during their operation, it is of interest to develop a deformation model and the of criteria and crack-resistance of the contact area of a composite flat stressed ferroconcrete structure.

Results. The model is suggested based on the deformation expressions for complex stressed reinforced concrete elements as well as a criterion of strength and crack resistance of a flat stressed corrosion-damaged element in the contact area of a ferroconcrete wall beam and physical equations for the reinforced concrete element with corrosion damages and intersecting cracks written in the form of relationships between finite stress and deformation increments. Compliance matrix of flat stressed reinforced concrete element is obtained that models the contact area of composite wall beam and takes in a count long-term deformation, corrosion damages and concentrated shear at intersections cracks formation.

Conclusions. The presented results can be used in practical methods of calculation of deformations and long-term strength of reinforced concrete composite structures operating with cracks. An example of calculating a reinforced concrete beam of a composite section is given. The calculation results are compared with the data of experimental studies and show effectiveness.

Keywords: reinforced concrete, flat-stressed element, corrosion, composite construction, scheme of intersecting cracks.

Introduction. The main problem of calculating ferroconcrete composites has to do with determining a specified shear modulus in the contact area of their elements. This parameter depends on a construction solution for adjoining elements of a composite, scheme and intensity of longitudinal reinforcement, type of stress-strain, crack formation patterns, concrete type of the elements, etc.

Despite a great number of studies of deformation of ferroconcrete composites, they mainly deal with beam and rod structures with a one-axial stress-strain for certain reinforcement schemes [1, 5, 11—14, 17—19]. The theory of complex strained ferroconcrete structures operating with cracks was examined in a fundamental monography by N. I. Karpenko [6] and in [2] its variations for increments as applied to flat-strained wall-beams are discussed. Considering a long-term mode nature of stress of such structures and their likely corrosion, it is of interest to develop a deformation model and criteria of strength and crack resistance of the contact area of the elements of a composite flat-strained ferroconcrete structure.

1. Determining the stress-strain of a typical element. Let us look at a composite flat-strained ferroconcrete structure of a wall-beam consisting of three beams B1, B2 and B3 adjoined by contact joints that are crossed by a longitudinal reinforcement of the beam A_{sw} (Fig. 1). The beam is loaded with an external load of concentrated forces P and an aggressive medium causing its corrosion.

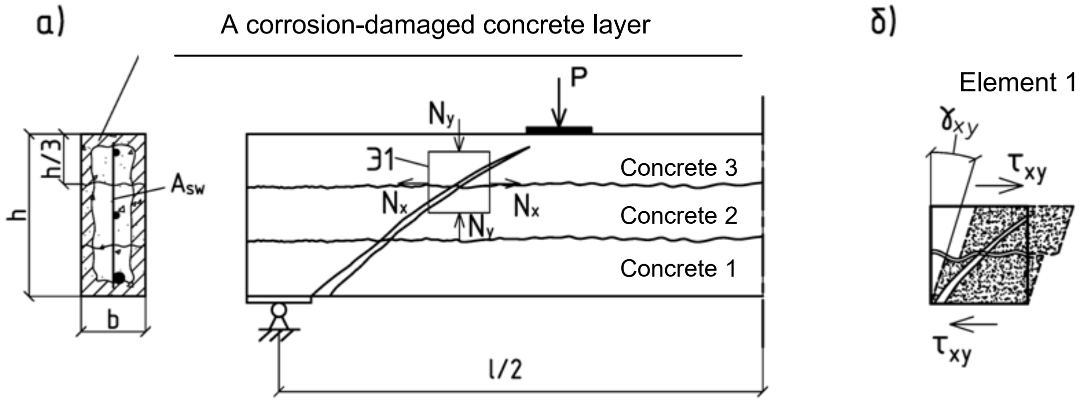


Fig. 1. Scheme of a corrosion-damaged ferroconcrete composite (wall-beam) (a) with a typical deformed finite element identified in the contact area (b): *h, b* is the height and width of a ferroconcrete beam; l is the length of the beam; τ_{xy} are shearing forces in a typical element E1

Let us identify a typical element E1 of a single size in the contact area of two concretes whose stress-strain is determined by the applied normal N_x, N_y, and shear N_{xy} efforts. According to

[7], a relation between the normal and shear efforts and deformations of a typical element can be written as

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{vmatrix} C_{11}C_{12}C_{13} \\ C_{21}C_{22}C_{23} \\ C_{31}C_{32}C_{33} \end{vmatrix} \times \begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix}, \qquad (1)$$

where ε_x, ε_y, γ_{xy} are relative deformations ; N_x, N_y, N_{xy} are normal and shear efforts in a typical flat-strained element of a single-size ; C_{ij} are the coefficients of a compliance matrix of ferroconcrete.

The dependencies of the deformation model [6, 18] hold true for all the values of an inclination angle of a crack in a typical element except α = 0° and α = 90° (Fig. 2) as the values of a relative shear of the reinforcement γ_{sxy} and shear efforts N_{xy} in these cases are zero. Therefore in order to design deformation dependencies in the specified anisotropic element E1 crossed by a horizontal crack along the contact joint of two beams, let us tunr the coordinate axis of the element *x* and *y* at the angle θ = 45° using the known transformation formulas (Fig. 2):

$$\begin{aligned} N_{x'} &= N_x cos^2\theta + N_y sin^2\theta + 2N_{xy} cos\theta sin\theta; \\ N_{y'} &= N_x \sin^2\theta + N_y \cos^2\theta - 2N_{xy} cos\theta sin\theta; \\ N_{x'y'} &= -N_x cos\theta sin\theta + N_y cos\theta sin\theta + N_{xy}(cos^2\theta - sin^2\theta); \end{aligned} \qquad (2)$$

$$\begin{aligned} \varepsilon_{x'} &= \varepsilon_x cos^2\theta + \varepsilon_y sin^2\theta + \gamma_{xy} cos\theta sin\theta; \\ \varepsilon_{y'} &= \varepsilon_x \sin^2\theta + \varepsilon_y \cos^2\theta - \gamma_{xy} cos\theta sin\theta; \\ \gamma_{x'y'} &= -2\varepsilon_x cos\theta sin\theta + 2\varepsilon_y cos\theta sin\theta + \gamma_{xy}(cos^2\theta - sin^2\theta). \end{aligned} \qquad (3)$$

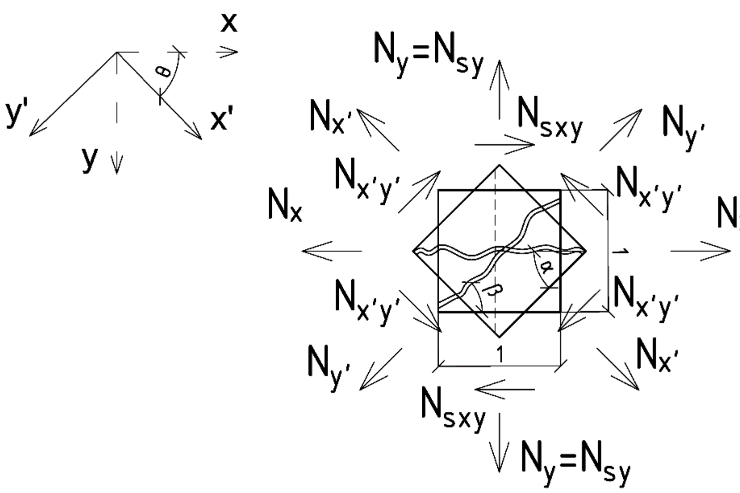


Fig. 2. Scheme of efforts in a typical flat-strained ferroconcrete element with crossing cracks: $N_{x'}$, $N_{y'}$, $N_{x'y'}$ are normal and shear efforts in the turned typical flat-strained element of a single size; N_{sy}, N_{sxy} are normal and shear efforts in the reinforcement bar

In the investigated typical element as it reaches the limit efforts of crack formation, cracks tear the concrete in the direction of the contact joint and main stretching efforts. As a result, concrete loses its capacity to independently perceive the acting efforts. In this case only some concrete relations between the crack edges are retained and the concrete is still capable of resist tangential movements of the reinforcement bars crossing the crack in the areas between the cracks.

Let us denote respectively:

— α, β is an inclination angle of the crack along the contact joint of two beams and the crack of the main stretching efforts to the axis *x*;

— *h* is the thickness of a typical element;

— f_{sy}^{*} is the area of the reinforcement in direction to the axis *y* that takes up a length unit of a typical element considering corrosion damage;

— μ_{sy}^{*} is the reinforcement coefficient for the reinforcement in direction to y : $\mu_{sy}^{*} = f_{sy}^{*} / h$.

As a longitudinal crack forms in the contact joint of two concretes and cracks of the stretching efforts, all of the acting efforts in the typical element are diverted onto the reinforcement. As a result, there are normal and tangential strains in it (Fig. 3).

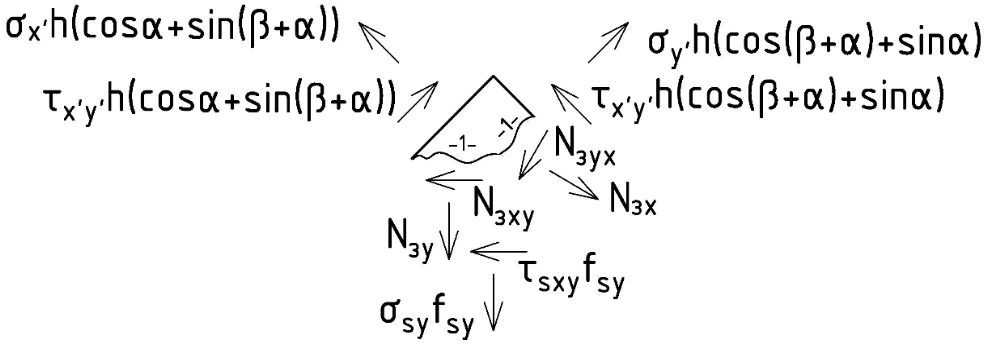


Fig. 3. Scheme of the efforts in the reinforcement following the formation of cracks in the typical element: $\sigma_{x'}$, $\sigma_{y'}$, $\tau_{x'y'}$ are normal and shear strains in the turned typical flat-strained element of a single size; σ_{sy}, τ_{sxy} are normal and shear strains in the reinforcement bar;

N_{3x}, N_{3xy}, N_{3y}, N_{3xy} are linear cohesions of the inclined and longitudinal cracks respectively

In order to determine these stresses, let us project all the forces applied to the edges of the element on the axis *x'* and *y'*:

— onto the axis *x'*:

$$\sigma_{x'} h(\cos\alpha + \sin(\beta+\alpha)) + \tau_{x'y'} h(\cos(\beta+\alpha) + \sin\alpha) =$$
$$= \sigma_{sy} f_{sy} \cos\alpha - \tau_{sxy} f_{sy} \cos\alpha + N_{3x'y'} \sin\alpha + N_{3x'} \cos\alpha - N_{3y'x'} \cos(\beta+\alpha) + N_{3y'} \sin(\beta+\alpha); \quad (4)$$

— onto the axis *y'*:

$$\sigma_{y'}h(\cos(\beta+\alpha)+\sin\alpha)+\tau_{x'y'}h(\cos\alpha+\sin(\beta+\alpha))=$$
$$=\sigma_{sy}f_{sy}\sin\alpha+\tau_{sxy}f_{sy}\sin\alpha-N_{зx'y'}\cos\alpha+N_{зx'}\sin\alpha+N_{зy'x'}\sin(\beta+\alpha)+N_{зy'}\cos(\beta+\alpha).$$

2. Cohesion of crack edges through concrete relations. According to [1, 6], while presenting the relations of cohesion of the crack edges as evently distributed along the crack area, we can linear cohesion forces that occur in the cracks using the formulas

$$\begin{aligned} N_{зx'y'} &= hE_{зx'y'}\Delta_{x'y'}/l_T; \\ N_{зy'x'} &= hE_{зy'x'}\Delta_{y'x'}/l_T; \\ N_{зx'} &= hE_{зx'}a_{x'}/l_T; \\ N_{зy'} &= hE_{зy'}a_{y'}/l_T. \end{aligned} \tag{5}$$

Using the dependence of axial movements on the crack opening a_i and a mutual displacement of its edges Δ_i and determining the axial displacements of the reinforcement as a function of average deformations of the reinforcement and concrete on the areas between the cracks, we get the following ratios for the components of the cohesion forces (see Fig. 3):

$$\begin{aligned} N_{зx'y'} &= -(hE_{зx'y'}\varepsilon_{sx'}\cos\alpha-hE_{зx'y'}\varepsilon_{sy'}\sin\alpha)/\sin 2\alpha; \\ N_{зy'x'} &= -(hE_{зy'x'}\varepsilon_{sx'}\cos(\beta+\alpha)-hE_{зy'x'}\varepsilon_{sy'}\sin(\beta+\alpha))/\sin(\beta+2\alpha); \\ N_{зx'} &= (hE_{зx'}\varepsilon_{sy'}\cos\alpha+hE_{зx'}\varepsilon_{sx'}\sin\alpha)/\sin 2\alpha; \\ N_{зy'} &= (hE_{зy'}\varepsilon_{sy'}\cos(\beta+\alpha)+hE_{зy'}\varepsilon_{sx'}\sin(\beta+\alpha))/\sin(\beta+2\alpha), \end{aligned} \tag{6}$$

where $E_{зx`y}$, $E_{зy`x}$, $E_{зx}$, $E_{зy`}$ are secant moduli of drformations of cohesion forces determined according to the strength criterion [6] that in the first approximation can be assumed to be $E_{зx`y} = E_{зx}$, $E_{зy`x} = E_{зy}$.

Let us express the elasticity using the dependencies suggested in [9] to determine the shear force following the formation of cracks considering the "peg" effect in the longitudinal reinforcement and occurring along the edges of the joint of the cohesion force displacement:

$$E_b = E_{s,tot} + E_{зn}. \tag{7}$$

Using the hypothesis by V. M. Bondarenko [3] on the invariability of damage functions describing deficit of the value of the investigated factor of a disbalanced force resistance of concrete in relation to all the physical and mechanical characteristics of force resistance of concrete for a specified cohesion modulus during deformation of the crack edges can be written as

$$E_{зn} = E_b\varphi. \tag{8}$$

3. Stresses occurring in the reinforcement bar. Assuming that in the investigated typical element the inclination angle of the longitudinal crack has a constant value $\alpha = 45$. Inserting the expression (6) into the equation system (4) and expressing the stresses in the reinforcement τ_{sy} and σ_{sy}, we get

$$\sigma_{sy} = A_1\sigma_{x'} + A_2\sigma_{y'} + A_3\tau_{x'y'};$$
$$\tau_{sy} = B_1\sigma_{x'} + B_2\sigma_{y'} + B_3\tau_{x'y'}, \tag{9}$$

where the coefficients of the equations are given by the formulas

$$A_1 = (-\sqrt{2}/4E'_{sy}\,\mu)E_{зx'} + (-\sqrt{2}/4E'_{sy}\,\mu)E_{зx'y'} +$$
$$+(\sqrt{2}(\sin 3\beta - 2\sin(3\beta/2)^2 - 2\sin(\beta/2)^2 + \sin\beta + 2)/4E'_{sy}\,\mu(2\sin\beta^2 - 2))E_{sy'} +$$
$$+(\sqrt{2}(\sin 3\beta - 2\sin(3\beta/2)^2 - 2\sin(\beta/2)^2 + \sin\beta + 2)/4E'_{sy}\,\mu(2\sin\beta^2 - 2))E_{зy'x'} -$$
$$-(\sqrt{2}(\sin(\pi/4 + 3\beta) - 4\sin\beta^2 - 4\sin(\beta/2)^2 + \sqrt{2}\sin(\pi/4 + \beta) + 6)/8\mu(\sin\beta^2 - 1);$$
$$A_2 = (-\sqrt{2}/4E'_{sy}\,\mu)E_{зx'} + (-\sqrt{2}/4E'_{sy}\,\mu)E_{зx'y'} +$$
$$+(-2\sqrt{2}\cos\beta^2 - 2\sqrt{2}\cos\beta\sin\beta/4E'_{sy}\,\mu\cos\beta)E_{зy'} +$$
$$+(2\sqrt{2}\cos\beta\sin\beta - 2\sqrt{2}\cos\beta^2 + 2\sqrt{2}/4E'_{sy}\,\mu\cos\beta)E_{зy'x'} + (\cos\beta - \sin\beta + 1)/2\mu;$$
$$A_3 = (\cos\beta + 1)/\mu; \tag{10}$$
$$B_1 = (\sqrt{2}/4E'_{sy}\,\mu)E_{зx'} + (\sqrt{2}/4E'_{sy}\,\mu)E_{зx'y'} +$$
$$+(-\sqrt{2}(\cos 2\beta - \sin 2\beta/4E'_{sy}\,\mu\cos\beta)E_{зy'} + (-\sqrt{2}(\cos 2\beta - \sin 2\beta/4E'_{sy}\,\mu\cos\beta)E_{зy'x'} -$$
$$-(\sqrt{2}(\sqrt{2}E'_{sy}/2 + E'_{sy}\sin(\pi/4 + 2\beta) + \sqrt{2}E'_{sy}\cos\beta)/4E'_{sy}\,\mu\cos\beta);$$
$$B_2 = (-\sqrt{2}/4E'_{sy}\,\mu)E_{зx'} + (-\sqrt{2}/4E'_{sy}\,\mu)E_{зx'y'} +$$
$$+(2\sqrt{2}(\cos\beta^2 + 2\sqrt{2}\cos\beta\sin\beta - 2\sqrt{2})/4E'_{sy}\,\mu\cos\beta)E_{зy'} +$$
$$+(2\sqrt{2}(\cos\beta^2 + 2\sqrt{2}\cos\beta\sin\beta)/4E'_{sy}\,\mu\cos\beta)E_{зy'x'} +$$
$$+(2E'_{sy}\cos\beta^2 + 2E'_{sy}\cos\beta - 2E'_{sy}\cos\beta\sin\beta)/4E'_{sy}\,\mu\cos\beta);\ B_3 = \sin\beta/\mu,$$

where l_{crc} is the size of the area of relative mutual displacements of concrete and reinforcement in the area adjoining the crack [2];

$$E'_{sy} = E_s/\psi_{st}, \tag{11}$$

E_s is the elasticity modulus of the reinforcement: ψ_{st} is the coefficient of averaging by V. I. Murashev [12].

4. Dependence of a flat-strained ferroconcrete in increments. As for intersecting cracks concrete loses its capacity to characterize the deformations of the element in any direction, the everage deformations of the reinforcement coincide with the general deformations of the element with cracks, for the investigated typical element in the axis *x'* and *y'* the deformations are

$$\Delta\varepsilon_{x'} = \Delta\varepsilon_{sx'};\quad \Delta\varepsilon_{y'} = \Delta\varepsilon_{sy'};\quad \Delta\gamma_{x'y'} = \Delta\gamma_{sx'y'}. \tag{12}$$

Considering the conditions of the compatibility of the displacements of the reinforcement bar in the crack and determining the axial displacements and those in direction of the axis *x*' and *y'* of the reinforcement as a function of the average deformations of the reinforcement ε_s and concrete ε_b in the areas between the cracks and then expressing tangential movements using the compliance to the axial displacements at $\varepsilon_b \approx 0$ and writing the deformations of the element with cracks in the axis *x'* and *y'*, following the insertion of the stresses in the reinforcement τ_{sy} and σ_{sy} we can write the following ratios:

$$\begin{aligned}\varepsilon_{sx'} &= (\sqrt{2}(E'_{sy}A_1-\eta_{\tau y}E_s^kB_1)/2E_s^kE'_{sy})\sigma_{x'}+(\sqrt{2}(E'_{sy}A_2-\eta_{\tau y}E_s^kB_2)/2E_s^kE'_{sy})\sigma_{y'}+\\&\quad+(\sqrt{2}(E'_{sy}A_3-\eta_{\tau y}E_s^kB_3)/2E_s^kE'_{sy})\tau_{x'y'};\\ \varepsilon_{sy'} &= (\sqrt{2}(E'_{sy}A_1+\eta_{\tau y}E_s^kB_1)/2E_s^kE'_{sy})\sigma_{x'}+(\sqrt{2}(E'_{sy}A_2+\eta_{\tau y}E_s^kB_2)/2E_s^kE'_{sy})\sigma_{y'}+\\&\quad+(\sqrt{2}(E'_{sy}A_3+\eta_{\tau y}E_s^kB_3)/2E_s^kE'_{sy})\tau_{x'y'}.\end{aligned} \tag{13}$$

where v^k_s is the elasticity coefficient that characterizes the ratio of the elastic deformations to the general deformations of the reinforcement; $\eta_{\tau y}$ is the coefficient that takes into account excessive compliance of reinforcement bars to tangential displacements that can be assumed to be 16 [6] in concrete at the boundary of the crack in the first approximation.

The dependencies (13) for two sequentially located loading steps *i* + 1 and *i* can be written in the increments of stresses and deformations in the reinforcement:

$$\begin{aligned}\Delta\varepsilon_{sx'} &= (\sqrt{2}(E'_{sy}A_1-\eta_{\tau y}E_s^kB_1)/2E_s^kE'_{sy})\Delta\sigma_{x'}+(\sqrt{2}(E'_{sy}A_2-\eta_{\tau y}E_s^kB_2)/2E_s^kE'_{sy})\Delta\sigma_{y'}+\\&\quad+(\sqrt{2}(E'_{sy}A_3-\eta_{\tau y}E_s^kB_3)/2E_s^kE'_{sy})\Delta\tau_{x'y'};\\ \Delta\varepsilon_{sy'} &= (\sqrt{2}(E'_{sy}A_1+\eta_{\tau y}E_s^kB_1)/2E_s^kE'_{sy})\Delta\sigma_{x'}+(\sqrt{2}(E'_{sy}A_2+\eta_{\tau y}E_s^kB_2)/2E_s^kE'_{sy})\Delta\sigma_{y'}+\\&\quad+(\sqrt{2}(E'_{sy}A_3+\eta_{\tau y}E_s^kB_3)/2E_s^kE'_{sy})\Delta\tau_{x'y'}.\end{aligned} \tag{14}$$

In order to get a complete system of physical equations, let us determine an increment in the inclination angle $\Delta\gamma_{x'y'}$ using the transformation formulas of relative deformations and stresses during the reverse turning of the coordinate axis. Inserting the values $\Delta\varepsilon_{sx'}$ and $\Delta\varepsilon_{sy'}$ from the equation (14) into the formulas we get the following initial inclination angle:

$$\Delta\gamma_{sx'y'} = -((\sqrt{2}A_1-1)/E_s^k)\Delta\sigma_{x'}-((\sqrt{2}A_2-1)/E_s^k)\Delta\sigma_{y'}+(\sqrt{2}A_3-2)/E_s^k). \tag{15}$$

Inserting the increments of the deformations (14) and shear deformations (15) into the expressions (12), we get the following systems of physical ratios in the increments:

$$\begin{cases}\Delta\varepsilon_{x'} = C_{11}\Delta\sigma_{x'}+C_{12}\Delta\sigma_{y'}+C_{13}\Delta\tau_{x'y'};\\ \Delta\varepsilon_{y'} = C_{21}\Delta\sigma_{x'}+C_{22}\Delta\sigma_{y'}+C_{23}\Delta\tau_{x'y'};\\ \Delta\gamma_{x'y'} = C_{31}\Delta\sigma_{x'}+C_{32}\Delta\sigma_{y'}+C_{33}\Delta\tau_{x'y'}.\end{cases} \tag{16}$$

Here the coefficients of the compliance matrix $[C_{ij}]$ of a flat element on the increments of stresses and deformations are given by the expressions

$$\begin{cases} C_{11} = \dfrac{\sqrt{2}(A_1E'_{sy} - B_1\eta_{\tau y}E_s^k)}{2E_s^kE'_{sy}}; C_{12} = \dfrac{\sqrt{2}(A_2E'_{sy} - B_2\eta_{\tau y}E_s^k)}{2E_s^kE'_{sy}}; C_{13} = \dfrac{\sqrt{2}(A_3E'_{sy} - B_3\eta_{\tau y}E_s^k)}{2E_s^kE'_{sy}}; \\ C_{21} = \dfrac{\sqrt{2}(A_1E'_{sy} + B_1\eta_{\tau y}E_s^k)}{2E_s^kE'_{sy}}; C_{22} = \dfrac{\sqrt{2}(A_2E'_{sy} + B_2\eta_{\tau y}E_s^k)}{2E_s^kE'_{sy}}; C_{23} = \dfrac{\sqrt{2}(A_3E'_{sy} + B_3\eta_{\tau y}E_s^k)}{2E_s^kE'_{sy}}; \\ C_{31} = -\dfrac{\sqrt{2}A_1 - 1}{E_s^k}; C_{32} = -\dfrac{\sqrt{2}A_2 - 1}{E_s^k}; C_{33} = -\dfrac{\sqrt{2}A_3 - 2}{E_s^k}. \end{cases} \quad (17)$$

5. Physical ratios for a corrosion-damaged ferroconcrete element. The above analytical dependencies allow the compliance matrix to be designed $\left[\bar{C}_{ij}\right]$ for a corrosion-damaged long-term deformed ferroconcrete element with cracks. A change in time of the deformation properties of a concrete neutralized with an aggressive medium during the formation of a matrix $\left[\bar{C}_{ij}\right]$ can be considered using a change in the specified tangential deformation modulus in time:

$$E^*_{зп}(t) = E^*_b(t)\varphi; E^*_{зпt}(t) = E^*_b(t)\varphi, \quad (18)$$

where $E^*_b(t)$ is the dependence of change in the deformation modulus of concrete in time due to the effect of an aggressive medium.

In order to take into account the effect of an aggressive medium on the reinforcement bar, it is necessary that we determine over which period the aggressive medium reaches the reinforcement bar following the initial contact of the element with the aggressive medium. Let us denote the neutralization time of the protective layer of concrete as τ. Then the corrosion losses of the section of the reinforcement bar over the time of the effect of the aggressive medium during the formation of the compliance matrix $\left[\bar{C}_{ij}\right]$ can be taken into account using a reduction in the reinforcement coefficient $\mu(t-\tau)$ due to a reduction in the area of the section of the operating reinforcement bar in by means of the formula

$$\mu_s(t-\tau) = f_s(t-\tau)/h, \quad (19)$$

where $f_s(t-\tau)$ is the area of the reinforcement in direction of y that accounts for a length unit of the typical element depending on the time of the effect of the aggressive medium

$$f_s(t-\tau) = 0,25\pi(d - 2\delta_k(t-\tau))^2, \quad (20)$$

d is the diameter of the specified reinforcement bar; $d-2\delta_к(t-\tau)$ is the diameter of a non-corrosion damaged reinforcement bar in time; $\delta_к$ is the depth of neutralization of the reinforcement bar in time; h is the thickness of the typical ferroconcrete element.

Disruption in the cohesion of the corrosive reinforcement with concrete between cracks due to steel corrosion products are characterized with change in the cohesion coefficient

ψ^k_s. Change in time of the tangential coefficient of cohesion of the concrete reinforcement $\psi^k_s(t-\tau)$ as some analogue of the coefficient by V. M. Murashev is determined as a function of the average deformations of the reinforcement in the areas between the cracks. In the first approximation it can be assumed that as a result of the effect of the aggressive medium as the depth of corrosion of the reinforcement increases $\delta_\kappa(t-\tau)$, the cohesion coefficient $\psi^k_s(t-\tau)$ decreases proportionately to the depth of corrosion of the reinforcement.

Then a tangential deformation modulus of the reinforcement and average deformation modulus of the reinforcement in the corrosion-damaged element with cracks can be determined using the following expressions respectively:

$$E_s^k(t-\tau) = E_s \nu_s^k / \psi_s^k(t-\tau), \tag{21}$$

$$E'_{sy}(t-\tau) = E_s / \psi_s(t-\tau), \tag{22}$$

where E_s is the deformation modulus of the non-corrosion damaged reinforcement. Using the dependencies (18)—(22), the coefficients of the compliance matrix for the corrosion-damaged ferroconcrete with cracks on the increments of stresses and deformations can be written as follows:

$$\begin{cases}
\overline{C}^k_{11}(t) = \dfrac{\sqrt{2}(A_1 E'_{sy}(t-\tau) - B_1\eta_{\tau y}E_s^k(t-\tau))}{2E_s^k(t-\tau)E'_{sy}(t-\tau)}; \\
\overline{C}^k_{12}(t) = \dfrac{\sqrt{2}(A_2 E'_{sy}(t-\tau) - B_2\eta_{\tau y}E_s^k(t-\tau))}{2E_s^k(t-\tau)E'_{sy}(t-\tau)}; \\
\overline{C}^k_{13}(t) = \dfrac{\sqrt{2}(A_3 E'_{sy}(t-\tau) - B_3\eta_{\tau y}E_s^k(t-\tau))}{2E_s^k(t-\tau)E'_{sy}(t-\tau)}; \\
\overline{C}^k_{21}(t) = \dfrac{\sqrt{2}(A_1 E'_{sy}(t-\tau) + B_1\eta_{\tau y}E_s^k(t-\tau))}{2E_s^k(t-\tau)E'_{sy}(t-\tau)}; \\
\overline{C}^k_{22}(t) = \dfrac{\sqrt{2}(A_2 E'_{sy}(t-\tau) + B_2\eta_{\tau y}E_s^k(t-\tau))}{2E_s^k(t-\tau)E'_{sy}(t-\tau)}; \\
\overline{C}^k_{23}(t) = \dfrac{\sqrt{2}(A_3 E'_{sy}(t-\tau) + B_3\eta_{\tau y}E_s^k(t-\tau))}{2E_s^k(t-\tau)E'_{sy}(t-\tau)}; \\
\overline{C}^k_{31}(t) = -\dfrac{\sqrt{2}A_1 - 1}{E_s^k(t-\tau)}; \\
\overline{C}^k_{32}(t) = -\dfrac{\sqrt{2}A_2 - 1}{E_s^k(t-\tau)}; \\
\overline{C}^k_{33}(t) = -\dfrac{\sqrt{2}A_3 - 2}{E_s^k(t-\tau)}.
\end{cases} \tag{23}$$

6. Calculation example. In order to test the obtained deformation dependencies of the typical flat-strained ferroconcrete element, a calculation of the ferroconcrete wall-beam of a composite section loaded with two symmetrically positioned cocnrentrated forces in the areas adjoining the support was performed. The sizes of the structure as well as the concrete type are accepted to be the same as in the experimental beam structures [8, 10].

For a calculation analysis the typical elements are investigated in the area of the formation of inclined cracks including the element E1 positioned at the boundary of contact of two concretes (Fig. 4). The strength and deformation characteristics of concrete for the typical element E1 and the sizes of this element are specified according to the guidelines [5, 8].

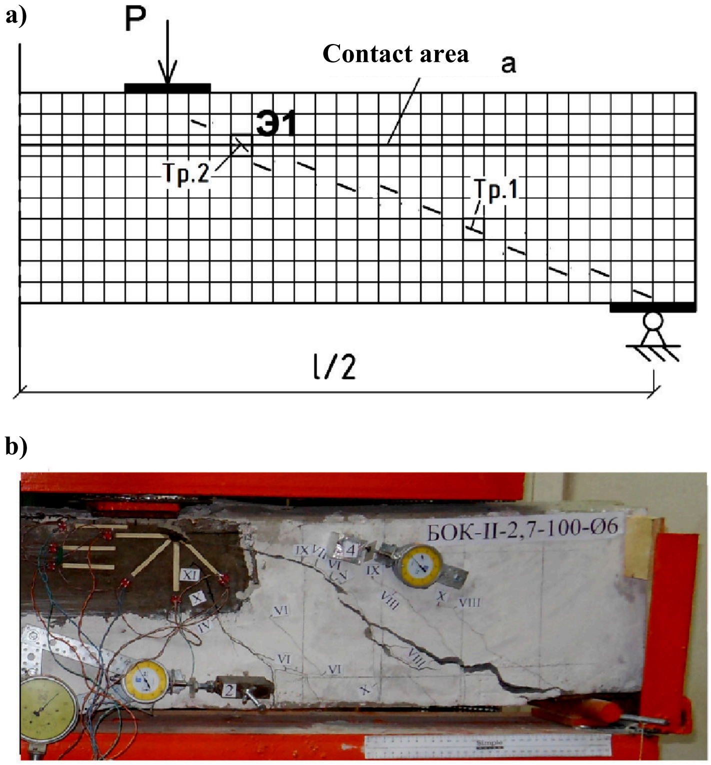


Fig. 4. Theoretical (a) and experimental (b) pictures of crack formation at different stages of laoding the structure

The calculation results as a theoretical picture of the formation of the inclined cracks in the wall-beam structure are presented in Fig. 4. Here in order to compare, there is a picture of the cracks in the same structure obtained experimentally [10]. As shown in Fig., in the investigated composite structure along with non-intersecting inclined cracks caused by the main stretching stresses (crack 1) there are intersecting cracks (crack 2) in the area of the contact joint of the composite.

Crack resistance and strength of the investigated flat-strained element in the area of the contact joint was evaluated for a beam structure of the new concrete damaged with corrosion

(Fig. 5). The resulting crack resistance criteria are in agreement with the calculation diagram of relative shear deformations of the typical element E1 (Fig. 6). The formation of cracks along the contact joint under the load $\overline{P}^*_{crc}$ leads to a significant increase in the shear deformations in the investigated element. As the load increases up to P^*_{crc} and there are intersecting cracks in the typical element (an inclined crack and cracks along the contact joint), there is another more significant increase in the shear deformation. At this laoding level P^*_{crc} the shear resistance is only observed in the reinforcement bars before there is fluidity in them (see the strength criteria of the element E1 in Fig. 5).

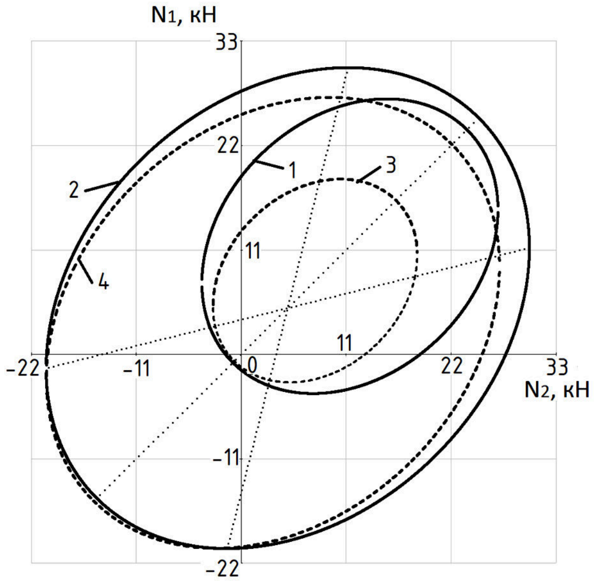
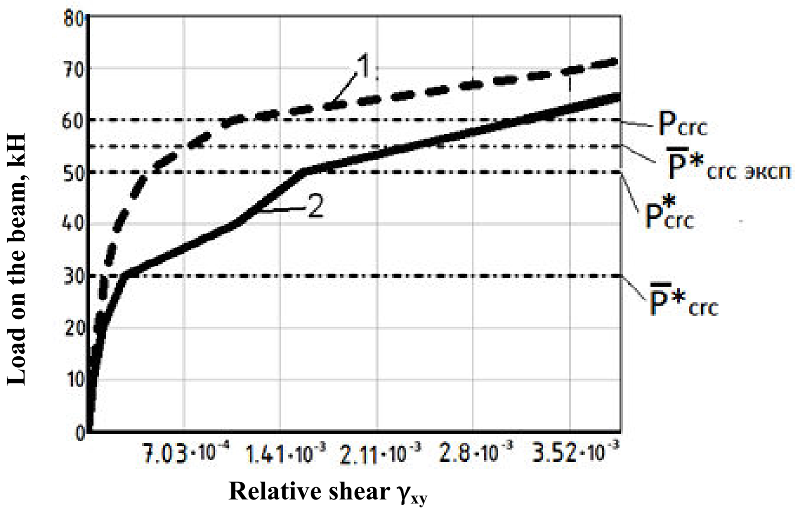


Fig. 5. Crack resistance and strength criteria of the typical flat-strained element in the contact area:
1 for the beam of the new concrete (*t* = 0);
2 for the beam of the new concrete (*t* = 0) following the formation of intersecting cracks;
3 for the beam of the concrete under the effect of the aggressive medium (*t* = 900);
4 for the beam of the concrete under the effect of the aggressive medium (*t* = 900) following the formation of intersecting cracks;
N_1, N_2 are the main efforts in the typical flat-strained element of a single size



1 for the new concrete (*t* = 0);
2 for the concrete under the effect of the aggressive medium (*t* = 900)

Fig. 6. Graph of changes in the relative shear caused by loading: $\overline{P}^*_{crc}$ is a calculation load of the formation of the longitudinal crack along the joint for a corrosion-damaged element; P^*_{crc} is a calculation load of the formation of the inclined crack for the corrosion-damaged element; $\overline{P}^*_{crc\ эксп}$ is an experimental load of the formation of the longitudinal crack along the joint of the corrosion-damaged element; P_{crc} is a calculation load of the formation of the inclined crack for the non-corrosion damaged element

Conclusions

1. The resulting calculation model of a long-term deformation of a flat-strained corrosion-damaged ferroconcrete element in the contact area of two concretes allows one to determine the stress-strain of the composite ferroconcrete flat-strained structure operating with cracks in the contact area of two beams considering a long-term deformation, corrosion of concrete and reinforcement.

2. The presented results can be employed in practical methods of calculating deformations and long-term strength of ferroconcrete composites operating with cracks. The efficiency of this calculation was shown compared to the results of a calculation involving experimental studies.

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