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CALCULATION MODELS OF DEFORMATION OF REINFORCED CONCRETE CONSTRUCTIONS WITH SPATIAL CRACKS

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Statement of the problem. A model of deformation of reinforced concrete structures with spatial cracks is set forth.
Results. In the article working prerequisites are shown and the identifying equations of the model of the resistance of reinforced concrete structures to torsion with bending are derived.
Conclusions. The methods for calculating the resistance of reinforced concrete structures, as well as calculating the distance between spatial cracks and the width of crack opening under the joint action of the bending moment, torque and lateral force in the second stage of the stress-strain are discussed for two cases (case 1 — when the spatial cracks of the first type appear on the lower edge of the reinforced concrete structure, case 2 — when the spatial cracks of the first type appear on the lateral edge of the reinforced concrete structure).

Keywords: calculation method, torsion with bending, stress – strain, reinforced concrete structures, spatial crack.

Introduction. The resistance of reinforced concrete structures in spatial sections is discussed as complex if, apart from bending moments, shear forces, torques act on the structure. Under the action of torques, a spiral crack is formed in the reinforced concrete forming a spatial section within the three faces of the element together with the compressed zone closing it along the fourth face.

The construction of computational models of complex torsional resistance with bending is beco-ming increasingly urgent [1, 2]. Firstly, because there are relatively few such studies [3–10],

and secondly, it is caused by the urgent need to take into consideration the spatial work of the great majority of reinforced concrete structures of growingly original buildings and structures that considerably change the architectural appearance of modern cities [13–15]; thirdly, it is already a generally recognized postulate that there is nothing more practical than a viable theory of their calculation [16–23].

In the existing standards [12], elaborate formulas are employed to calculate the work of structures in torsion with bending. In spite of this, the norms do not consider the effect of a complex stress state on the value of ultimate stresses in compressed concrete R_b. Due to this, not only the engineering visibility of the solution is lost, but also its essential accuracy.

The article by A. S. Zalesov [8] made a considerable step forward, i.e., the equilibrium equations for the problem under discussion are written in relation to the transverse and longitudinal vertical planes, which greatly simplifies the calculation formulas compared to the normative ones; axial forces in the transverse reinforcement located at the lateral edges of the element are considered (in the norms, only forces are considered in the reinforcement located at the edge opposite to that with the compressed zone). However, the equations of deformation of the spatial section employed here and the replacement of this section by a simplified diagonal section have not been confirmed in a number of experiments. Acceptance of longitudinal stresses in concrete of a compressed zone, equal to R_b has no appropriate justification for the complex stress state of this zone; while identi-fying the projection of a dangerous spatial crack, the condition of the minimum of a function of many variables is not employed, etc.

In this article, the authors suggest a calculation method which has no above disadvantages.

1. Working prerequisites for the torsion-bending resistance model of reinforced concrete structures. The construction of the proposed method is based on the following design prerequisites:

— the formation of a spatial crack on the lower face of a reinforced concrete element occurs perpendicular to the direction of the main deformations of concrete elongation and the location of the end of the front of a spatial crack at a compressed face of a reinforced concrete element coincides with the direction of the major deformations of shortening of concrete — thus, a spatial crack has a spiral shape with three possible layouts of the compressed zones (Fig. 1);

— a design scheme is taken consisting of a support block formed by a spatial crack and a vertical section passing through the end of the front of this crack in compressed concrete and a second block formed by a vertical section passing perpendicular to the longitudinal axis of the reinforced concrete element along the edge of the spatial crack (Fig. 1);
— as the design forces in the spatial section, the following are considered: normal and tangential forces in the concrete of the compressed zone; components of axial forces in the reinforcement located at the edge opposite to that with the compressed zone; components of axial forces in transverse reinforcement located at the side faces of a reinforced concrete element;
— for average fiber deformations of compressed concrete and tensile reinforcement in section *I-I*, the hypothesis of their proportionality to the heights of the compressed and stretched zones of the section is assumed to be valid;
— the relationship between the intensity of deformations ε_i and the intensity of stresses σ_i of concrete.

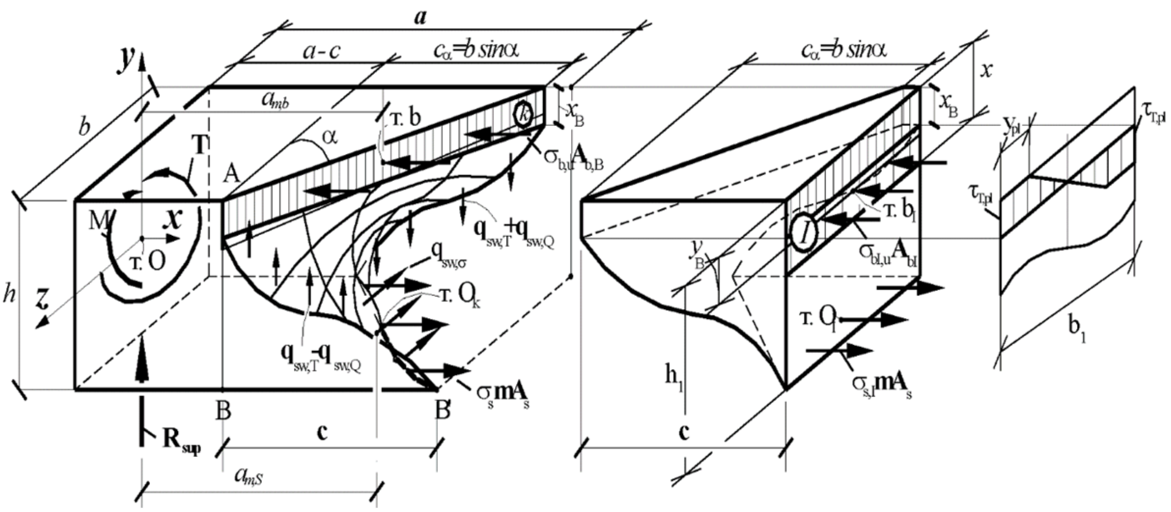

Fig. 1. Calculation scheme of the resistance of a reinforced concrete structure under a combination of bending moment, rolling moment and lateral force (***case 1***):

k is a compressed zone of the spatial section; *I* is a compressed zone with the section *I–I*

When solving a direct engineering problem, their ratio (*Q:M:T*) is always specified between external influences. Thus, having identified one of them, e.g., a support reaction, the remaining influences, e.g., M and T, are easily found.
Based on the equilibrium conditions in section *I-I* and in the spatial section, the following calculated parameters are sought (Fig. 1): limiting support reaction R_{sup}; the height of the compressed zone x in the section *I-I*; stresses in longitudinal reinforcement σ_s at the point of its

intersection with a spatial crack; the height of the compressed zone of concrete $x_в$ in the vertical plane passing through the end of the front of the spatial crack; linear force in the transverse reinforcement located at the lateral faces of the spatial section $q_{sw,Q}$ caused by the shear force; linear force in the transverse reinforcement located at the lateral faces of the spatial section $q_{sw,T}$ caused by the torque; linear force in transverse reinforcement located at the lower edge of the spatial section $q_{sw,\sigma}$ caused by the torque.

The shear stress τ_Q and shear torsional stress in compressed concrete τ_T are determined by projec-ting the $\sigma_i - \varepsilon_i$ diagram onto the plane $\tau - \gamma$ (taking into account the distribution in proportion to the $Q{:}T$ ratio) and onto the *I–I* plane and projecting the stress components of the k plane onto a plane perpendicular to the longitudinal axis of the reinforced concrete element.

In order to construct the design equations, two blocks are separated from the reinforced concrete element using the section method (Fig. 1). The first block is separated by the cross-section *I-I* pas-sing at the end of the spatial crack. This block is balanced under the influence of external forces.

2. Determining equations of the model of resistance of reinforced concrete structures to torsion with bending. Based on the equilibrium equation of the moments of internal and external forces in this *I-I* section with respect to the z axis relative to the point of application of the resultant forces in the tensioned reinforcement ($\sum M_{\mathrm{O,I}} = 0$), we get:

$$\sigma_{b,\mathrm{I}} A_b [h_0 - \phi_y (x_{b,} x) \cdot x] - M - R_{\sup} a = 0, \quad (1)$$

where $\varphi_y(x_{b,} x)$ is a static-geometric parameter that considers the location of the center of gravity of the compressed zone of concrete in the section *I-I* (at section x_b, the compressive stress diagram is rectangular, at section x–x_b triangular); R_{sup} is a support reaction in the first block (Fig. 1) (for the second group of limit states this parameter is known); a is the horizontal distance from the support to the section *I-I.* The unknown is given by this equation.

It should be noted that for the limiting states of the second group, the reference response in the first block R_{sup} is a known parameter. We shall see that a similar parameter is needed at the moment of the formation of the first spatial crack, $R_{sup,crc}$, which is known from the solution of the problem of the formation of spatial cracks.

Based on the equilibrium equation of the projections of all forces acting in section *I-I* on the x axis, we find the height of the compressed concrete zone x in this section ($\sum X = 0$;):

$$\sigma_b \cdot \phi_{np}(\sigma_i, \varepsilon_i) \cdot \phi_\alpha(c) \cdot \phi_{y,1}(x_b, x) \cdot b \cdot x - \sigma_{s,1} m A_{s,1} = 0, \quad (2)$$

where $\varphi_{np}(\sigma_i, \varepsilon_i)$ is a parameter that considers the projection of the diagram on the direction perpendicular to the plane *k* (Fig. 1); φ_α is a parameter that considers the projection of the stress components in the *k* plane onto the *I-I* plane, perpendicular to the longitudinal axis of the reinforced concrete element; $\varphi_{y,1}(x_{b,}x)$ is a parameter equal to the parameter up to a numerical coefficient.

Based on the equilibrium equation of the moments of internal and external forces acting in section *I-I* relative to the axis perpendicular to this section and passing through the point of application of the resultant forces in the compressed zone ($T_{b,I} = 0$), we get:

$$2 \times 0.5\tau_T \cdot \frac{b}{2} \cdot \frac{2}{3} \cdot \frac{b}{2} \cdot x - T = 0, \tag{3}$$

where τ_T is shear torsional stress in compressed concrete determined by projecting the diagram $\sigma_i - \varepsilon_i$ on the plane $\tau - \gamma$ and on the plane $I - I$ and distributed proportionally to the ratio Q:T.

Based on this equation, we determine the unknown T (for the first group of limit states) and if T is specified through the ratio R_{sup}:T and when considering the second group of limit states, this equation is used to determine τ_T.

The following condition is checked:

$$\tau_T \le \tau_{T,pl}, \tag{4}$$

where $\tau_{T,pl}$ is shear torsional stress in compressed concrete (considering the ratio *Q:T*) corresponding to the maximum on the $\sigma_i - \varepsilon_i$ diagram. If the condition (4) is not met, then τ_T is assumed to be equal to $\tau_{T,pl}$ and from the transformed equation (3), the parameter y_{pl} (Fig. 1):

$$2 \cdot [\tau_{\mathrm{T,pl}} \cdot y_{pl} \left(b/2 - 0.5y_{pl}\right) + 0.5\tau_{T,pl}(b/2 - y_{pl})2/3\left(b/2 - y_{pl}\right)] \cdot x - T = 0. \tag{5}$$

Based on the hypothesis of proportionality of the mean longitudinal deformations, we find

$$\sigma_{s,I} = \frac{\sigma_b \cdot E_s(\lambda)}{E_b(\lambda)} \cdot \frac{h_0 - x}{x}. + \sigma_0, \tag{6}$$

where σ_0 is prestressing in the stressed reinforcement at the moment when the prestressing value in concrete is reduced to zero by the external forces acting on the structure considering prestressing losses in prestressing reinforcement, corresponding to the considered stage of the operation of the structure.

In this case, it is necessary to check the condition:

$$\sigma_{s,I} \le R_s. \tag{7}$$

If the condition (7) is not met, $\sigma_{s,i}$ is assumed to equal R_s.

The second support block is separated from the reinforced concrete element by a spatial section formed by a spiral crack and a vertical section passing through the compressed concrete zone through the end of the spatial crack front. The balance of this block is ensured by satisfying the following conditions. The sum of the moments of all the internal and external forces acting in the vertical longitudinal plane relative to the z axis passing through the point of application of the resultant forces in the compressed zone is equal to zero ($\sum M_b=0$, block II):

$$\sigma_s m A_s (h_0 - 0.5x_b) - M - R_{\sup} \cdot a_{m,\mathrm{b}} = 0, \tag{8}$$

where $a_{m,b}$ is the horizontal distance from the support to the center of gravity of the compressed concrete zone in section k.

It should be noted that in this equation the moments $q_{sw,T}\,\dfrac{c^2}{8}$ arising on the lateral faces from the longitudinal forces in the transverse reinforcement are mutually balanced relative to the point B. The same should be attributed to the moments caused by the “dowel” components in the longitudinal reinforcement [11]. The unknown σ_s is given by the equation (8).

Here it is also necessary to point out the parameter σ_b (the stress in compressed concrete of section k, Fig. 1). This parameter is given by the equation of equilibrium of the moments of internal and external forces in the spatial section with respect to the z axis relative to the point O_K of application of the resultant forces (section k, Fig. 1) in compressed concrete ($\sum M_{O,K} = 0$); we get:

$$\sigma_b A_b (h_0 - 0.5x_b) - M - R_{\sup} \cdot a_{m,S} = 0, \tag{9}$$

where $a_{m,S}$ is the horizontal distance from the support to the center of gravity of all longitudinal reinforcement in section k. The unknown σ_b is given by the equation (9).

The sum of the projections of all forces acting in a spatial section on the x axis is zero ($\sum X = 0$, block II):

$$\sigma_b \phi_{np}(\sigma_i, \varepsilon_i) \cdot \phi_\alpha(c) \cdot x_b \cdot \sqrt{c^2 + b^2} - \sigma_s m A_s - 2q_{2sw} \cdot \sqrt{(h_0 - x_b)^2 + c^2} = 0, \tag{10}$$

where $\phi_\alpha(c)$ is a parameter equal to the parameter up to a numerical coefficient ϕ_α; q_{2sw} is the linear “dowel” force in the clamps [11], which occurs on the lateral faces of the reinforced concrete element (not shown in Fig. 1). The unknown x_b is given by the equation (10).

The sum of the projections of all the forces acting in the spatial section on the y-axis is zero ($\sum Y = 0$, block II):

$$-\tau_Q \cdot \sqrt{b^2 + c^2} \cdot x_b - 2q_{sw,Q} \cdot \sqrt{(h_0 - x_b)^2 + c^2} - Q_s + R_{\sup} = 0, \tag{11}$$

where τ_Q is a shear stress in compressed concrete determined by projecting the σ_i–ε_i diagram onto the plane $\tau-\gamma$ (considering the distribution in proportion to the ratio $Q{:}T$) and projecting the stress components of the k plane onto the plane perpendicular to the longitudinal axis of the reinforced concrete element; $q_{sw,Q}$ is the linear force in the clamps arising on the lateral faces of the reinforced concrete element from the transverse force Q (Fig. 1); Q_s are "dowel" forces in longitudinal reinforcement [11] (not shown in Fig. 1). The unknown $q_{sw,Q}$ is given by the equation (11).

The sum of the moments of internal and external forces in the vertical transverse plane relative to the x-axis passing through the point of application of the resultant forces in the compressed zone is equal to zero ($\sum T_b = 0$, block *II*):

$$q_{sw,\sigma}\sqrt{c^2+b^2}\cdot(h_0-0{,}5x_b)-2q_{sw,T}\cdot b/2\cdot\sqrt{(h_0-x_b)^2+c^2}-2\tau_T\omega\cdot b/2\cdot x_b-T=0, \quad (12)$$

where τ_T is a shear stress caused by torsion in compressed concrete, determined by projecting the σ_i–ε_i diagram onto the plane $\tau-\gamma$ (considering the distribution in proportion to the ratio $Q{:}T$) and projecting the stress components of the k plane onto a plane perpendicular to the longitudinal axis of the reinforced concrete element; ω is the coefficient of filling the diagram of shear torsional stresses in compressed concrete; $q_{sw,T}$ is the linear force in the clamps arising on the lateral edges of the reinforced concrete element from the torque T (Fig. 1, a); $q_{sw,\sigma}$ is a linear force in the clamps arising on the lower edge of the reinforced concrete element from the torque T (Fig. 1). The unknown $q_{sw,T}$ is given by the equation (12).

The sum of the projections of all forces acting in the spatial section on the z axis is zero ($\sum Z = 0$, block II):

$$q_{sw,\sigma}\cdot\sqrt{c^2+b^2}+\sigma_b\cdot\varphi_{np}(\sigma_i,\varepsilon_i)\cdot\varphi_{\alpha1}(c)\cdot x_b\cdot\sqrt{c^2+b^2}-\tau_T\cdot x_b(\sqrt{c^2+b^2})=0, \quad (13)$$

where $\varphi_{\alpha1}$ is a parameter that considers the projection of the stress components in the k plane onto a plane parallel to the longitudinal axis of the reinforced concrete element and equal to the parameter up to a numerical coefficient.

The unknown $q_{sw,\sigma}$ is given by the equation (13). Hence the above method for calculating the resistance of reinforced concrete structures under the combined action of a bending moment, torque and shear force for the second stage of the stress-strain state (case 1) can be employed when spatial cracks of the first type appear on the lower edge of the structure. The second scheme is implemented with the resistance of reinforced concrete elements subject to the combined action of torques and shear forces. The design diagram of the resistance of a reinforced concrete structure under the combined action of a bending moment, torque and shear force (case 2) is shown in Fig. 2.

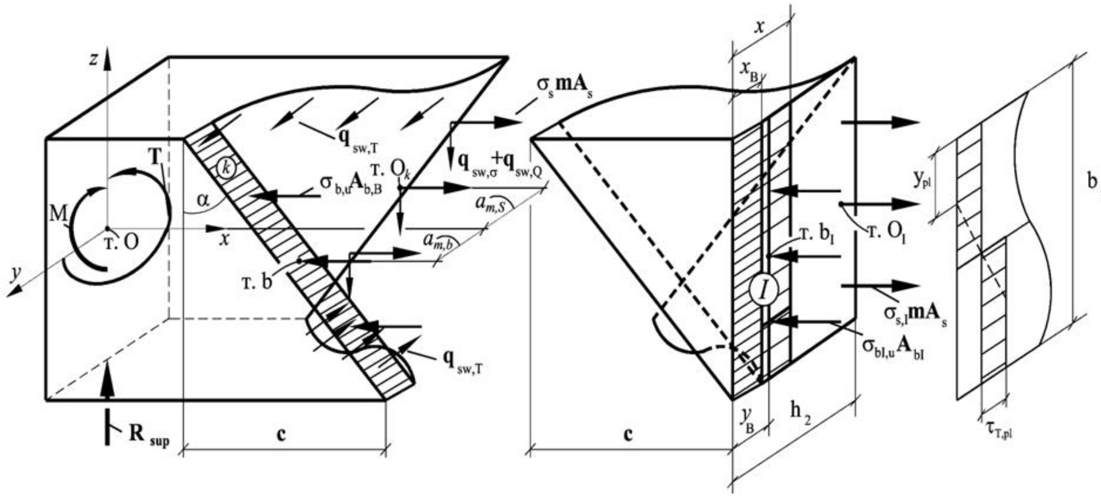

Fig. 2. Design diagram of the resistance of a reinforced concrete structure under the combined action of bending moment, torque and shear force (***case 2***):
k is a compressed zone of the spatial section; I is a compressed zone of the section I–I

In order to construct the design equations, two blocks are separated from the reinforced concrete element using the section method (Fig. 2). The first block is separated by the cross-section *I–I* pas-sing at the end of the spatial crack. This block is in equilibrium under the action of external forces applied to the block from the support side and internal forces arising at the site of the section.

Based on the equilibrium equation of the moments of internal and external forces in this section *I-I* relative to the point passing through the point of application of the resultant forces in the stretched reinforcement ($\sum M_{\mathrm{O,I}} = 0$), we get:

$$\sigma_{b,\mathrm{I}} A_b [h_0 - \varphi_{y,2}(x_{b,} x) \cdot x] - M - R_{\sup} a_{\mathrm{m,S}} = 0, \qquad (14)$$

where $a_{\mathrm{m,S}}$ is a horizontal distance from the support in the direction of the *y*-axis to the center of gravity of the working longitudinal reinforcement in the section *I-I* (point O_1). In this case, it should be emphasized that the moment created by $R_{\sup} \cdot a_{m,S}$ will be torsional relative to the *x* axis and relative to point O_1; the moment *M* will be bending with respect to the *y*-axis and with respect to the point O_1; the moment created by $\sigma_{b,I} A_b \left[h_0 - \varphi_{y,2}\left(x_b, x\right) \cdot x \right]$ will be bending about the *z*-axis and about the point O_1. Here $\varphi_{y,2}\left(x_b, x\right)$ is a static-geometric parameter that considers the location of the center of gravity of the compressed zone of concrete in section *I-I* (in section x_b, the diagram of compressive stresses is rectangular, in section x-x_b triangular); R_{sup} is a support reaction in the first block (Fig. 2), (for the second group of limiting states this parameter is known). The unknown $\sigma_{b,I}$ is given by this equation.

Based on the equilibrium equation of the projections of all forces acting in section $I - I$ on the x axis, the height of the compressed concrete zone x in this section is found. The equation takes the following form:

$$\sigma_b \cdot \varphi_{np}(\sigma_i, \varepsilon_i) \cdot \varphi_\alpha(c) \cdot \varphi_{y,2}\left(x_{b,}x\right) \cdot b \cdot x - \sigma_{s,1} m A_{s,1} = 0\,, \tag{15}$$

where $\varphi_{np}\left(\sigma_i, \varepsilon_i\right)$ is a parameter that considers the projection of the $\sigma_i - \varepsilon_i$ diagram on the direction perpendicular to the plane k (Fig. 2); φ_α is a parameter that considers the projection of stress components in the k plane onto the I-I plane, perpendicular to the longitudinal axis of the reinforced concrete element; $\varphi_{y,2}\left(x_{b,}x\right)$ is a parameter equal to the parameter up to a numerical $\varphi_y\left(x_{b,}x\right)$.

Based on the equilibrium equation of the moments of internal and external forces acting in section I-I relative to the axis perpendicular to this section and passing through the point of application of the resultant forces in the compressed zone ($T_{b,I} = 0$), we get:

$$2 \times 0.5\tau_T \cdot \frac{b}{2} \cdot \frac{2}{3} \cdot \frac{b}{2} \cdot x - T = 0\,. \tag{16}$$

Using this equation, the unknown T (for the first group of limit states) is identified and if T is specified through the ratio R_{sup}:T and when considering the second group of limit states, this equation is used to determine τ_T .

This equation is the same as equation (16). The condition (17) is checked:

$$\tau_T \le \tau_{T,pl}\,. \tag{17}$$

If the condition (17) is not met, τ_T is assumed to be equal to $\tau_{T,pl}$ and the transformed equation (16) is used to identify the parameter y_{pl} (see equation (18) and Fig. 2):

$$2 \cdot [\tau_{\mathrm{T,pl}} \cdot y_{pl}\left(b/2 - 0.5y_{pl}\right) + 0.5\tau_{T,pl}(b/2 - y_{pl})2/3\left(b/2 - y_{pl}\right)] \cdot x - T = 0\,. \tag{18}$$

It should be noted that for scheme II, the τ_T diagram is commonly close to rectangular.

Based on the hypothesis of proportionality of longitudinal deformations (the equation is similar to (19), we find $\sigma_{s,I}$:

$$\sigma_{s,I} = \frac{\sigma_b \cdot E_s(\lambda)}{E_b(\lambda)} \cdot \frac{h_0 - x}{x}. + \sigma_0\,. \tag{19}$$

The condition (20) must be checked. If the condition (20) is not met, $\sigma_{s,I}$ is assumed to be equal to R_s.

$$\sigma_{s,I} \le R_s. \tag{20}$$

While considering the second block, in the same way as in the first scheme, equilibrium equations are drawn up. The sum of the moments with respect to the *z* axis relative to the point of application of the resultant forces in the compressed zone is equal to zero ($\sum M_b = 0$, block *II*):

$$\sigma_s m A_s (h_0 - 0.5 x_b) - M - R_{\sup} \cdot a_{m,\mathrm{b}} = 0. \tag{21}$$

The equation (21) is used to identify the unknown σ_s. Using the equation of equilibrium of the moments of internal and external forces in the spatial section with respect to the *z*-axis relative to the point O_K of application of the resultant forces (section *k*, Fig. 2) in compressed concrete ($\sum M_{O,K}=0$), the stresses in compressed concrete of the section k σ_b are calculated:

$$\sigma_b A_b (h_0 - 0.5 x_b) - M - R_{\sup} \cdot a_{m,S} = 0. \tag{22}$$

Using the equation of the sum of the projections of all forces acting in the spatial section on the *x* axis, the unknown x_b ($\sum X=0$, block II).

$$\sigma_b \varphi_{np}(\sigma_i, \varepsilon_i) \cdot \varphi_\alpha(c) \cdot x_b \cdot \sqrt{c^2 + b^2} - \sigma_s m A_s - 2 q_{2sw} \cdot \sqrt{(h_0 - x_b)^2 + c^2} = 0. \tag{23}$$

The sum of the moments of internal and external forces in the vertical transverse plane relative to the *x*-axis passing through the point of application of the resultant forces in the compressed zone is equal to zero ($\sum T_b = 0$, block *II*):

$$\left(q_{sw,\sigma} + q_{sw,Q}\right)\sqrt{c^2 + b^2} \cdot (h_0 - 0{,}5 \cdot x_b) - 2 q_{sw,T} \sqrt{(h_0 - x_b)^2 + c^2} - 2 \tau_T \omega \cdot x_b - T = 0. \tag{24}$$

The equation (24) is used to identify the unknown $q_{sw,T}$.

The sum of the projections of all forces acting in the spatial section on the *z* axis is zero ($\sum Z = 0$, block II):

$$\left(q_{sw,\sigma} + q_{sw,Q}\right) \cdot \sqrt{c^2 + b^2} + \sigma_b \cdot \varphi_{np}(\sigma_i, \varepsilon_i) \cdot \varphi_{\alpha_1}(c) \cdot x_b \cdot \sqrt{c^2 + b^2} - \tau_T x_b (\sqrt{c^2 + b^2}) = 0. \tag{25}$$

The equation (25) is used to identify the unknown $q_{sw,Q}$.

The unknown $q_{sw,\sigma}$ is found using the following considerations. This linear force arises on the la-teral face from the action of $T + R_{sup}\, e_Q$ as well as the linear force $q_{sw,T}$ arising on the upper and lower edges, thus its difference from the latter will consist in taking into account the ratio $b_2 : h_2$ and the characteristics of the reinforcement used. Then

$$\frac{q_{sw,\sigma}}{q_{sw,Q}} = \frac{h_2}{b_2} \frac{R_{sw,\sigma} A_{sw,\sigma}}{R_{sw,T} A_{sw,T}} = n_T. \tag{26}$$

Hence

$$q_{sw,\sigma} = q_{sw,T} n_T \,. \tag{27}$$

The "dowel forces" in the longitudinal Q_s and transverse reinforcement $q_{sw,2}$ are determined from a special model of the "dowel effect" considered in [11]. Thus, the considered method for calculating the resistance of reinforced concrete structures under the combined action of a bending moment, torque and shear force for the second stage of the stress-strain state (case 2) can be used when spatial cracks of the first type appear on the side face of the structure.

3. Calculation of the distance between spatial cracks and the width of their opening in reinforced concrete structures in torsion with bending (case 1).

While calculating the resistance of reinforced concrete structures to the action of transverse forces, bending and torque moments, it becomes necessary to assess the complex stress-strain state, which is even more complicated in the presence of spatial cracks.

Following the formation of cracks, the continuity of concrete is violated and the application of the formulas of the mechanics of a solid deformed body is no longer legitimate. Nevertheless, in order to determine the actual stress-strain of reinforced concrete structures, it becomes necessary to consider the complete picture of cracking during loading. In this case, it is important not only to have different levels of cracking of spatial cracks, but also to have their complete picture.

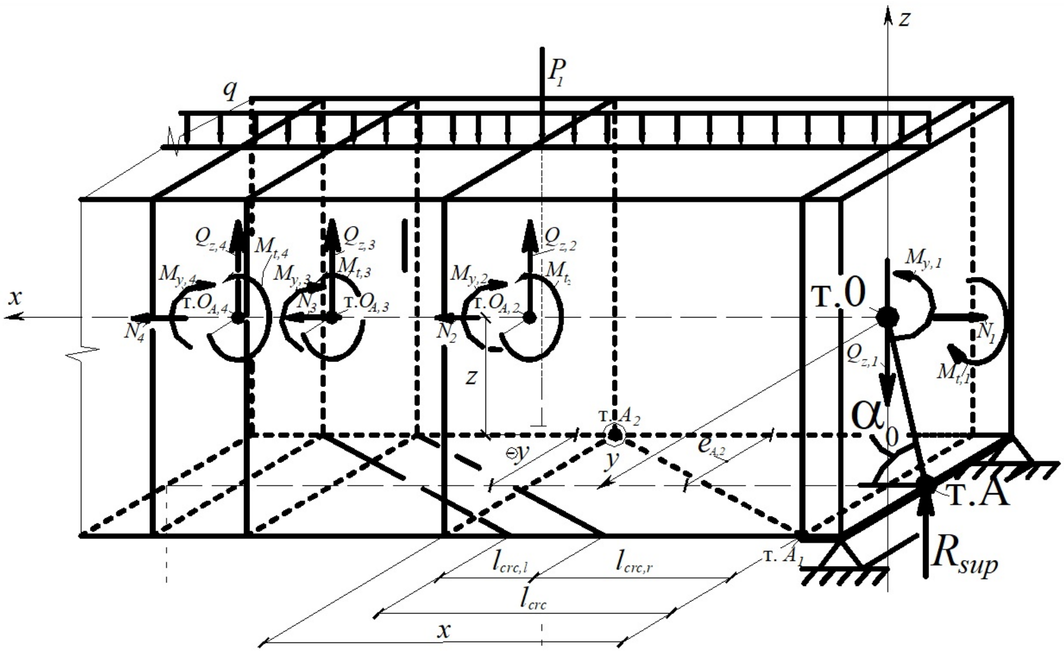

Fig. 3. Calculation scheme for determining the distance between cracks of the first type (case 1):
a is a scheme of efforts and the choice of a coordinate system for the formation of the first spatial crack

First of all, it is essential to apply the entire fan of spatial cracks of all types. After identifying a dangerous spatial crack according to the criterion of formation or the largest width of their opening, it is necessary to apply the whole picture of spatial cracks.

At the same time, as the practice of calculations and design of reinforced concrete structures has shown, the distance between spatial cracks of the first type for the first level of cracking $l_{crc,1}$ located along the transverse or longitudinal reinforcement, can be determined from the following relationship (Fig. 3, Fig. 4).

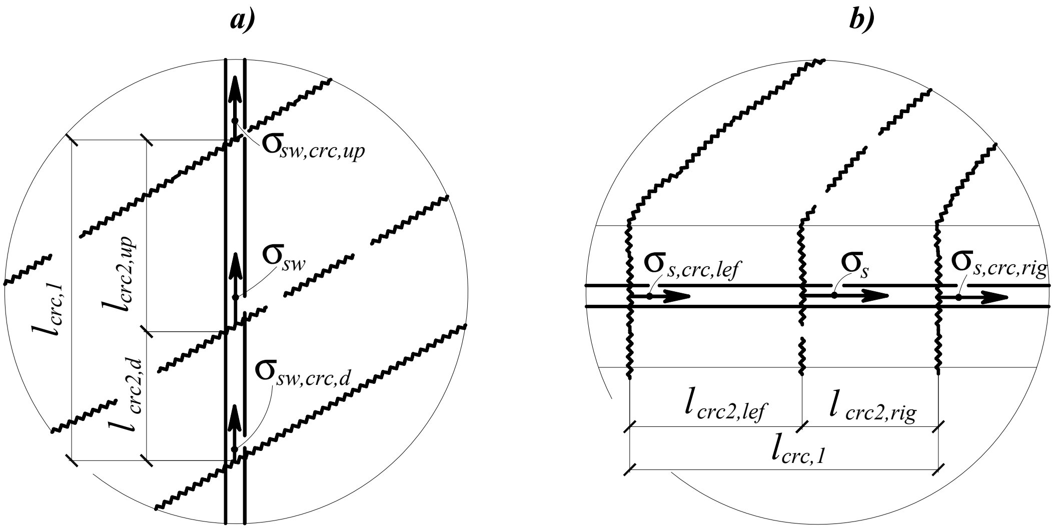

Fig. 4. Location of the adjacent crack of the next level between two cracks of the previous level: *a* is along the axis of the transverse reinforcement; *b* is along the axis of the longitudinal reinforcement

$$\frac{a}{a\text{-}l_{crc,1}} = \frac{\sigma_{\mathrm{S,I}}}{\sigma_{S,crc}}. \tag{28}$$

Hence

$$l_{crc,1} = \frac{a \cdot (\sigma_{\mathrm{S,I}} - \sigma_{S,crc})}{\sigma_{\mathrm{S,I}}}. \tag{29}$$

In order identify the distance between spatial cracks of the second level of their formation, the ratio between the stresses in the reinforcement in section I – I and in the section with a dangerous spatial crack is used, which is given by the criterion of the maximum width of their opening:

$$\frac{a}{a\text{-}l_{crc,2}} = \frac{\sigma_{\mathrm{S,I}}}{\sigma_{S,C}}. \tag{30}$$

Hence

$$l_{crc,2} = \frac{a \cdot (\sigma_{\mathrm{S,I}} - \sigma_{S,C})}{\sigma_{\mathrm{S,I}}}. \tag{31}$$

In this case, the emergence of a new level of cracking corresponds to the level of load where the following inequality is observed:

$$l_{crc,i} \leq \eta \cdot l_{crc,i\text{-}1}, \tag{32}$$

where η along the transverse reinforcement from a dangerous inclined crack is given by the following ratios (Fig. 4, *b*):

$$\frac{\sigma_{\mathrm{Sw,crc,d}}}{\sigma_{\mathrm{Sw,crc,up}}} = \frac{l_{crc,2,up}}{l_{crc,2,\mathrm{d}}} = \eta_{\mathrm{w}}; \tag{33}$$

$$l_{crc,2,up} + l_{crc,2,\mathrm{d}} = l_{crc,1}. \tag{34}$$

While moving to the right along the longitudinal reinforcement, the following ratios are obtained (Fig. 4, *b*):

$$\frac{\sigma_{S,crc,rig}}{\sigma_{S,crc,lef}} = \frac{l_{crc,2,lef}}{l_{crc,2,rig}} = \eta, \tag{35}$$

$$l_{crc,2,lef} + l_{crc,2,\mathrm{rig}} = l_{crc,1}. \tag{36}$$

While moving to the left of a dangerous spatial crack, the functional l_{crc} and l_* are compared (see Fig. 4) and, if necessary, similar ratios are used: We do not move beyond the section where $\varepsilon_{\mathrm{bt}}(y) \leq \varepsilon_{\mathrm{bt,u}}$:

$$\frac{l_{crc,lef,*}}{l_{crc,rig,*}} = \frac{\sigma_{\mathrm{S,C}}}{\sigma_{\mathrm{s,crc}}} = \eta_*, \tag{37}$$

$$l_{crc,lef,*} + l_{crc,rig,*} = l_*. \tag{38}$$

We do not move beyond the section restrained by $l_{crc,1}$ (Fig. 4), either.

In the case of breaks of longitudinal reinforcement in the section of spatial inclined cracks, relations (35) and (37) are somewhat modified, i.e., in addition to the ratio of stresses in the reinforcement, the ratios of the areas of longitudinal reinforcement (prior and following the break) are also consi-dered. As a result, while moving to the right, these formulas will take the following form:

$$\frac{l_{crc,lef}}{l_{crc,rig}} = \frac{\sigma_{\mathrm{S,I}}}{\sigma_{\mathrm{S,C}}} \cdot \frac{A_{S,rig}}{\mathrm{A}_{\mathrm{S},lef}} = \eta; \tag{39}$$

while moving to the left:

$$\frac{l_{crc,lef,*}}{l_{crc,rig,*}}=\frac{\sigma_{\mathrm{S,C}}}{\sigma_{\mathrm{S,crc}}}\cdot\frac{\mathrm{A}_{\mathrm{S},rig,*}}{\mathrm{A}_{\mathrm{S},lef,*}}=\eta_{*}. \tag{40}$$

Hence cracking continues until the moment of destruction. At the same time, not one stands out (as is common in a number of known techniques), but several levels of cracking:

$$\left.\begin{array}{ll} l_{crc} > l_{crc,1} & -\text{ No cracks;} \\ l_{crc,1} \geq l_{crc} > l_{crc,2} & -\text{ First level;} \\ l_{crc,2} \geq l_{crc} > l_{crc,3} & -\text{ Second level;} \\ \ldots\ldots\ldots\ldots\ldots\ldots & \\ l_{crc} \geq 6t_{*} & -\text{ Last level.} \end{array}\right\}. \tag{41}$$

While comparing the functional and level values l_{crc}, possible implementation of the emergence of subsequent levels of fracturing is analyzed.

Having the levels of cracking along the longitudinal and transverse reinforcement of the reinforced concrete structure, it is possible to obtain a complete picture of various cracks adjacent to the concentrated force and to the support (Fig. 3) and identify the width of their opening. In this case, crack opening is defined as the accumulation of relative conditional concentrated mutual displacements of reinforcement and concrete in areas located on both sides of the crack:

$$a_{crc}=2\int_{0}^{t_{*}}\varepsilon_{g}(x_{1})dx_{1}+2\int_{0}^{0.5l_{crc}-t_{*}}\varepsilon_{g}(x)dx, \tag{42}$$

4. Calculation of the distance between spatial cracks and the width of their opening in reinforced concrete structures in torsion with bending (case 2). For case 2, as in case 1, the distance between spatial cracks of the first type can be given by the ratio (28) (Fig. 5, Fig. 6).

Hence

$$l_{crc,1}=\frac{a\cdot\left(\sigma_{\mathrm{S,I}}-\sigma_{S,crc}\right)}{\sigma_{\mathrm{S,I}}}. \tag{43}$$

In order to identify the distance between spatial cracks of the second level for case 2 as well as for case 1, relation (30) is used hence

$$l_{crc,2}=\frac{a\cdot\left(\sigma_{\mathrm{S,I}}-\sigma_{S,C}\right)}{\sigma_{\mathrm{S,I}}}. \tag{44}$$

In case 2, as in case 1, the emergence of a new level of cracking corresponds to the load level at which inequality (32) is observed where η along the transverse reinforcement from a dangerous inclined crack is determined from the ratios (33)—(40).

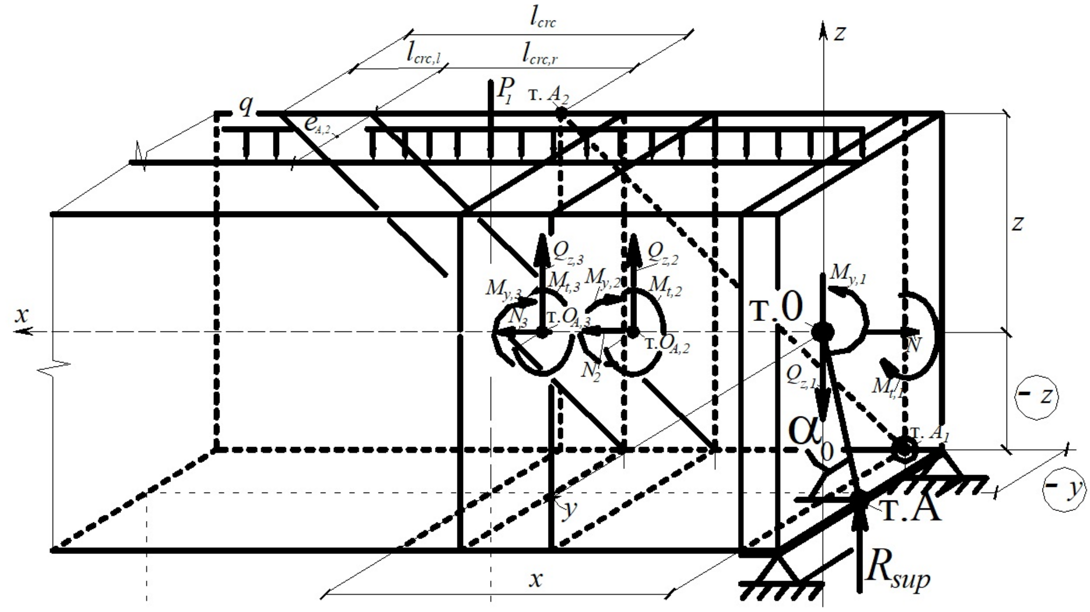

Fig. 5. Design scheme for identifying the distance between cracks of the first type (case 2): *a* is a scheme of efforts and the choice of a coordinate system for the formation of the first spatial crack

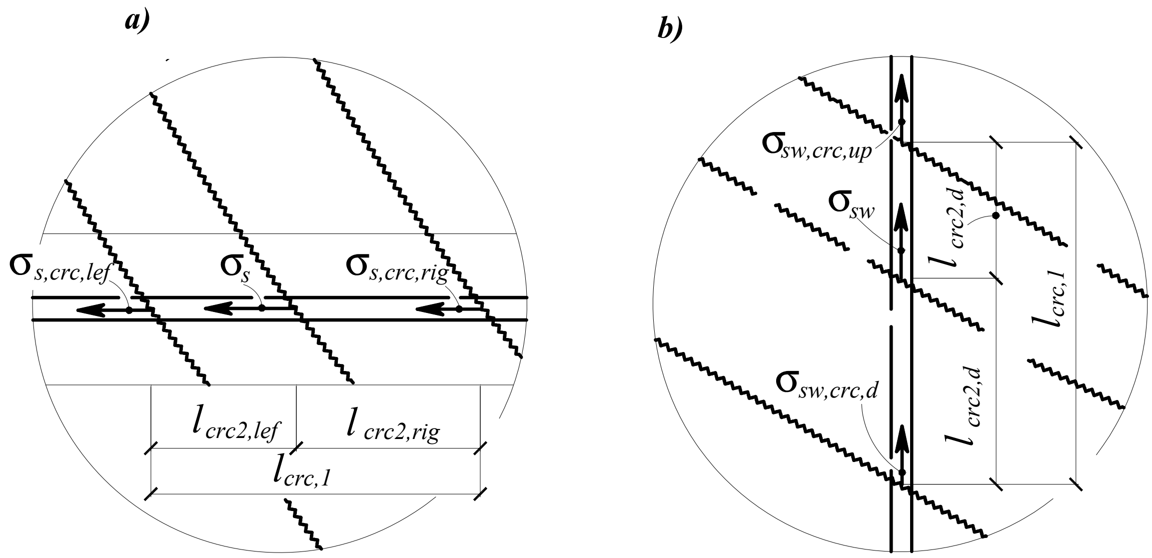

Fig. 6. Location of the adjacent crack of the next level between two cracks of the previous level: *a* is along the axis of the transverse reinforcement; *b* is along the axis of the longitudinal reinforcement

Therefore cracking continues until fracture occurs. In this case, not one is distinguished (as is common in a number of known techniques), but several levels of fracturing (see system (41)).

The distance between cracks l_{crc} is identified from the condition according to which concrete elongations are observed on the surface of the structure in the middle section (in the area between the cracks).

The analysis shows that an increase in deformations in reinforcement with increasing load causes a decrease in the distance between cracks. In this case, the emergence of a new level of cracking corresponds to the load level where the following inequality is observed

$$l_{crc,i} \le 0.5 l_{crc,i-1} . \tag{45}$$

Cracking continues until the moment of destruction (see system (41)).

The value ψ_s is calculated prior to conducting special studies according to the method of norms.

As a result, the general calculation algorithm is as follows:

1. in accordance with the developed technique, the parameters of the stress-strain state of the design section are identified;

2. the functional value l_{crc} is identified, then the level value l_{crc} is found using the inequality (41);

3. the crack opening width (discussed in more detail in Section 5) is identified.

5. Calculation of the crack opening width in reinforced concrete structures under central tension considering the effect of discontinuity.

Crack opening is the accumulation of relative conditional concentrated mutual displacements of reinforcement and concrete in areas located on both sides of the crack (development of the Thomas-Golyshev hypothesis).

Now, in accordance with the third premise, the expression (42) takes the following form:

$$a_{crc} = \delta \cdot \varphi_l \cdot \eta \cdot k_r \cdot \left[\frac{2\Delta T}{G} + \frac{2B_3}{B}\left(1 - e^{-B\cdot(0.5 l_{crc} - t_*)}\right) \right] \tag{46}$$

While conducting practical calculations, the crack opening width, calculated by formula (46), should be multiplied by the coefficient k_r which considers the deplanation of concrete in the section with a crack and also multiplied by the coefficients φ_l, η that consider the duration of the load and the profile of the reinforcement surface, respectively, and are identified in accordance with the normative documents.

Deplanation in a section with a crack (see Fig. 7) is considered using the k_r coefficient. The k_r coefficient is identified in accordance with the research results [34]. Their analysis shows that for practical calculations a simplified dependence can be recommended:

$$k_r = -0.088533\left(\frac{r}{d_s}\right)^2 + 0.522666\left(\frac{r}{d_s}\right) + 0.308801, \qquad (47)$$

where d_s is the diameter of the reinforcement, r is the radius of the boundary layer.

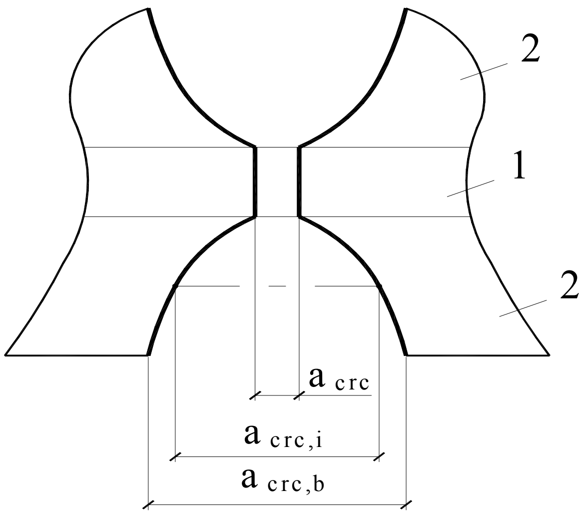

Fig. 7. Deplanation in the section with a crack

The resulting dependence (46) considers the influence of a number of important factors such as the deformation of the reinforcement in the section with a crack, the distance between the cracks, the parameters of adhesion *B* of the reinforcement to concrete, the geometric characteristics of the section and the characteristics of concrete.

As a result, the general calculation algorithm is the following:

1. in accordance with the developed technique, the parameters of the stress-strain of the design section are identified;

2. the functional value l_{crc} is identified, then the level value l_{crc} is found using the inequality (41);

3. the value a_{crc} is calculated using the formula (46)/

Hence a method is set forth for calculating the resistance of reinforced concrete structures under the combined action of a transverse force, bending and torque (case 2) for the second stage of the stress-strain state, which makes it possible to identify the actual stress-strain in the presence of spatial cracks with identifying of the distance between the width of their disclosure.

Conclusions

1. A method for calculating the resistance of reinforced concrete structures under the combined action of a bending moment, torque and transverse force for the second stage of the stress-strain state (case 1 — when spatial cracks of the first type appear on the lower edge of the structure).

The prerequisites underlying the suggested calculation method are indicated. The layouts of the compressed zone occurring in the spatial section of a reinforced concrete structure under the action of bending with torsion are discussed. Analytical ratios are shown for identifying the internal forces occurring in two blocks: a cut-off section passing at the end of a spatial crack; formed by a spiral crack, and a vertical section passing through the compressed zone of concrete through the end of the front of the spatial crack.

2. A method is discussed for calculating the resistance of reinforced concrete structures under the combined action of a bending moment, torque and shear force for the second stage of the stress-strain (case 2 — with the emergence of spatial cracks of the first type on the lateral face of the structure).

3. A method is discussed for calculating the distance between spatial cracks and the width of their opening in reinforced concrete structures with torsion with bending (case 1 — the compressed zone of concrete is located at the upper edge of the reinforced concrete structure).

Analytical dependences are obtained for identifying the internal forces arising in two blocks: a cut-off section passing at the end of a spatial crack; formed by a spiral crack and a vertical section passing through the compressed zone of concrete through the end of the front of the spatial crack.

The projection of a dangerous spatial crack is identified as a function of a lot of variables and has a clear physical interpretation in the form of a set of spatial sections whose the equilibrium is affected by the parameters included in the equations. Among this set of sections, there is one that will correspond to the maximum width of the opening of spatial cracks.

The analysis shows that in order to identify the actual stress-strain of reinforced concrete structures, it becomes essential to obtain a complete picture of cracking during loading. Not only various levels of cracking of spatial cracks are discussed, but also formulas for identifying the distances between them are designed. In order to obtain a complete picture of the development and opening of spatial cracks, a representative volume from a reinforced concrete structure was selected in the form of a design diagram of the second and subsequent levels. In compliance with the resulting design scheme, equations were obtained for identifying the distance between spatial cracks of various types and the width of their opening.

4. A method is discussed for calculating the distance between spatial cracks and the width of their opening in reinforced concrete structures during torsion with bending (case 2 – the compressed zone of concrete is located at the side face of the reinforced concrete structure).
Analytical dependencies are obtained for identifying the internal forces arising in two blocks: a cut-off section passing at the end of a spatial crack; formed by a spiral crack and a vertical section passing through the compressed zone of concrete through the end of the front of the spatial crack.
The projection of a dangerous spatial crack is identified as a function of a lot of variables and has a clear physical interpretation in the form of a set of spatial sections whose the equilibrium is affected by the parameters included in the equations. Among this set of sections, there is one that will correspond to the maximum width of the opening of spatial cracks.
In order to identify the actual stress-strain of reinforced concrete structures, it becomes essential to obtain a complete picture of cracking during loading. Not only different levels of cracking of spatial cracks are discussed, but also formulas for identifying the distances between them are designed. In order to obtain a complete picture of the development and opening of spatial cracks, a representative volume from a reinforced concrete structure was selected in the form of a design diagram of the second and subsequent levels. In compliance with the resulting design scheme, equations were obtained for identifying the distance between spatial cracks of various types and the width of their opening.

References

1. Bondarenko V. M., Kolchunov V. I. *Raschetnye modeli silovogo soprotivleniya zhelezobetona* [Computational models of reinforced concrete resistance]. Moscow, ASV Publ., 2004. 472 p.

2. Golyshev A. B. , Kolchunov V. I. *Soprotivlenie zhelezobetona* [Resistance of reinforced concrete]. Kiev, Osnova Publ., 2009. 432 p.

3. Dem'yanov A. I., Kolchunov Vl. I., Yakovenko I. A. Razrabotka universal'nogo korotkogo dvukhkonsol'nogo elementa k soprotivleniyu zhelezobetonnykh konstruktsii pri kruchenii s izgibom [Development of a universal short two-pole element for the resistance of reinforced concrete structures during torsion with bending]. *Izvestiya VUZov. Tekhnologiya tekstil'noi promyshlennosti*, 2017, no. 4 (367), pp. 258—263.

4. Dem'yanov A. I., Kolchunov V. I., Sal'nikov A. S., Mikhailov M. M. Raschetnye modeli statiko-dinamicheskogo deformirovaniya zhelezobetonnoi konstruktsii pri kruchenii s izgibom v moment obrazovaniya prostranstvennoi treshchiny [Computational models of static-dynamic deformation of a reinforced concrete structure during torsion with bending at the moment of formation of a spatial crack]. *Stroitel'stvo i rekonstruktsiya*, 2017, no. 3 (71), pp. 13—22.

5. Dem'yanov A. I., Kolchunov V. I., Pokusaev A. A. Eksperimental'nye issledovaniya deformirovaniya zhelezobetonnykh konstruktsii pri kruchenii s izgibom [Experimental studies of deformation of reinforced concrete con-

structions in torsion with bending]. *Stroitel'naya mekhanika inzhenernykh konstruktsii i sooruzhenii*, 2017, no. 6, pp. 37—44.

6. Dem'yanov A. I., Sal'nikov A. S., Kolchunov Vl. I. Eksperimental'nye issledovaniya zhelezobetonnykh konstruktsii pri kruchenii s izgibom i analiz ikh rezul'tatov [Experimental studies of reinforced concrete structures under torsion with bending and analysis of their results]. *Stroitel'stvo i rekonstruktsiya*, 2017, no. 4 (72), pp. 17—26.

7. Dem'yanov A. I., Pokusaev A. A., Kolchunov V. I. Eksperimental'nye issledovaniya zhelezobetonnykh konstruktsii pri kruchenii s izgibom [Experimental studies of reinforced concrete structures under torsion with bending]. *Stroitel'stvo i rekonstruktsiya*, 2017, no. 5 (73), pp. 5—14.

8. Zalesov A. S., Khozyainov B. P. Prochnost' zhelezobetonnykh elementov pri kruchenii i izgibe [Strength of reinforced concrete elements during torsion and bending]. *Izvestiya vuzov, razd. Stroitel'stvo i arkhitektura*, 1991, no. 1, pp. 1—4.

9. Kolchunov V. I., Dem'yanov A. I., Yakovenko I. A., Garba M. O. Problema privedeniya v sootvetstvie opytnykh dannykh treshchinostoikosti zhelezobetonnykh konstruktsii ikh teoreticheskim znacheniyam [The problem of bringing experimental data on the crack resistance of reinforced concrete structures into line with their theoretical values]. *Nauka ta budivnitstvo*, 2018, no. 1 (15), pp. 42—49.

10. Kolchunov V. I., Sal'nikov A. S. Eksperimental'nye issledovaniya treshchinoobrazovaniya zhelezobetonnye konstruktsii pri kruchenii s izgibom [Experimental studies of crack formation of reinforced concrete structures during torsion with bending]. *Stroitel'stvo i rekonstruktsiya*, 2016, no. 3 (65), pp. 24—32.

11. Kolchunov V. I., Zazdravnykh E. I. Raschetnaya model' "nagel'nogo effekta" v zhelezobetonnom elemente [Calculation model of the "Nagel effect" in a reinforced concrete element]. *Izvestiya vuzov. Ser. Stroitel'stvo*, 1996, no. 10, pp. 25—29.

12. *SP 63.13330.2012 Betonnye i zhelezobetonnye konstruktsii. Osnovnye polozheniya. Aktualizirovannaya redaktsiya SNiP 52-01-2003* [SP 63.13330.2012 Concrete and reinforced concrete structures. Fundamentals. Updated version of SNiP 52-01-2003]. Moscow, 2012. 155 p.

13. Adheena Thomas, Afia S Hameed. An Experimental Study On Combined Flexural And Torsional Behaviour Of RC Beams. International Research Journal of Engineering and Technology, 2017, vol. 04, iss. 05, pp. 1367—1370.

14. Demyanov A., Kolchunov Vl. The dynamic loading in longitudinal and transverse reinforcement at instant emergence of the spatial crack in reinforced concrete element under the action of a torsion with bending. Journal of Applied Engineering Science, 2017, vol. 15(2017)3, article 456, pp. 375—380. doi:10.5937/jaes15-14663

15. Iakovenko I., Kolchunov V., Lymar I. Rigidity of reinforced concrete structures in the presence of different cracks. MATEC Web of Conferences. 6th International Scientific Conference «Reliability and Durability of Railway Transport Engineering Structures and Buildings». Transbud-2017. Kharkiv, Ukraine, April 19–21, 2017, vol. 0216. 12 p.

16. Iakovenko I., Kolchunov Vl. The development of fracture mechanics hypotheses applicable to the calculation of reinforced concrete structures for the second group of limit states. Journal of Applied Engineering Science, 2017, vol. 15(2017)3, article 455, pp. 366–375. doi:10.5937/jaes15-14662

17. Ilker Kalkan, Saruhan Kartal. Torsional Rigidities of Reinforced Concrete Beams Subjected to Elastic Lateral Torsional Buckling. International Journal of Civil and Environmental Engineering, 2017, vol. 11, no. 7, pp. 969—972.

18. Khaldoun Rahal. Combined Torsion and Bending in Reinforced and Prestressed Concrete beams Using Simplified Method for Combined Stress-Resultants. ACI Structural Journal, 2007, vol. 104, no. 4, pp. 402—411.
19. Nahvi H., Jabbari M. Crack detection in beams using experimental modal data and finite element model. International Journal of Mechanical Sciences, 2005, vol. 47, pp. 1477—1497.
20. Pettersen J. S. Non-Linear Finite Element Analyses of Reinforced Concrete with Large Scale Elements: Including a Case Study of a Structural Wall. Norwegian University of Science and Technology, 2014. 85 p.
21. Salnikov A., Kolchunov Vl., Yakovenko I. The computational model of spatial formation of cracks in reinforced concrete constructions in torsion with bending. Applied Mechanics and Materials, 2015, vol. 725—726, pp. 784—789.
22. Santhakumar R., Dhanaraj R., Chandrasekaran E. Behaviour of retrofitted reinforced concrete beams under combined bending and torsion : A numerical study. Electronic Journal of Structural Engineering, 2007, no. 7, pp. 1—7.
23. Vishnu H. Jariwalaa, Paresh V. Patel, Sharadkumar P. Purohit. Strengthening of RC Beams subjected to Combined Torsion and Bending with GFRP Composites. Procedia Engineering, 2013, vol. 51, pp. 282—289.