BASES AND FOUNDATIONS, UNDERGROUND STRUCTURES

DOI 10.25987/VSTU.2018.41.1.001
UDC 539.3 : 624.073.2

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PRESENTATION OF A KERNEL AND TRANSFORMANTS OF A NONCLASSICAL ELASTIC FOUNDATION THROUGH ITS INHOMOGENEITY FUNCTION*

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Statement of the problem. The paper is devoted to the question of practical applicability of a mathematical model by Aleinikov-Snitko which describes the contact interaction of a subbase and a shallow foundation. The main results of this method were announced with no proof in 2009 in a joint article [3].

Results. In this paper, based on the methods of integration and the Hankel transform, all the formulas and tools of the model by Aleinikov-Snitko are fully proven and optimized. Here we calculate and compare the results obtained by means of this method and the classical Mindlin method based on the theory of elasticity for a foundation whose elastic modulus is given by a power function. Interestingly, the results obtained are similar in form, but differ in magnitude. Using these results, we obtain simple approximate formulas for finding the precipitation of the base surface from the action of a point vertical load for both methods.

Conclusions. The formulas proved in the article can be useful in modeling the interaction of the soil base and the foundation, which is crucial in construction and design.

Keywords: inhomogeneous linearly deformable half-space, influence matrix, base kernel, transformant, base non-uniformity function, quasitransformant, Bessel function, Hankel transform.

Introduction. The problem of mathematical modeling of the interaction of a foundation with a subbase is a quite challenging one and has no universal solutions for all types of soils. The simplest for engineering calculations is the known Winkler model with one foundation coefficient. The main disadvantage of this model is that is fundamentally impossible to reflect a distribution capacity of soil in transmitting a vertical load in a horizontal direction while involving those layers of soil that are beyond the loading area [11, 13].

* The work is supported by the Russian Fund for Basic Research, grant № 17-01-00138.

Unlike the Winkler model, homogeneous elastic half-space modeling a subbase foundation using the elasticity theory causes an increase in the distribution capacity of actual soil.

In practice in order to mitigate the above disadvantages, the Winkler model is supplemented by new parameters (extra foundation coefficients) and a model of an elastic half-space is assumed to have elastic characteristics (elasticity modulus or the Poisson coefficient) that change as does the depth of a foundation point.

In [1, 3] S. M. Aleynikob following N. K. Snitko's suggestion [15] developed a calculation scheme for the interaction of a foundation and heterogeneous foundation base. In this paper an analytical aspect of the method is developed and compared with the classical approach based on the methods of the elasticity theory and the Fourier transformation. The problem of identifying the main part of the deposit of the surface of a soil foundation under the effect of a concentrated force is discussed as well as the options for practical implementation of the method.

1. Statement of the problem and preliminary data. A model of a linearly deformed foundation (half space) will be used where settling w and Ω distributed along the area *on a daytime surface* of a half space and the load q are connected with a ratio

$$w(x,y)=\iint\limits_{\Omega} G(x,y,u,v)\; q(u,v)\,du\,dv, \tag{1}$$

where $G(x,y,u,v)$ is a so-called *nuclear of an elastic foundation* [12]. The latter is an influence function that equals the displacement of the point $P(x,y)$ of a daytime surface of an elastic half-space caused by a single vertical concentrated force applied to the point $Q(u,v)$ of the surface.

Note that for the half-space which is isotropic in a horizontal area when its deformation characteristics depend only on the vertical coordinate z, nuclear $G(x,y,u,v)$ depend only on the distance r between P and Q. It is the case that will be dealt with as we proceed.

1.1. Representation of the influence function based on the methods of the elasticity theory. Let a single load be concentrated at the beginning of the coordinate system XOY on a daytime surface and thus $r=\sqrt{x^2+y^2}$. The use of the methods of the elasticity theory and Fourier transformation lead to the following formula [10, 12, 14] while searching for an influence function:

$$w(x,y)=\omega(r)=\frac{1}{2\pi}\int_0^\infty sc(s)J_0(sr)ds, \tag{2}$$

where $J_0(x)$ is the first-class Bessel function of order zero and $c(s)$ is a so-called *transform* that meets (according to the Fourier transformation) the equation

$$c(s)=2\pi\int_0^\infty r\omega(r)J_0(sr)dr. \tag{3}$$

Note that the integrals (2) and (3) included in the formulas are nothing but a direct and reverse Hankel transformation which for the Bessel function of order k J_k is given by the formula

$$H_k(f)(r) = \int_0^\infty sf(s)J_k(sr)ds. \tag{4}$$

1.2. Calculation of the nuclear using the function of heterogeneity of a foundation. Following the example of N.K. Snitko [15], S. M. Aleynikov developed an alternative method for identifying the function ω that determines the nuclear for an isotropic, heterogeneous linearly deformed half-space. Let $E(z)$ be the elasticity modulus of a foundation depending on the depth z, and ν is the Poisson coefficient of the half-space that will be considered constant. Then according to [1, 3], settling of a daytime surface caused by a single load can be specified with the equation

$$\Omega(r) = \frac{(1-\nu^2)}{2\pi}\int_0^\infty \frac{z^3 dz}{e(R)R^5}, \quad R = \sqrt{r^2+z^2}, \tag{5}$$

where the function

$$e(R) = \int_0^1 E(Rt)t^2 dt = \frac{1}{R^3}\int_0^R E(z)z^2 dz = \frac{1}{3R^3}\int_0^R E(z)dz^3, R \ge 0 \tag{6}$$

is generated by the law of change $E(z)$ and called *the function of heterogeneity of a foundation*. For a homogeneous space when $E(z) = E_0 = const$, the formulas (2) and (5) yield the same results while in other cases (e.g., when $E(z) = E_n z^n$) these formulas yield similar-looking but different expressions for $\omega(r)$.

The comparison of the formulas (2) and (5) shows that the functions $c(s)$ and $e(R)$ play a similar role, i.e. they generate a nuclear of an elastic foundation. Certainly there is an issue with identifying a function similar to $c(s)$ that corresponds with a new method. I.*e. using the specified function* $e(R)$ *we will be identifying such a function* $C(s)$ *so that with the formula* (2) *it yielded the same result that the formula* (5). Such a function $C(s)$ will be called a *quasi-transform* one.

2. Main results

2.1. Presentation of a quasi-transform using a function of heterogeneity of a foundation. The first formula is obtained by inserting the expression (5) into the formula (3):

$$C(s) = 2\pi\int_0^\infty r\Omega(r)J_0(sr)dr = 2\pi\frac{(1-\nu^2)}{2\pi}\int_0^\infty r\int_0^\infty \frac{z^3 dz}{e(R)R^5} J_0(sr)dr,$$

or

$$C(s) = \frac{(1-\nu^2)}{4}\int_0^\infty r\left(\int_0^\infty \frac{dz^4}{e(R)R^5}\right)J_0(sr)dr, \quad R = \sqrt{r^2+z^2}. \tag{7}$$

Changing the integration order, we get the second formula:

$$C(s)=\frac{(1-\nu^2)}{4}\int_0^\infty\left(\int_0^\infty \frac{r}{e(R)R^5}J_0(sr)dr\right)dz^4,\quad R=\sqrt{r^2+z^2}. \tag{8}$$

In the formula (7) in the internal integral $r=\mathrm{const}$. Thus

$$dz^4=2z^2dz^2=2((z^2+r^2)-r^2)d(z^2+r^2).$$

Replacing the variable $z^2+r^2=R^2,\ \ 0\le z<\infty,\ \ r\le R<\infty$ by (7) we get

$$\begin{aligned}C(s)&=\frac{(1-\nu^2)}{2}\int_0^\infty r\left(\int_r^\infty \frac{(R^2-r^2)dR^2}{e(R)R^5}\right)J_0(sr)dr=\\&=(1-\nu^2)\int_0^\infty r\left(\int_r^\infty \frac{(R^3-r^2R)dR}{e(R)R^5}\right)J_0(sr)dr.\end{aligned} \tag{9}$$

Similar transformations applied for the formula (5) result in the equation

$$\Omega(r)=\frac{1-\nu^2}{2\pi}\int_r^\infty \frac{(R^3-r^2R)dR}{e(R)R^5}=\frac{1-\nu^2}{2\pi}\left(\int_r^\infty \frac{dR}{e(R)R^2}-r^2\int_r^\infty \frac{dR}{e(R)R^4}\right). \tag{10}$$

Let us change the order of integration in the formula (9) and repalce $r=tR,\ 0\le t\le 1$:

$$\begin{aligned}C(s)&=(1-\nu^2)\int_0^\infty\left(\int_0^R \frac{rR^3-r^3R}{e(R)R^5}J_0(sr)dr\right)dR=\\&=(1-\nu^2)\int_0^\infty\left(\int_0^1 \frac{tR^4-t^3R^4}{e(R)R^5}J_0(sRt)Rdt\right)dR=(1-\nu^2)\int_0^\infty \frac{1}{e(R)}\left(\int_0^1(t-t^3)J_0(sRt)dt\right)dR.\end{aligned} \tag{11}$$

Let us use the formula (6.2) as suggested by V. G. Korenev [7, p. 23]:

$$\frac{d}{zdz}(z^kJ_k(z))=z^{k-1}J_{k-1}(z),$$

according to which we get the known equation

$$\int_0^u z^{k+1}J_k(z)dz=u^{k+1}J_{k+1}(u).$$

Then assuming that $sR=a$, we get

$$\int_0^1 tJ_0(sRt)dt=\int_0^1 tJ_0(at)dt=\frac{1}{a^2}\int_0^1 atJ_0(at)dat=\frac{1}{a^2}zJ_1(z)\,\Big|_0^a=\frac{J_1(a)}{a}=\frac{J_1(sR)}{sR}. \tag{12}$$

Integrating individual parts and using the previous formula we have

$$\begin{aligned}\int_0^u t^3J_0(t)dt&=t^2\int_0^t sJ_0(s)ds\,\Big|_0^u-\int_0^u 2t\int_0^t sJ_0(s)dsdt=\\&=u^2\int_0^u tJ_0(t)dt-2\int_0^u t^2J_1(t)dt=u^3J_1(u)-2u^2J_2(u).\end{aligned}$$

Hence

$$\int_0^1 t^3J_0(sRt)dt=\frac{1}{(sR)^4}\int_0^1(sRt)^3J_0(sRt)dsRt=\frac{1}{(sR)^4}((sR)^3J_1(sR)-2(sR)^2J_2(sR)).$$

Deducting this formula from (12) and considering (11) we get the expression we need

$$C(s)=\frac{2(1-\nu^2)}{s^2}\int_0^\infty \frac{J_2(sR)}{e(R)R^2}dR. \tag{13}$$

2.2. Representation of nuclei and quasi-transforms using a function of heterogeneity of a foundation for a power law of change $E(z)$. The main results regarding the nuclei determined using the formulas (2), (3) are related to the power claw of change of the elasticity modulus with the depth $E(z)=E_n z^n, n\geq 0$ (the Poisson coefficient $\nu = const$). Hence in [6, 7] the following expressions for the nuclei and transforms were obtained:

$$\omega_n(r)=\frac{1-\nu^2}{\pi E_n r^{n+1}},\quad c_n(s)=\frac{s^{n-1}2^{1-n}(1-\nu^2)\Gamma(\frac{1-n}{2})}{E_n\Gamma(\frac{1+n}{2})}, \tag{14}$$

where $\Gamma(z)$ is the Euler gamma-function.

Let us look at how this can be applied to the method based on the function of heterogeneity of the foundation. According to the formula (6) we have

$$e(R)=\frac{1}{R^3}\int_0^R E(z)z^2dz=\frac{1}{R^3}\int_0^R E_n z^{n+2}dz=\frac{E_nR^n}{n+3}. \tag{15}$$

Hence using the formula (10) equivalent to the formula (5), we get

$$\begin{aligned}\Omega_n(r)&=\frac{1-\nu^2}{2\pi}\left(\int_r^\infty \frac{dR}{e(R)R^2}-r^2\int_r^\infty\frac{dR}{e(R)R^4}\right)=\frac{1-\nu^2}{2\pi}\left(\int_r^\infty \frac{(n+3)dR}{E_nR^{n+2}}-r^2\int_r^\infty\frac{(n+3)dR}{E_nR^{n+4}}\right)=\\ &=\frac{(1-\nu^2)(n+3)}{2\pi E_n}\left(\frac{1}{r^{n+1}n+1}-\frac{1}{r^{n+1}(n+3)}\right)=\frac{(1-\nu^2)}{\pi E_n r^{n+1}(n+1)}.\end{aligned} \tag{16}$$

The comparison of the formulas (14) and (16) shows that in case when $n=0$ (homogeneous half-space) both methods yield the same result and at $n>0$ settling of a daytime surface of a foundation under a single load calculated using the formulas (5), (6) turns out to be $n+1$ times smaller. This indicates that in the foundation model designed using the function of heterogeneity of the foundation the distribution capacity of the soil turns out to be smaller than that of the model designed based on the laws of the theory of elasticity. Let us show that the quasi-transform calculated using the formula (13) is also $n+1$ times smaller. It is true that the integral emerging in (13) is a table integral for the Hankel transformation [11]:

$$\begin{aligned}C_n(s)&=\frac{2(1-\nu^2)}{s^2}\int_0^\infty\frac{J_2(sR)}{e(R)R^2}dR=\frac{2(1-\nu^2)}{s^2}\int_0^\infty\frac{(n+3)J_2(sR)}{E_nR^{n+2}}dR=\\ &=\frac{2(1-\nu^2)(n+3)}{s^2E_n}\frac{2^{-n-2}\Gamma(1/2-(n+2)/2+1)}{s^{-n-1}\Gamma(1/2+(n+2)/2+1)}=\frac{(1-\nu^2)(n+3)s^{n-1}}{E_n}\frac{2^{-n-1}\Gamma((1-n)/2)}{\Gamma((n+3)/2+1)}=\\ &=\frac{(1-\nu^2)s^{n-1}}{E_n}\frac{2^{-n+1}\Gamma((1-n)/2)}{\Gamma((n+1)/2)(n+1)}.\end{aligned} \tag{17}$$

The last equation is obtained using the known property $\Gamma(z+1) = z\Gamma(z)$ two times [8].

2.3. Combined law of changes in the elastiticity modulus with evaluation of the behavior of a quasi-nuclear (settling) in zero and infinity. As at $n > 0$ the elasticity modulus $E_n z^n$ turns to zero on a daytime surface of the half-space, which does not correspond with the actual properties of soil, we will look at a feasible case

$$E(z) = E_0 + E_n z^n,$$

where $E_n = \dfrac{E_H - E_0}{H}$ and E_H is the elasticity modulus at the depth H. Then

$$e(R) = \frac{E_0}{3} + \frac{E_n}{n+3} R^n,$$

and using the formula (10) we get the expression for the quasi-nuclear

$$\begin{aligned}\Omega_n^0(r) &= \gamma\left(\int_r^\infty \frac{dR}{e(R)R^2} - r^2 \int_r^\infty \frac{dR}{e(R)R^4}\right) = \\ &= \gamma\frac{3}{E_0}\left(\int_r^\infty \frac{dR}{(1+(R/Q)^n)R^2} - r^2\int_r^\infty \frac{dR}{(1+(R/Q)^n)R^4}\right),\end{aligned} \tag{18}$$

where $Q = \sqrt[n]{\dfrac{(n+3)E_0}{3E_n}}$, a $\gamma = \dfrac{1-v^2}{2\pi}$. At $n = 1$ after the integration we get

$$\begin{aligned}\Omega_1^0(r) &= \gamma\frac{3}{E_0}\left(\left(\frac{1}{Q} - \frac{r^2}{Q^3}\right)\ln\frac{r/Q}{1+r/Q} + \frac{2}{3r} - \frac{r}{Q^2} + \frac{1}{2Q}\right) = \\ &= \gamma\frac{3}{E_0}\cdot\frac{2}{3r} + \gamma\frac{3}{E_0}\left(\left(\frac{1}{Q} - \frac{r^2}{Q^3}\right)\ln\frac{r/Q}{1+r/Q} - \frac{r}{Q^2} + \frac{1}{2Q}\right) = \\ &= \omega_0(r) + \gamma\frac{3}{E_0}\left(\left(\frac{1}{Q} - \frac{r^2}{Q^3}\right)\ln\frac{r/Q}{1+r/Q} - \frac{r}{Q^2} + \frac{1}{2Q}\right),\end{aligned}$$

where as previously $\omega_0(r) = \dfrac{1-\nu^2}{\pi E_0 r}$ is the nuclear of a homogeneous space with the elasticity modulus E_0. At $r \to 0$ dropping the members converging to zero we get

$$\Omega_1^0(r) \approx \frac{1-\nu^2}{\pi E_0 r} + \gamma\frac{9E_1}{4{E_0}^2}\left(\ln r/Q + \frac{1}{2}\right) = \omega_0(r) + \gamma\frac{9E_1}{4{E_0}^2}\left(\ln r/Q + \frac{1}{2}\right).$$

Hence at small r the quasi-nuclear is $\Omega_1^0(r) < \omega_0(r)$, but it has the same growth order as $\omega_0(r)$.

At $n = 2$ after the integration we get

$$\Omega_2^0(r) = \gamma\frac{3}{E_0}\left(\frac{2}{3r} + \frac{r}{Q^2} - \left(\frac{1}{Q} + \frac{r^2}{Q^3}\right) arctg\frac{Q}{r}\right) = \omega_0(r) + \gamma\frac{3}{E_0}\left(\frac{r}{Q^2} - \left(\frac{1}{Q} + \frac{r^2}{Q^3}\right) arctg\frac{Q}{r}\right).$$

At $r \to 0$ dropping the members converging to zero we get

$$\Omega_2^0(r) \approx \omega_0(r) - \frac{(1-\nu^2)}{4E_0} \cdot \sqrt{\frac{27E_2}{5E_0}}.$$

Let us now examine the behavior of $\Omega_n^0(r)$ at $r \to \infty$ and $r \to 0$ in a general case. Using the formula (18) and replacing with $t = R/Q$ we get

$$\Omega_n^0(r) = \gamma \frac{3}{E_0}\left(\int_r^\infty \frac{dR}{(1+(R/Q)^n)R^2} - r^2 \int_r^\infty \frac{dR}{(1+(R/Q)^n)R^4}\right) =$$
$$= \frac{3\gamma}{E_0}\left(\frac{1}{Q}\int_{r/Q}^\infty \frac{dt}{(1+t^n)t^2} - \frac{r^2}{Q^3}\int_{r/Q}^\infty \frac{dt}{(1+t^n)t^4}\right).$$

Let $r > Q$, then $t > r/Q > 1$ and thus $\dfrac{t^{-n}}{(Q/r)^n+1} < \dfrac{t^{-n}}{t^{-n}+1} = \dfrac{1}{1+t^n} < t^{-n}$.

Therefore

$$\frac{3\gamma}{E_0}\left(\frac{1}{Q}\int_{r/Q}^\infty \frac{dt}{(t^{-n}+1)t^{n+2}} - \frac{r^2}{Q^3}\int_{r/Q}^\infty \frac{dt}{t^{n+4}}\right) < \Omega_n^0(r) <$$
$$< \frac{3\gamma}{E_0}\left(\frac{1}{Q}\int_{r/Q}^\infty \frac{dt}{t^{n+2}} - \frac{r^2}{Q^3}\int_{r/Q}^\infty \frac{dt}{(t^{-n}+1)t^{n+4}}\right).$$

Replacing t^{-n} with $(r/Q)^{-n}$, we will strengthen the inequality. Thus after the integration in the left and right parts of the inequality we get

$$\frac{3\gamma}{E_0}\left(\frac{1}{Q}\frac{Q^{n+1}}{((Q/r)^n+1)(n+1)r^{n+1}} - \frac{r^2}{Q^3}\frac{Q^{n+3}}{(n+3)r^{n+3}}\right) < \Omega_n^0(r) <$$
$$< \frac{3\gamma}{E_0}\left(\frac{1}{Q}\frac{Q^{n+1}}{(n+1)r^{n+1}} - \frac{r^2}{Q^3}\frac{Q^{n+3}}{((Q/r)^n+1)(n+3)r^{n+3}}\right),$$

or

$$\frac{3\gamma Q^n}{E_0}\left(\frac{2}{(n+1)(n+3)r^{n+1}} - \frac{(Q/r)^n}{((Q/r)^n+1)(n+1)r^{n+1}}\right) < \Omega_n^0(r) <$$
$$< \frac{3\gamma Q^n}{E_0}\left(\frac{2}{(n+1)(n+3)r^{n+1}} + \frac{(Q/r)^n}{((Q/r)^n+1)(n+3)r^{n+1}}\right).$$

Ultimately considering that $Q^n = \dfrac{(n+3)E_0}{3E_n}$, we conclude that at larger r

$$\Omega_n^0(r) \approx \frac{3\gamma Q^n}{E_0} \cdot \frac{2}{(n+1)(n+3)r^{n+1}} = \frac{(1-\nu^2)}{\pi E_n (n+1)r^{n+1}} = \frac{\omega_n(r)}{n+1}.$$

Now let α be some small fixed number and $r < \alpha Q$.

Then

$$\Omega_n^0(r) = \frac{3\gamma}{E_0}\left(\frac{1}{Q}\int_{r/Q}^{\alpha}\frac{dt}{(1+t^n)t^2} - \frac{r^2}{Q^3}\int_{r/Q}^{\alpha}\frac{dt}{(1+t^n)t^4}\right) + \Omega_n^0(\alpha Q), \tag{19}$$

and for examining the behavior of $\Omega_n^0(r)$ at $r \to 0$ all we have to do is to evaluate $\Omega(r)$ that is behind the bracket. We have

$$\Omega(r) = \left(\frac{1}{Q}\int_{r/Q}^{\alpha}\frac{dt}{(1+t^n)t^2} - \frac{r^2}{Q^3}\int_{r/Q}^{\alpha}\frac{dt}{(1+t^n)t^4}\right) < \left(\frac{1}{Q}\int_{r/Q}^{\alpha}\frac{dt}{t^2} - \frac{r^2}{Q^3}\int_{r/Q}^{\alpha}\frac{dt}{t^4} + \frac{r^2}{Q^3}\int_{r/Q}^{\alpha}\frac{\alpha^n dt}{(1+\alpha^n)t^4}\right).$$

Similarly, we get the estimate below

$$\left(\frac{1}{Q}\int_{r/Q}^{\alpha}\frac{dt}{t^2} - \frac{r^2}{Q^3}\int_{r/Q}^{\alpha}\frac{dt}{t^4} - \frac{1}{Q}\int_{r/Q}^{\alpha}\frac{\alpha^n dt}{(1+\alpha^n)t^2}\right) < \Omega_n(r).$$

After the integration we conclude that

$$\frac{2}{3r} - \frac{1}{\alpha Q} + \frac{r^2}{3(\alpha Q)^3} - \frac{\alpha^n}{1+\alpha^n}\left(\frac{1}{r} - \frac{1}{\alpha Q}\right) < \Omega(r) < \frac{2}{3r} - \frac{1}{\alpha Q} + \frac{r^2}{3(\alpha Q)^3} + \frac{\alpha^n}{1+\alpha^n}\left(\frac{1}{3r} - \frac{r^2}{3(\alpha Q)^3}\right).$$

As $r \to 0$ and α is a randomly small fixed number, we conclude that $\Omega(r)$ is mostly made up of $\frac{2}{3r}$.

Therefore going back to the formula (19) we get the estimate

$$\Omega_n^0(r) = \frac{3\gamma}{E_0}\Omega(r) + \Omega_n^0(\alpha Q) = \frac{3\gamma}{E_0}\left(\frac{2}{3r} + o\left(\frac{1}{r}\right)\right) + \Omega_n^0(\alpha Q) = \omega_0(r) + o\left(\frac{1}{r}\right).$$

I.e. at any n $\Omega_n^0(r)$ of almost zero is mostly made up of *the nuclear of the elastic half-* $\omega_0(r)$.

As seen from the above results, even at $n = 1, 2$ the expressions for the quasi-nuclear $\Omega_n^0(r)$ obtained based on the formula (18) are complex and not informative (as they contain growing and mutually destructive summands). A more simple but quite precise formula for the quasi-nuclear is certainly necessary.

This is the following formula:

$$\Omega_n^0(r) \approx \frac{1-\nu^2}{\pi(E_0 r + (n+1)E_n r^{n+1})}, \tag{20}$$

which obviously keeps $\Omega_n^0(r)$ at the same level of decrease for infinity and growth at zero. Tests show that at other arguments the formula proves more accurate for calculating $\Omega_n^0(r)$ as shown in the following figures obtained for $n = 1, 2$ at $Q = 1$ (Fig. 1, 2).

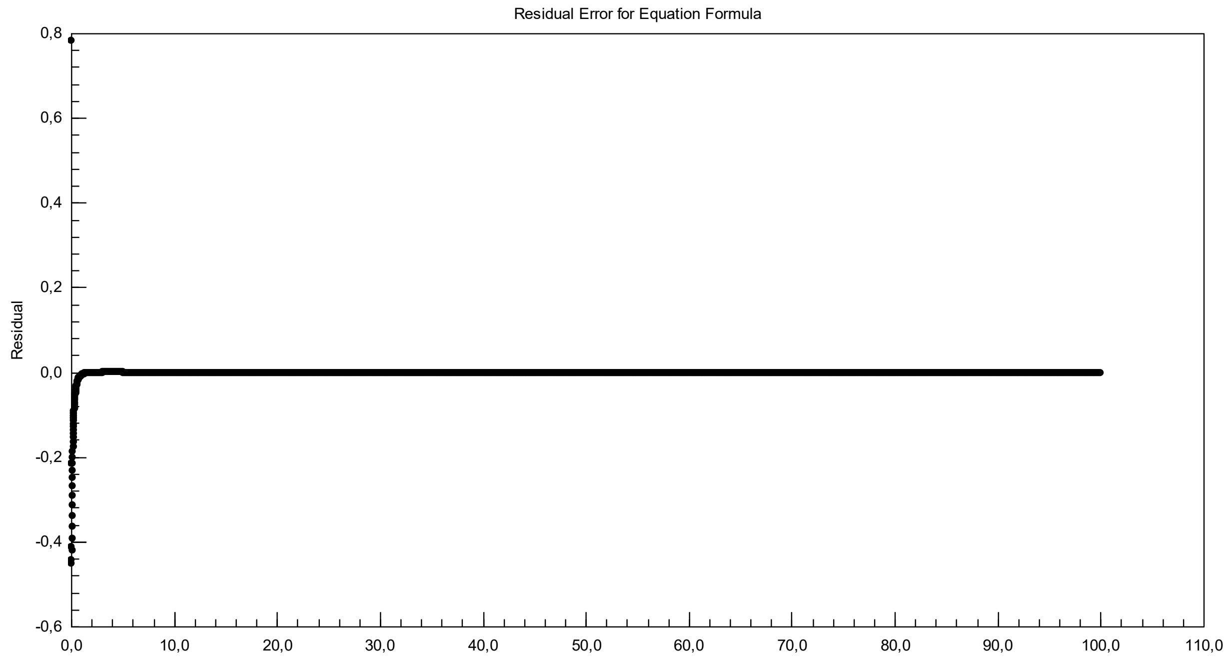

Fig. 1. Absolute error of the formula (20) at $n = 1$ and $Q = 1$

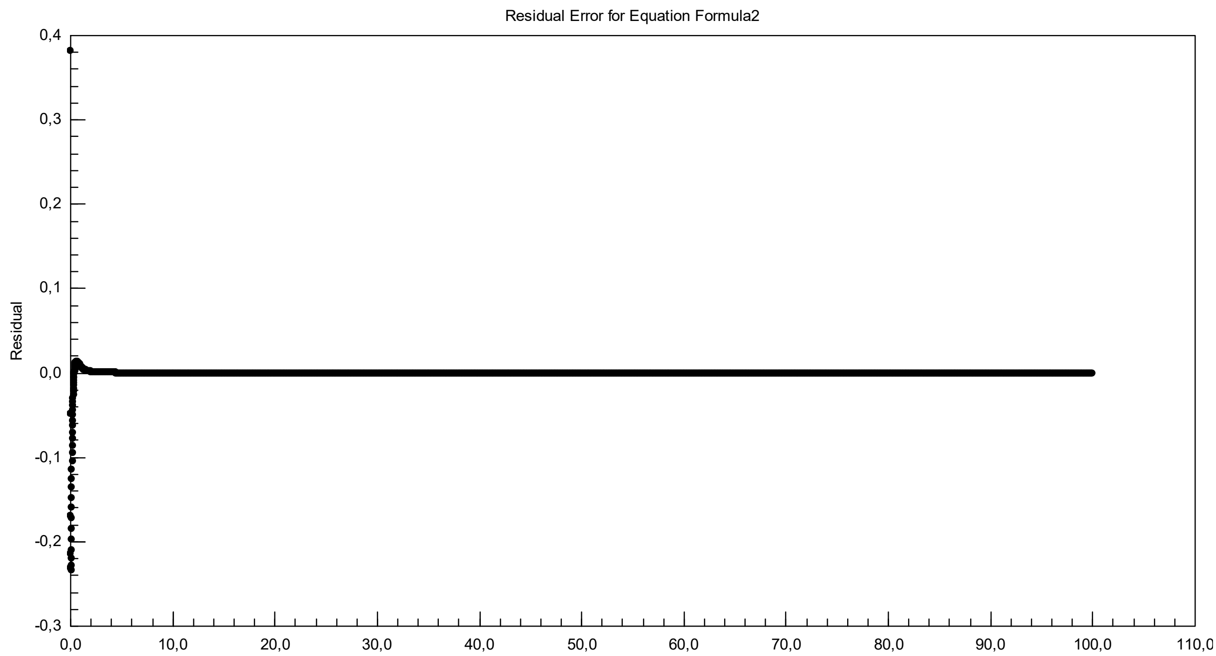

Fig. 2. Absolute error of the formula (20) at $n = 2$ and $Q = 1$

A somewhat lower accuracy at almost zero is due to the fact that there is not an asymptotic approximate equality $\Omega_n^0(r) \approx \omega_0(r)$ but an equality of the growth rate (equivalency of two infinitely large values).

Conclusions. The paper looks at a very important problem that has to do with mathematical modeling of the interaction between a base and a foundation which is also closely related to practical tasks facing foundation design and strength calculations of buildings and structures.
The model of a soil foundation suggested by Aleynikov-Snitko has been considered in detail. By modifying the Hankel transformation the author was able to calculate quasi-transforms that correspond with the quasi-nuclear of the foundation. New formulas for the function of heterogeneity and a quasi-nuclear of an isotropic, heterogeneous linearly deformed foundations were obtained.
For the combined power law of changes in the elasticity modulus of an *n*-order foundation an asymptotic behavior of the quasi-nuclear at zero and infinity was studied.
A simple approximate formula describing a (quasi) nuclear in case of a combined power law was obtained.
The use of the method by Aleynikov-Snitko was found to show a large distribution capacity of soil compared to that by Winker and a smaller one than in that of the elastic half-space.
The obtained formulas will be instrumental in calculating the strength of construction structures and solutions of spatial tasks of contact interaction of a base and a foundation.

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