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MATHEMATICAL MODELING OF COLD PRESSING THE SHEET COMPOSITE

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Statement of the problem. The article examines the problem of cold pressing, which is the most important technological component in the production of sheet composite, which is widely studied in the repair and construction works in the interior decoration of residential and industrial premises. The solution to this problem is carried out on the basis of a physical and mathematical model under the assumption that the rheological properties of the deformable medium correspond to the principles of ideal plasticity and a flat deformable state. Within the framework of the problem, in two dimensions of quasistatic compression between absolutely rigid parallel-approaching plates of a thin ideally plastic layer, the stress-strain state of a composite medium is studied. It is believed that in the absence of volumetric loads, the condition of incompressibility of the medium and the associated flow law is fulfilled. Based on the hypothesis of the linear distribution of tangential stresses over the thickness of the deformable layer, analytical expressions for the statistical and kinematic characteristics of the deformation are obtained, and the condition at the edges of the rough plates makes it possible to determine the coefficient of slip thorns, which makes it possible to control the pressing process.

Results and conclusions. It was established that the components of the strain rate are directly proportional to the plate approach speed, and the normal stresses acting in the pressing direction are independent of the loading speed, decreasing in magnitude from the center to the periphery.

Keywords: yield strength, pressing, plasticity condition, mathematical model.

Introduction. The development and improvement of modern building technologies, aviation industry and electronics cannot be conceived without the use of new construction materials including composites in particular [3, 8, 13, 14].

A composite is a structural material consisting of several components: a matrix (binder) and a reinforcing medium (filler) in the form of threads, fibers, particles, etc. The mechanical behavior of composites is given by the ratio of the properties of the reinforcing elements and the matrix, as well as the bond strength between them.

Methods for designing new composite materials and investigating the stress-strain state in the process of pressing them is an urgent problem of mechanics of a deformable solid [13]. The first step to solving any problem suitable for practical research is commonly the development of a physical and mechanical model that allows for multifunctional analysis and selection of the optimal strategy for conducting the technological process [8, 15].

To date, a number of mathematical models have been developed that describe the processes occurring under the conditions of pressing composite materials [3, 10, 14]. A mathematical model is set forth that describes the processes occurring in a viscous incompressible fluid located in a thin layer between parallel planes moving towards them. The parameters of the stress-strain state obtained in this case make it possible to control the pressing process of plywood and other laminated plastics [10, 15]. The results of theoretical and experimental studies of the process of powder materials in the conditions of self-propagating high-temperature synthesis were previously presented [3,8]. A model based on the fundamental laws of conservation of mass, momentum and energy, within the framework of the mechanics of a deformable solid under conditions of wood pressing considering anisotropy was suggested in [6]. The results of modeling the process of flat pressing of plates from wood waste and thermoplastics were obtained in [11, 12, 13] where a qualitative assessment of the temperature across the thickness of the plates, degree of solidification of the thermoplastic as well as normal and tangential stresses at the interface was performed. The above analysis of these and other known models [9, 16, 18] shows that most of them describe the behavior of the bonding agent under pressing conditions without affecting the base material. In addition, the suggested models are based on the viscous and elastic properties of the medium which do not fully correspond to actual materials. The purpose of the study is to develop a mathematical model that within the framework of a deformable solid and the equations of ideal plasticity potentially describe the rheological properties of a composite material under conditions of cold flat pressing. The solution to this problem is presented in accordance with the methodology [7] on the example of semi-dry pressing of gypsum plates (GP).

1. Methods of mathematical modeling. First let the space between two rigid rough parallel plates moving towards each other at a constant velocity u be filled with a composite material whose the rheological properties correspond to the ideal plasticity model [7]. A schematic diagram of the deformation of the medium is presented in Fig. 1.

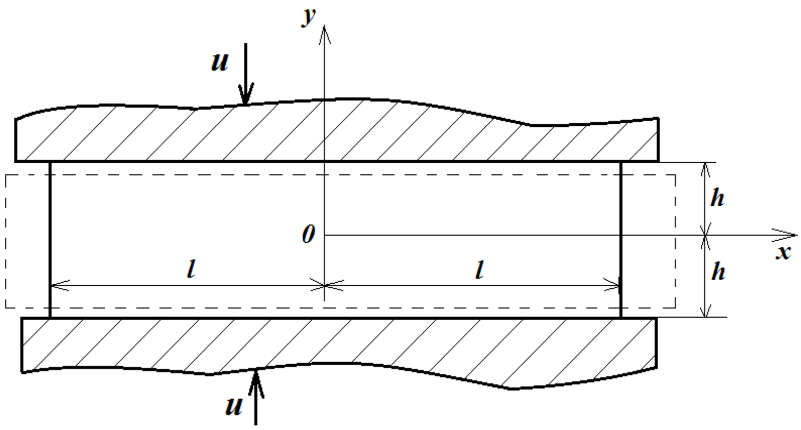

Fig. 1. Compression of a thin layer of a composite sample between the approaching plates ($-l < x < l;\ y = \pm h$)

Let us assume that the flow plane of the medium in question is determined by the plane *(x, y)*, there are no volumetric forces and the velocity u of the closure of the plates is insignificant. Neglecting elastic deformations provided that all quantities do not depend on z, let us consider a plastic flow in two dimensions [17].

Then the equilibrium equations are as follows:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0, \tag{1}$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} = 0, \tag{2}$$

where σ_x, σ_y, τ_{xy} are the stress components related to the axes x, y.

Differentiating (1) and (2) for the second time with subsequent subtraction and using the von Mises plasticity condition [1, 4, 13]

$$\frac{1}{4}(\sigma_x - \sigma_y)^2 + \tau_{xy}{}^2 = k^2, \tag{3}$$

we obtain the following expression:

$$\frac{\partial^2 \tau_{xy}}{\partial x^2} - \frac{\partial^2 \tau_{xy}}{\partial y^2} = \pm 2 \frac{\partial^2 \left(\sqrt{k^2 - \tau^2{}_{xy}}\right)}{\partial x \partial y}. \tag{4}$$

Here k is the yield stress equal to the maximum value of the shear stresses which for the Mises yield condition takes the value

$$k = \frac{\sigma_0}{\sqrt{3}},$$

where σ_0 is the material constant.

We will search for a solution (4) where the stress τ_{xy} is a function that depends only on *y*. In this case, it will be as follows:

$$\frac{\partial^2 \tau_{xy}}{\partial y^2} = 0, \tag{5}$$

which is a linear function [7]:

$$\tau_{xy} = -\frac{ky}{\mathrm{h}}, \tag{6}$$

with the boundary conditions $\tau_{xy} = \pm k$ at $y = \mp h$. This corresponds to the state of flow of a deformable medium along the contact plane to the right.

Considering that the differential equilibrium equations (1) and (2) are as follows:

$$\frac{\partial \sigma_x}{\partial x} = \frac{k}{h}, \qquad \frac{\partial \sigma_y}{\partial y} = 0, \tag{7}$$

whose integration yields:

$$\sigma_x = \frac{k}{h} x + F(y), \qquad \sigma_y = G(x), \tag{8}$$

where *F(y)* and *G(x)* are arbitrary functions identified from the plasticity condition (3) which considering (6) can be written as:

$$F(y) - G(x) + \frac{kx}{h} = \pm 2k\sqrt{1 - \frac{y^2}{h^2}}. \tag{9}$$

Since the equation (9) is satisfied for any values of *x* and *y*, the following relations are valid for *F(y)* and *G(x)*:

$$F(y) = -P \pm 2k\sqrt{1 - \frac{y^2}{h^2}}, \tag{10}$$

$$G(x) = -P + \frac{kx}{h}, \tag{11}$$

where *P* is an arbitrary constant. Under this condition, the expressions (8) are satisfied by two stress systems of this type:

$$\sigma_x = -P + \frac{kx}{h} \pm 2k\sqrt{1 - \frac{y^2}{h^2}}, \tag{12}$$

$$\sigma_y = -P + \frac{kx}{h}. \tag{13}$$

In order to identify the distribution of velocities corresponding to the stresses (6), (12), (13), it is necessary that we determine the trajectory of the slip lines [2, 15]. For that, we assume that the principal axes are generally inclined at the angle α and α+π/2, n it is obvious that the shear directions will be inclined to it at the angles β and β ± π/2 where β=α+π/4 (Fig. 2).

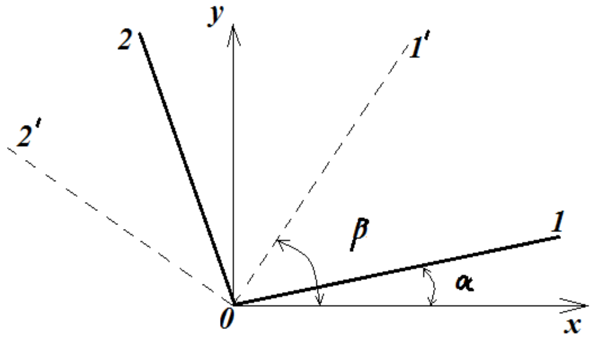

Fig. 2. General case of the mutual location of the axis coordinates (*x, y*), directions of the main axes (1, 2) and shear axes (1', 2')

Using the known equations

$$\sigma_0 = \frac{1}{2}(\sigma_1 + \sigma_2) \text{ and } \sigma_1 - \sigma_2 = 2k$$

and replacing α by (β - π/4), the stress components (12), (13) and (6), respectively, can be the following taking into account only the positive sign of the stress (12):

$$\sigma_x = \sigma_0 + k \sin 2\beta,$$
$$\sigma_y = \sigma_0 - k \sin 2\beta, \qquad (14)$$
$$\tau_{xy} = -k \cos 2\beta.$$

Then the differential equations of two systems of slip lines will be written:

$$\frac{dx}{dy} = tg\beta = \frac{1 - \cos 2\beta}{\sin 2\beta} = \frac{2(k + \tau_{xy})}{\sigma_x - \sigma_y}, \qquad (15)$$

$$\frac{dy}{dx} = -ctg\beta = -\frac{\sigma_x - \sigma_y}{2(k + \tau_{xy})}. \qquad (16)$$

Substituting the stress components (6), (12), (13) into (15), we obtain differential equations for a system of the slip lines:

$$\frac{dy}{dx} = -\frac{h - y}{\sqrt{h^2 - y^2}}, \qquad (17)$$

whose integration in the substitution $y = h\cos\Theta$ gives an equation of the family of cycloids, for which the derivative (17) is always positive, therefore, it is necessary to consider only the part where y increases as does x:

$$\begin{cases} y = h\cos\Theta, \\ x = -h(\Theta + \sin\Theta) + A. \end{cases} \tag{18}$$

Similarly, the second system of slip lines defined by the equation (16) yields the cycloids:

$$\begin{cases} y = h\cos\Theta, \\ x = h(\Theta - \sin\Theta) + B, \end{cases} \tag{19}$$

for which the sections where y decreases should be considered. The arbitrary constants A and B must be identified based on the boundary conditions.

2. Calculation of the rates of stress and deformation. For the purpose of physical interpretation of the results, let us consider a part of the medium deformed according to the scheme in Fig. 1 and bounded by the lines $x = 0$, $x = l$, $y = \pm h$ (Fig. 3).

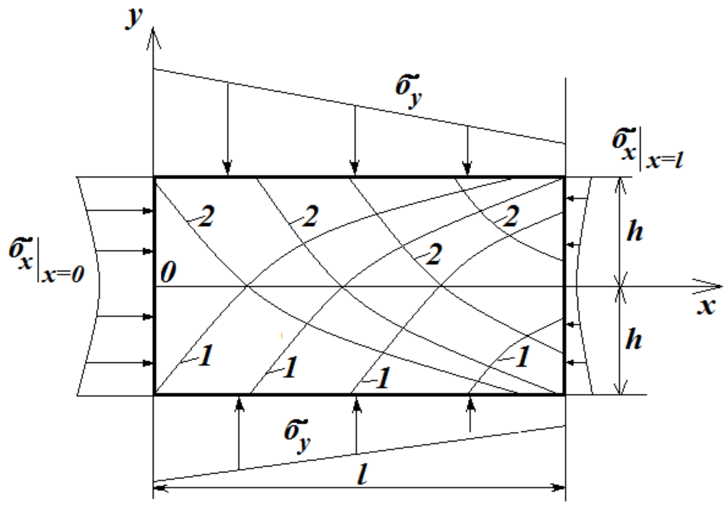

Fig. 3. Scheme of the distribution of the slip lines and normal stresses in the deformed environment between the rough plates. 1 is a family of slip lines (18); 2 is a family of slip lines (19)

Obviously, the planes $y = \pm h$ will be under the stresses (13) where the constant P is found using the boundary condition:

$$\sigma_y\big|_{x=l} = 0 = -P + \frac{kl}{h},$$

hence

$$P = \frac{kl}{h}.$$

Then the expression (13) is as follows:

$$\sigma_y = \frac{k}{h}(x - l). \tag{20}$$

In the planes $x = 0$ and $x = 1$ there are the following stresses

$$\sigma_x\big|_{x=0} = \frac{kl}{h} + 2k\sqrt{1 - \frac{y^2}{h^2}}, \tag{21}$$

$$\sigma_x\big|_{x=l} = 2k\sqrt{1 - \frac{y^2}{h^2}}, \tag{22}$$

whose directions of action and their qualitative distribution are presented in Fig. 3. It can be seen that there is a linear decrease in the stresses σ_y from the center to the periphery of the compressing plates and a power-law dependence of σ_x over the thickness of the deformed layer ensuring the movement of the material to the right. The same figure schematically shows families of orthogonal slip lines which are cycloids in accordance with the equations (18) and (19).

In order to find the distribution of the strain rates of the corresponding stresses (6), (12), (13), we use the dependences between stress and strain, which for the case of an absolutely incompressible body within the framework of plane strain [1, 16, 18] are

$$S_x = 2\phi \frac{\partial v_x}{\partial x}, \tag{23}$$

$$S_y = 2\phi \frac{\partial v_y}{\partial y}, \tag{24}$$

$$\tau_{xy} = \phi\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right), \tag{25}$$

where v_x and v_y are the components of the loading speed; φ is an unknown function containing the strain rate; S_x and S_y are the components of the stress deviator which for the case of coincidence of the main axes of the strain rate with the main axes of the stress deviator and the previously introduced designations are

$$S_x = k\sqrt{1 - \frac{y^2}{h^2}}, \tag{26}$$

$$S_y = -k\sqrt{1 - \frac{y^2}{h^2}}. \tag{27}$$

As a value is chosen,

$$\phi = \frac{k\sqrt{1 - \frac{y^2}{h^2}}}{2u}, \tag{28}$$

the equation (24) is satisfied by the solution

$$v_x = -\frac{y}{h}u, \tag{29}$$

which corresponds to the approximation of the planes with the velocity u. Inserting a similar value into (23), we get

$$v_x = \frac{ux}{h} + \xi(y), \tag{30}$$

where $\xi(y)$ is determined based on the condition that v_x and v_y satisfy (25). For that it is necessary that

$$d\left[\xi(y)\right] = -\frac{2uydy}{h\sqrt{h^2 - y^2}},$$

hence

$$\xi(y) = 2u\sqrt{1 - \frac{y^2}{h^2}},$$

and the expression (30) for the velocity is

$$v_x = \frac{ux}{h} + 2u\sqrt{1 - \frac{y^2}{h^2}}. \tag{31}$$

It can be seen that the velocity components (29) and (31) satisfy the required conditions and correspond to the plastic material, which is squeezed to the right during flat pressing and the velocity (31) at the exit from the plates is

$$v_x\big|_{x=l} = \frac{ul}{h}. \tag{32}$$

It should be noted that the obtained velocity components v_x and v_y are proportional to the speed of approach of the plates, while their corresponding stresses do not depend on the loading rate. This is contradictory to the conclusions of the problem [10] which employs the model of a viscous fluid. Additionally, the effect of slipping of the deformable medium on the rough surfaces of the pressing plates and the satisfaction of the boundary conditions $\tau_{xy} = \pm k$ at $y = \mp h$ means [5, 17] the equilibrium of the friction forces and pressing pressure per unit length of the deforming plates, i.e.,

$$\int_0^l \sigma_y dx = \frac{kl}{\mu}, \tag{33}$$

where μ is the coefficient of the sliding friction between the surface of the plate and the pressed medium whose the value makes it possible to vary the composition and moisture content of the composite.

The analysis of the results of the presented study allows the following conclusions to be drawn.

Conclusions

1. A physical and mathematical model has been set forth which describes the properties of the material in the process of direct pressing of the composite within the framework of plane deformation and the principles of ideal plasticity.

2. The parameters of the stress-strain are obtained under the assumption of a quasi-static pressing process making it possible to obtain the distribution of normal stresses over the thickness and plane of the deformed layer. The stresses in the direction of approaching the plates were found to decrease linearly from the center to the periphery and not to depend on the deformation rate.

3. If negative values for σ_x in (12) are used, for the practical use of this solution one should remember the state when the deformable material will be injected through the section $x = l$ forcing the compressive planes outward and away from each other.

4. The kinematic characteristics of the pressing process were obtained: the components of the deformation rate which are tangent at each point of the slip lines and their value is proportional to the speed of convergence of the plates.

5. For the resulting theoretical calculations to be practically employed for investigating the stress-strain of a composite material, it is necessary to conduct experiments to identify the coefficient of sliding friction between the surface of the slab and the deformable medium to be able to control the processes of compression and spreading of the layer.

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