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# MATHEMATICAL ANALYSIS OF HUMIDITY URE TRANSFER OF UNDERGROUND WATER IN THE SUBBASE OF ROAD SURFACING AND PREDICTING THE PARAMETERS OF A DRAINAGE SYSTEM

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**Statement of the problem**. The problems of searching for an accelerated solution of the problem of calculating the parameters of drainage systems are studied by means of mathematical modeling of water transfer in a porous subbase of a road.

**Results.** Based on an exact solution of a humidity ure transport equation in porous soils, a technique for investigating vertical filtration has been developed. It is shown that the rate of a rise in water levels along the height of the soil layer is unstable and changes as a maximum value of the height of a subbase is reached.

**Conclusions.** The technique sufficiently describes the transfer of humidity ure in a porous soil and is useful for accelerated analysis of the parameters of drainage systems in the subbase of a road. An example of the application of the procedure is given for a new scheme of water removal from under the roadway, which reduces the negative effects of frost heaving.

**Keywords:** road surfacing, destruction, soil freezing, filtration, fluid migration, mathematical modeling.

**Introduction**. Motorway foundations are water-resistant media, the water filtration rate for a single pumping gradient changes from several hundreds of m/day (for pebbles and gravel with coarse sand) to a one tenth proportion m/day (for weakly penetrated loam, sandy loam) [9, 14]. The height of a capillary lift over a mirror of underground water in sand can be 1 m and more and it can be 8 m in clays [14], the porosity of sandy soils varies in the range of 0.55…0.80 [4].

Low deposition of underground waters causes porous water to come into the upper part of a base foundation. For negative Celsius temperatures this water freezes and it goes from liquid to solid accompanied by forces of frosty swelling [15] that result in the failure of roadway surfacing.

In order to dry water-saturated soil, the upper part of a drowned slope is drained using high-porous sand and gravel drains [5] and anti-filtration screens [13], oil waste hydrophobisators, preventing water migration in the freezing zone [15]. In [12] there is a suggestion for water drainage systems using draining pipes.

A high-speed solution for calculating the parameters of drainage systems is achieved by mathematical modeling of water transfer in a porous motorway foundation. There are also papers that deal with modeling in porous media [1, 9, 11, 17]. In [16] a mathematical modeling describing an improbable imbalanced heat and thermal transfer in a three-dimensional porous medium is set forth.

There are methods of mathematical modeling of filtration in soils considering film humidity transfer [6], evaporation from its free surface [2], horizontal filtration increasing as does the depth of underground water [19], non-stationary vertical filtration based on the Fokker-Planck equation [18] based on vertical capillary cylinders [20].

In the present paper a mathematical model is suggested that allows only for vertical filtration and major typical porosity of soils, an accurate solution for the equation of the humidity transfer and the way it could be applied for calculating a drainage system.

1. Statement of the problem. The Figure presents a scheme of a road foundation that is a porous medium. The humidity ure transfer of underground water is presented which is a porous medium. The humidity ure transfer of underground water in the vertical direction occurs under the effect of capillary forces and gravity.

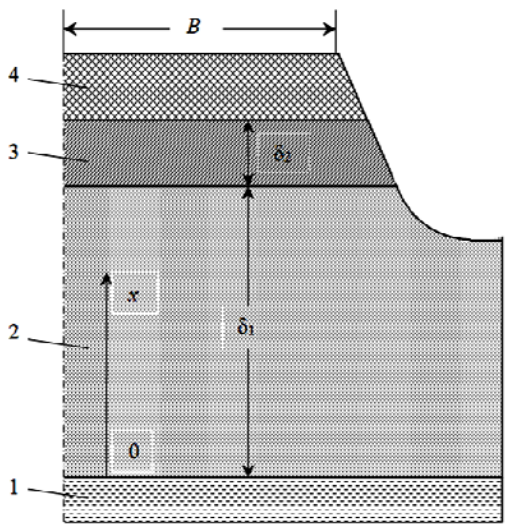


**Fig.** Scheme for calculating humidity ure transfer of underground water in a motorway foundation:
1 is underground water;
2 is a soil foundation;
3 is a draining layer;
4 is roadway surfacing

In this case a differential equation of humidity ure transfer, which is a particular case of the movement equation presented in [16] takes the following form:

$$\frac{1}{m^2} u \frac{du}{dx} = \frac{\nu}{k} u - g, \tag{1}$$

where $m = V_{por}/V$ is the porosity of the soil; $k$ is the coefficient of the penetrability of the soil that has a dimensional area; $x$ is the coordinate calculated from the vertical up to the underground water level; $u$ is a projection of the water lifting rate up to the axis $x$; $\nu$ is a kinematic viscosity of water; $g$ is the free fall acceleration. In engineering calculations of water penetrability of soils the filtration coefficient is employed

$$K_{\phi} = gk/\nu, \tag{2}$$

characterizing the filtration rate related to a dimensionless pumping gradient [3].

We transform the equation (1):

$$\frac{du^2}{dx} = 2m^2 \left( \frac{\nu}{k} u - g \right), \tag{3}$$

by integrating it

$$\int_0^u \frac{du^2}{\left( \frac{\nu}{k} u - g \right)} = 2m^2 \int_0^x dx. \tag{4}$$

we get an accurate solution:

$$u = u_g - u_\sigma = m\sqrt{2g\,x} - m^2 \nu\, x/k, \tag{5}$$

where $u_\sigma$ and $u_g$ are the humidity  transfer rates in a porous soil only under the effect of capillary forces and gravity.

An extremum study of the function (5) shows that the rate of water lifting along the height of the soil level is not stable and changes from zero to the underground water level up to the maximum at

$$x = 2gk^2 / \left( m^2 \nu^2 \right),$$

and then drops down to zero. The maximum and average length in the range of a layer with the thickness δ of the rate is given by the ratios

$$u_{\max} = gk/(2\nu), \tag{6}$$

$$u_{cp} = \frac{1}{\delta} \int_0^\delta u dx = \frac{2m}{3} \sqrt{2g\delta} - \frac{m^2 \nu \delta}{2k}. \tag{7}$$

For a multi-layer structure of a motorway foundation along the height the local rates at the contact boundaries are equal. Therefore the distribution of the water lifting rate in the *i*-th layer is determined with the formula:

$$u_{i+1} = u_i + m_i \sqrt{2g\left(x - \sum_{i=1}^{n} \delta_i\right) - \frac{m_i^2 \nu}{k_i}\left(x - \sum_{i=1}^{n} \delta_i\right)}, \quad (8)$$

where $i$ is the number of the layer, $i = 1, 2, \ldots, n$; $n$ is the number of the layers of the thickness $\delta_i$ with homogeneous porosity along the height of a motorway foundation.

The complete water consumption ρ in any layer of a motorway foundation for one running meter of a motorway with the halfwidth $B$ (Fig.) is given by the formula

$$G_i = u_i \rho B. \quad (9)$$

2. The results of the calculation analysis. Let us look at the example of calculating humidity transfer for three types of homogeneous subbase foundation with varying penetrability for fixed halfwidths of a roadway $B = 10$ m and thermal and physical properties of water: $\nu = 10^{-6}$ m$^2$/sec; $\rho = 10^3$ kg/m$^3$.

The tables shows that the maximum water filtration rate (6) is the same as the filtration coefficient, which proves that the calculation analysis based on the solution of the movement equation is viable (1).

Table

| Type of soil | Characteristics of porosity and penetrability of soil | | | $u_{max}$, m/sec | $G$, kg/(m·sec) |
|---|---|---|---|---|---|
| | $m$ | $K_ф$, m/day<br>м/с | $k$, m$^2$ | | |
| Coarse crushed stone with sand | | 1000<br>$6.3 \cdot 10^{-3}$ | $1.26 \cdot 10^{-9}$ | $6.3 \cdot 10^{-3}$ | 63 |
| Penetrable pebbles with fine sand | 0.65 | 100<br>$6.3 \cdot 10^{-4}$ | $1.26 \cdot 10^{-10}$ | $6.3 \cdot 10^{-4}$ | 6.3 |
| Pebbles with sand and clay | | 10<br>$6.3 \cdot 10^{-5}$ | $1.26 \cdot 10^{-11}$ | $6.3 \cdot 10^{-5}$ | 0.63 |
| Sand, sandy loam | | 1<br>$6.3 \cdot 10^{-6}$ | $1.26 \cdot 10^{-12}$ | $6.3 \cdot 10^{-6}$ | 0.063 |

3. Calculation of the parameters of a new drainage system. In [12] a new scheme for road drainage in areas with high humidity humidity is suggested which is effective for small deposits of underground water for constantly flooded areas as well as those with a great amount of

rainfall and where water is accumulated and coming from the surface submerges roadways. In a drainage layer of a subbase foundation with the filtration coefficient of no less than 1 m/day there are perforated plastic pipes with the diameter $d$ = 70—110 mm filled with coarse crushed stone. The pipes are positioned along the entirety of the road.

If a draining level is not too thick, pipes are positioned horizontally with the distance $L$ between them. These intervals then have to be identified using the conditions of equal water consumption in a subbase foundation and drainage pipes.

The water consumption in a subbase foundation of sand and sandy loam for one running meter of a roadway with the halfwidth $B$ is given by the formula (9):

$$G_{Soil} = u_{\max}\rho B = 6.3\cdot 10^{-6}\cdot 10^{3}\cdot 10 \approx 6.3\cdot 10^{-2},\ \ \text{kg}/(\text{m}\times\text{Sec}). \qquad (10)$$

The mass water consumption per second along a pipe with the diameter $d$ = 0.1 m filled with coarse crushed stone and sand is determined only with capillary forces thus

$$G_{Pipe} = u_{\max}\cdot\rho\cdot\frac{\pi d^2}{4} = 6.3\cdot 10^{-3}\cdot 10^{3}\,\frac{3.14\cdot 10^{-2}}{4} \approx 4.9\cdot 10^{-2},\ \ \text{Kg/Sec}. \qquad (11)$$

The distance between the drainage pipes is

$$L = G_{Pipe}\big/G_{Soil} = 4.9\cdot 10^{-2}\,/\,6.3\cdot 10^{-2} \approx 0.8\ \ \text{m}. \qquad (12)$$

**Conclusions**

1. The developed method considering only vertical filtration and the major characteristics of the porosity of soil yields the accurate solution for the humidity transfer equation. The water lifting rate along the height of a soil layer is shown to be unstable and changeable as a maximum is reached along the height of a subbase foundation.
2. The method provides an accurate description of humidity transfer in porous soil and is instrumental in high-speed calculation analysis of the parameters of drainage systems in roadway subbase foundations.
3. The example is provided of the use of the method of evaluating a distance between drainage pipes in a new drainage system from under a roadway foundation that mitigates the negative effects of swelling.

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