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V. I. Kolchunov [1], S. N. Karpenko [2]

RIGIDITY OF REINFORCED CONCRETE STRUCTURES UNDER COMPLEX RESISTANCE

Research Institute of Structural Physics (NIISF) [1, 2]
Russia, Moscow
Southwestern State University (SWU) [1]
Russia, Kursk

[1] *D. Sc. in Engineering, Prof., Prof. of the Dept. of Unique Buildings and Structures, RAASN Academician, e-mail: vlik52@mail.ru*
[2] *D. Sc. in Engineering, Prof., RAASN advisor, RAASN Academician*

Statement of the problem. The article provides an approximation of rectangular cross-sections using small squares in the elements a_{pq} of the matrix.
Results. From the rigidity characteristics, forward (for internal forces) and reverse transition has been obtained, — the known curvature, twisting angle, linear deformations and displacement from shear force, including in elastoplastic compressed concrete, crack opening widths and shears. The stages of reinforced concrete during the formation of spatial cracks in works from experimental studies have been determined. The authors have got the convergence of physical phenomena and models of the theory of reinforced concrete. In the strips between the cracks, for the shear of the crack edges for their linear rigidity along the entire length of the crack, there is force “meshing” or a new effect of reinforced concrete in the works of Vl. I. Kolchunov. The forces in the longitudinal stretched reinforcement in the works of N.I. Karpenko, the parameters of the distance between the cracks l_{crc} and the coefficient ψ_c of V. I. Murashev, taking into account the effect of the coupling of reinforcement with concrete on its deformation on pliability, have been obtained based on the analysis of horizontal — normal (U_s) and vertical — tangential (V_s) displacements. The "dowel" force was determined for the ratio of normal stresses (and area) of reinforcement to experimental coefficients.
Conclusions. As a result, the rigidity physical characteristics have been obtained in the system of equations, where the elements of the rigidity matrix have been developed for the compressed area of concrete and working reinforcement from the equations (static, geometric and physical).

Keywords: rigidity, torsion, matrices, curvature, twisting angle, linear deformations, transverse displacements, crack opening, shears, the effect of reinforced concrete, pliability, "dowel" forces, experimental coefficients.

Introduction. A modern technical progress in the field of capital construction and the main directions for improving reinforced concrete structures in a complex stress strain are associated with the development of design models in torsion with bending [1—10].

Therefore, it is necessary to use new technologies for experimental research from the convergence of physical phenomena, theory and practice of reinforced concrete [11—16] where a decrease in the material consumption of its use is common, which is important in the production of efficient structures of buildings and structures.

To determine the rigidity of reinforced concrete structures, it is necessary to calculate the values of longitudinal deformations $\varepsilon_{0,\,x,\,j,\,i,\,*}$ (from the neutral axis), curvature $\frac{1}{r_{x,j,i,*}}$, torsion angles $\varphi_{A,j,i,*}$ and transverse displacements $\Delta_{Q,j,i,*}$. Rigidity characteristics have been obtained from the action of bending moment, torque, longitudinal and transverse forces. The determination of the stage of reinforced concrete for the formation of spatial cracks, longitudinal and transverse shear between cracks in reinforced concrete structures plays an important role in this.

In order to develop a credible model of deformation of reinforced concrete in the form of inclined strips, it is necessary to obtain the displacement from transverse force Δ_Q, the displacement of reinforcement Δ_s, and the forces from the dowel effect in the longitudinal Q_s and transverse reinforcement $Q_{s,Q}$. In the works by N. I. Karpenko, S. N. Karpenko for stripes separated by a crack, one side of the crack is stretched, and the other one is compressed, $n_{q1} = g_{crc}\Delta_1$. g_{crc} is the linear shear rigidity of the concrete edges of the crack, which largely depends on the width of the crack opening. $n_{q2} = g_{crc}\Delta_2$ is the shear for an oblique crack. It is also necessary to adopt average values $n_q = n_{q1} + n_{q2}$ for the whole length of the crack.

1. Rigidity matrix. The rigidity of reinforced concrete structures in bending with torsion has been obtained from the approximation of rectangular cross-sections using small squares in matrix elements D_{pq} for the distribution of bending and torque moments as well as longitudinal and transverse forces with lateral and normal cracks (Fig. 1). A direct transition, where M_x, M_t, N, Q is known but $\frac{1}{r_x}$, φ, ε_0, Δ_Q is unknown, takes the form:

$$\left.\begin{aligned} M_x &= D_{11,****}\cdot\frac{1}{r_x} + D_{12,****}\cdot\varphi + D_{13,****}\cdot\varepsilon_0 + D_{14,****}\cdot\Delta_Q;\\ M_t &= D_{21,****}\cdot\frac{1}{r_x} + D_{22,****}\cdot\varphi + D_{23,****}\cdot\varepsilon_0 + D_{24,****}\cdot\Delta_Q;\\ N &= D_{31,****}\cdot\frac{1}{r_x} + D_{32,****}\cdot\varphi + D_{33,****}\cdot\varepsilon_0 + D_{34,****}\cdot\Delta_Q;\\ Q &= D_{41,****}\cdot\frac{1}{r_x} + D_{42,****}\cdot\varphi + D_{43,****}\cdot\varepsilon_0 + D_{44,****}\cdot\Delta_Q. \end{aligned}\right\} \quad (1)$$

Here the coefficients of the first to fourth algebraic levels are from D_{11} to D_{44}.

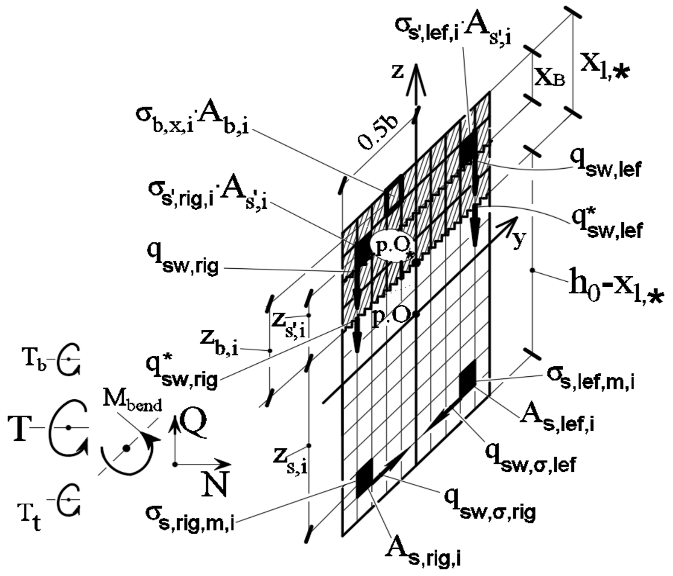

Fig. 1. Approximation of any rectangular midsections in compressed concrete and tensile reinforcement using small squares for the distribution of bending and torque moments, as well as longitudinal and shear forces with lateral and normal cracks

A reverse transition, where $\frac{1}{r_x}$, φ, ε_0, Δ_Q is known but M_x, M_t, N, Q is not, takes the form:

$$\left.\begin{aligned}
\frac{1}{r_x} &= D_{11,**}\cdot M_x + D_{12,**}\cdot M_t + D_{13,**}\cdot N + D_{14,**}\cdot Q;\\
\varphi &= D_{21,**}\cdot M_x + D_{22,**}\cdot M_t + D_{23,**}\cdot N + D_{24,**}\cdot Q;\\
\varepsilon_0 &= D_{31,**}\cdot M_x + D_{32,**}\cdot M_t + D_{33,**}\cdot N + D_{34,**}\cdot Q;\\
\Delta_Q &= D_{41,**}\cdot M_x + D_{42,**}\cdot M_t + D_{43,**}\cdot N + D_{44,**}\cdot Q.
\end{aligned}\right\} \quad (2)$$

Thus, we have the elements of the matrix $D_{p,q}$ (from $p=1$ to $p=4$ and $q=1$ to $q=4$), as well as transitions from the first to the third one: direct transition 1 from internal forces M_x, M_t, N, reverse transition 2 relative deformations $\frac{1}{r_x}$, φ, ε_0, Δ_Q and direct transition 3 internal forces M_x, M_t, N, Q. The algebraic levels from first to fourth, respectively, are obtained.

2. Determining the relative deformations. For small squares in compressed concrete, longitudinal reinforcement and cracks, we obtained longitudinal deformations $\varepsilon_{0,x,j,i}$ (from its neutral axis), curvature $\frac{1}{r_{x,j,i}}$, their torsion angles $\varphi_{A,j,i}$ and transverse displacements $\Delta_{Q,\sum}$.

An approximation of a rectangular section was performed using *n* small squares (for linear and nonlinear concretes — Fig. 2) in cross sections 1—6 (*j* is from 1 to 6 for a spatial crack from the beginning to its end).

The coefficient of the reduction in a curvilinear diagram to a trapezoid (Fig. 2) is:

$$k_{\omega} = \frac{A_{cur}}{A_{trap}}. \tag{3}$$

Here A_{cur} is the area of the diagram under the graph of the curve (Fig. 2), A_{trap} is the area of the diagram of the trapezium drawn through the points of the upper and lower fibers of this curve.

The equation of a straight line approximating this curve can be written similarly to the equation of a straight line $\varepsilon_{n,i}(z)$ multiplied by a coefficient k_{ω}. Then the value (Fig. 2) has been obtained from:

$$\varepsilon_{n,i}(z_{n,b,A,0,i}) = k_{\omega} \cdot \varepsilon_{n,i}(z_{n,b,A,0,i} + \Delta z_{\omega}). \tag{4}$$

Here k_{ω} is the coefficient of reduction of the curvilinear diagram to a trapezoid, other parameters are shown in Fig. 2.

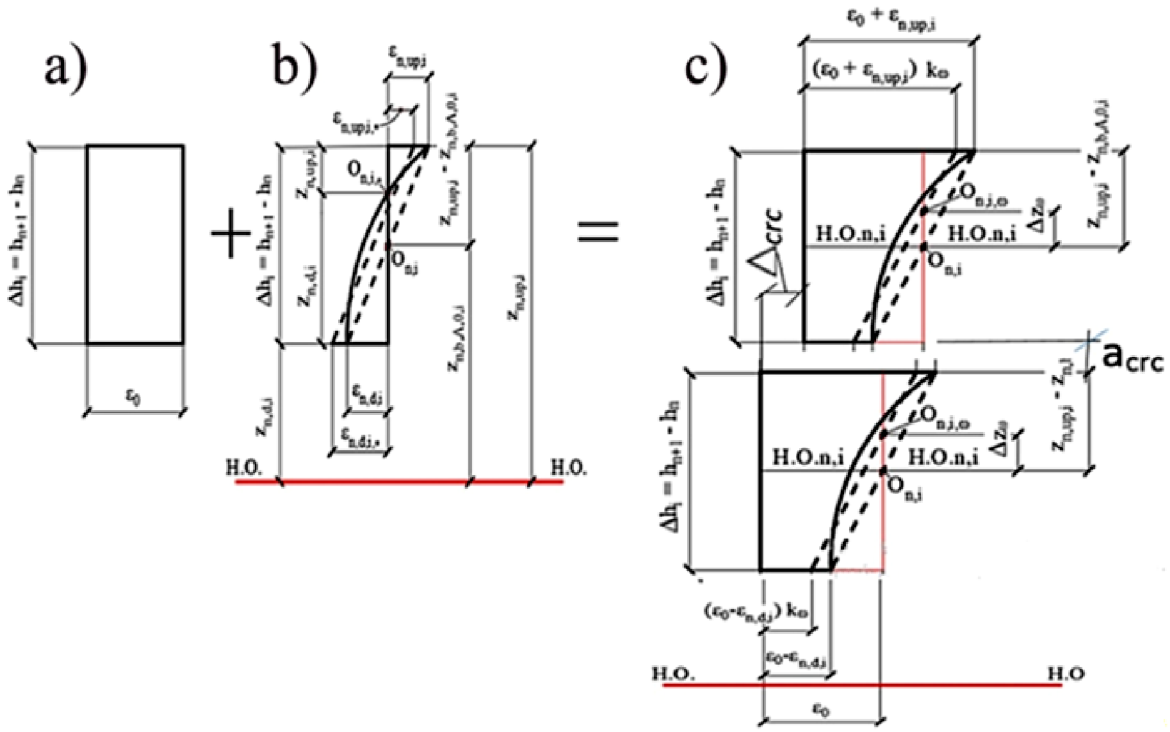

Fig. 2. Small square of concrete for longitudinal relative deformations ε_0 (a), relative deformation from bending (b), total relative deformations of small squares from the neutral axis of a common rectangular cross section (c)

The relative linear deformation $\varepsilon_{b,0,x,j,i}$ from bending of the neutral axis for the distance increment Δz (Fig. 2) takes the form (the second functional and the new hypothesis of linear deformations [17]):

$$\varepsilon_{b,0,x,j,i} = \pm\left[B_1\cdot\left(z_{n,b,A,0,i}+\Delta z - z_c\right)+B_2\cdot\left(h_0+z_{n,b,A,0,i}+\Delta z - z_c\right)\right]\cdot B_3\cdot B_4\cdot x \pm$$

$$\pm D_1\cdot y\cdot\left(z_{n,b,A,0,i}+\Delta z\right)\cdot\left[-D_2\cdot x\cdot e^{-\lambda_{***}\left(\frac{x}{l}+A_{***}\right)}+D_3\cdot e^{-\lambda_{***}\left(\frac{x}{l}+A_{***}\right)}+D_4\right]+\frac{a_{crc,0}}{l_{crc,0}}. \qquad (5)$$

Here $\Delta\varepsilon_{b,0,x,j,i}$ is an internal addition to the crack open width a_{crc}, $\Delta\varepsilon_{b,0,x,j,i} = a_{crc,0}/l_{crc,0}$ are crack edges of the concrete, $\Delta\varepsilon_{s,0,x,j,i} = a_{crc,0}\cdot k_r/l_{crc,0}$ are crack edges of the reinforcement. Internal (crack) additions are located in the neutral axis (i.e. point O*) of concrete or reinforcement.

The curvature takes the form (the second functional and the new hypothesis of linear deformations [17]):

$$\frac{1}{r_{x,j,i}} = \frac{\varepsilon_{n,up,i}}{z_{n,up,i}-\left(z_{n,b,A,0,i}+\Delta z_{\omega}\right)} = \left[\frac{\varepsilon_{n,up,i}+\varepsilon_{n,d,i}}{\Delta h_i}\right]\cdot k_{\omega}\pm\Delta_{j,i}\left(\frac{1}{r_x}\right)=$$

$$=\left[\frac{\varepsilon_{n,up,i}+\varepsilon_{n,d,i}}{\Delta h_i}\right]\cdot k_{\omega}\pm\left[\frac{\left(\frac{a_{crc}\cdot k_r}{l_{crc}}\right)_{n,up,i}+\left(\frac{a_{crc}\cdot k_r}{l_{crc}}\right)_{n,d,i}}{\Delta h_i}\right]\cdot k_{\omega}. \qquad (6)$$

Here $\varepsilon_{n,up,1,i}$ and $\varepsilon_{n,d,1,i}$ are the relative longitudinal deformation of the upper and lower regions of 1 square; Δh_i is the height of the square; $\pm\Delta_{j,i}\left(\frac{1}{r_x}\right)$ is an internal addition to crack open width a_{crc}; $\Delta\varepsilon_{s,d,x,j,i} = a_{crc}/l_{crc}$; $\Delta A_{j,i} = A_s/\mu$; $\delta = \Delta b_i/\Delta h_i$; $\Delta A_{j,i} = \Delta b_{j,i}\cdot\Delta h_{j,i} = \delta\cdot\Delta h_{j,i}^2$; $\Delta h_{j,i} = \left(A_{s,j,i}/(\mu\cdot\delta)\right)^{0.5}$. The twist angle takes the form (the first functional and a new hypothesis of concrete angular deformation (or analogue for reinforcement) [17]:

$$\varphi_{A,b,j,i} = \frac{M_{t,i}\cdot l_j}{\nu_b(\lambda)\cdot G_b\cdot A_{b,c,i}\cdot z_{b,t,i}}\times$$

$$\times\frac{1}{\sqrt{\left(b^2\left(487h-5640z\right)+16y^2\left(-101h+1320z\right)\right)^2+256y^2\left(229h^2+202hz-1320z^2\right)^2}}\times \qquad (7)$$

$$\times\frac{125h^2\pi^3}{2}\pm\left(\left(\frac{\Delta_{crc,zx,rig}\cdot l_j}{l_{crc}\cdot 0.5h_{j,i}}\right)^2+\left(\frac{\Delta_{crc,yx,rig}\cdot l_j}{l_{crc}\cdot 0.5b_{j,i}}\right)^2\right)^{\frac{1}{2}}\pm\left(\left(\frac{\Delta_{crc,zx,lef}\cdot l_j}{l_{crc}\cdot 0.5h_{j,i}}\right)^2+\left(\frac{\Delta_{crc,yx,lef}\cdot l_j}{l_{crc}\cdot 0.5b_{j,i}}\right)^2\right)^{\frac{1}{2}}.$$

Here

$$M_{t,b,i} = \gamma_{t,b,sum,u}\cdot\nu_b(\lambda)\cdot\omega_{\gamma,i}\cdot A_{b,c,i}\cdot z_{\eta,b,i},$$

i is from 1 to m;

$$M_{t,s,i} = \varepsilon_{s,rig,m,i} \cdot v_{s,m,i} \cdot E_{s,m,i} \cdot A_{s,rig,i} \cdot b_{s,i} + \varepsilon_{s',lef,m,i} \cdot v_{s,m,i} \cdot E_{s,m,i} A_{s',lef,i} \cdot b_{s',i},$$

i is from 1 to *n-m*, contour for longitudinal tensile reinforcement, *i* is from 1 to *m*, contour for compression reinforcement, from 1 to *m*; A_s is an area of a small square of longitudinal reinforcement, $\Delta A_{j,i} = A_s / \mu$, μ is the coefficient of reinforcement;

$$M_{t,sw,i} = \frac{\varepsilon_{sw,rig,i} \cdot v_{sw} \cdot E_{sw} \cdot A_{sw}}{S} \times (a_j - c_j) \times \times z_{\eta,i} \pm \frac{\varepsilon_{sw,lef,i} \cdot v_{sw} \cdot E_{sw} \cdot A_{sw}}{S} \cdot (a_j - c_j) \cdot z_{\eta,i,*},$$

j is from 1 to *j*, contour for lateral right and left reinforcement;

$$M_{t,sw,\sigma,i} = \frac{\varepsilon_{sw,\sigma,rig,i} \cdot v_{sw} \cdot E_{sw} \cdot A_{sw}}{S_\sigma} \cdot (a_j - c_j) \cdot z_{\eta,\sigma,i} \pm \frac{\varepsilon_{sw,\sigma,lef,i} \cdot v_{sw} \cdot E_{sw} \cdot A_{sw}}{S_\sigma} \cdot (a_j - c_j) \cdot z_{\eta,\sigma,i,*},$$

j is from 1 to j, contour for transverse reinforcement; $\omega_{\gamma_{t,sum}}$ is a filling the deformation diagram; $A_{b,i}$ are areas for *i*-th squares ; *n* is the total number of small squares ; *m* is the number of squares of the compressed area ; *n-m* is the number of squares for the longitudinal reinforcement ; *k* is tensile reinforcement, compressed reinforcement, transverse reinforcement with normal cracks and lateral cracks, *j* are cross-sections (*j* = 1 – 6); $z_{\eta,b,i}$, $b_{s,i}$, $b_{s',i}$, $z_{\eta,\sigma,i}$, $z_{\eta,\sigma,i,*}$ are arms of force from point O*; internal addition $\pm\Delta\varphi_{j,i}$ for cracks from longitudinal and transverse shear of concrete (or analogue for reinforcement), $\pm\Delta\varphi_{b,j,i} = \left(\left(\frac{\Delta_{crc,zx,rig} \cdot l_j}{l_{crc} \cdot 0.5h_{j,i}}\right)^2 + \left(\frac{\Delta_{crc,yx,rig} \cdot l_j}{l_{crc} \cdot 0.5b_{j,i}}\right)^2\right)^{0,5} + \left(\left(\frac{\Delta_{crc,zx,lef} \cdot l_j}{l_{crc} \cdot 0.5h_{j,i}}\right)^2 + \left(\frac{\Delta_{crc,yx,lef} \cdot l_j}{l_{crc} \cdot 0.5b_{j,i}}\right)^2\right)^{0,5}$.

The displacement from the transverse force *Q* in [18] takes the form:

$$\Delta_{Q,b,\sum} = \Delta_{Q,j,i} + \Delta_{add,b} = \frac{Q_{j,i}}{G_b(\lambda) \cdot A_{b,j,i}} \cdot \eta_{Q,b} + \Delta_{add,zx,b}(k_r); \tag{8}$$

$$\Delta_{Q,s,\sum} = \frac{Q_{j,i}}{G_s(\lambda) \cdot \frac{A_{s,j,i}}{\mu_s}} \cdot \eta_{Q,s} + \Delta_{add,zx,s}(k_r). \tag{9}$$

Here $\Delta_{Q,\sum}$ are the total displacements from the lateral force (from structural mechanics); internal addition $\Delta_{add,zx,b}(k_r)$ ($\Delta_{crc,zx}$ in [18—20]); internal addition includes displacement (transverse and longitudinal shears) from shear force $\Delta_{crc,zx,Q}$ and torque $\Delta_{crc,zx,Mt}$; $\Delta_{crc,Q} = \Delta_{crc,Mt} / \chi$ ($\Delta_{crc,Q}(x,z) = \Delta_{crc,Mt}(x,z) / \chi$), $\Delta_{crc,Q}(y,z) = \Delta_{crc,Mt}(y,z) \cdot / \chi$; $\chi = M_t / Q$; $\phi = M_t / N$; $\eta = M_t / M_x$.

3. Displacements from the shear force Δ_Q in an inclined section. In the section crack T_2, forces N_b, N_s, have increments of ΔN_b and ΔN_s, respectively. Changes in other quantities can be neglected when *Q=const* (on the segment $T_1 - T_2$).

For each element of the inclined section (and Fig. 3 a), there are three equilibrium equations: the equation of the projections of forces on the vertical (Z) and horizontal (Y) axes, and the equation of the moments of forces (traditionally compiled relative to the center of gravity of the concrete in the compressed zone are points O_1 and O_2, Fig. 3 b). The transverse forces Q_1, Q_2 $(Q_1 = Q_2 = Q)$ and moments M_1, M_2 taken by the corresponding inclined sections T_1 и T_2 are equal to the transverse forces and moments in the normal sections passing through the tops of the indicated inclined cracks. The loading stages of crack propagation have contained a new criterion [21—23]:

$$\frac{d\zeta_{c,u}}{dh_{crc}} = 0. \qquad (10)$$

Here $\zeta_{c,u}$ is a new criterion for the development of a crack for an increment in the specific surface (in potential energy and work); h_{crc} is the crack length.

The possible action of the moment is taken into account by replacing 0.5x to ω_x where the actual coefficient of the completeness of the shear stress diagram in the concrete of the compressed zone on the segment x is which will increase under the action M_z, the limitation is $0.5 \le \omega \le 1.1$. The equation for the transverse force is

$$Q = \left(q_w ctg\theta + n_q + \frac{Q_s}{h_0 - x} \right) h_0 \beta, \qquad (11)$$

where β is a generalized parameter that takes into account the influence of additional factors $(\beta \approx 1 - 1.2)$.

The design diagrams of inclined sections shown in Fig. 3a are not the only possible ones. The scheme with a through section of the compressed zone and its modification with a partial through inclined section (Fig. 3 b) correspond criterion (11) at $\omega^* = 1$. However, it is necessary to take into account two features of calculating the strength of such sections.

The first feature is associated with the need to take into account the additional moment m_c in the inclined section of the compressed zone. The moment arises due to the eccentricities of the application of forces and on the inclined face relative to the selected center which is the point O'_2 (in Fig. 3, b). The moment of all the forces relative to the point O'_2 takes the form:

$$m_c = 0.5x^2\omega^*\left(n_q + q_{sw}ctg\theta + \frac{Q_s}{h_0 - x}\right)ctg\theta\,. \quad (12)$$

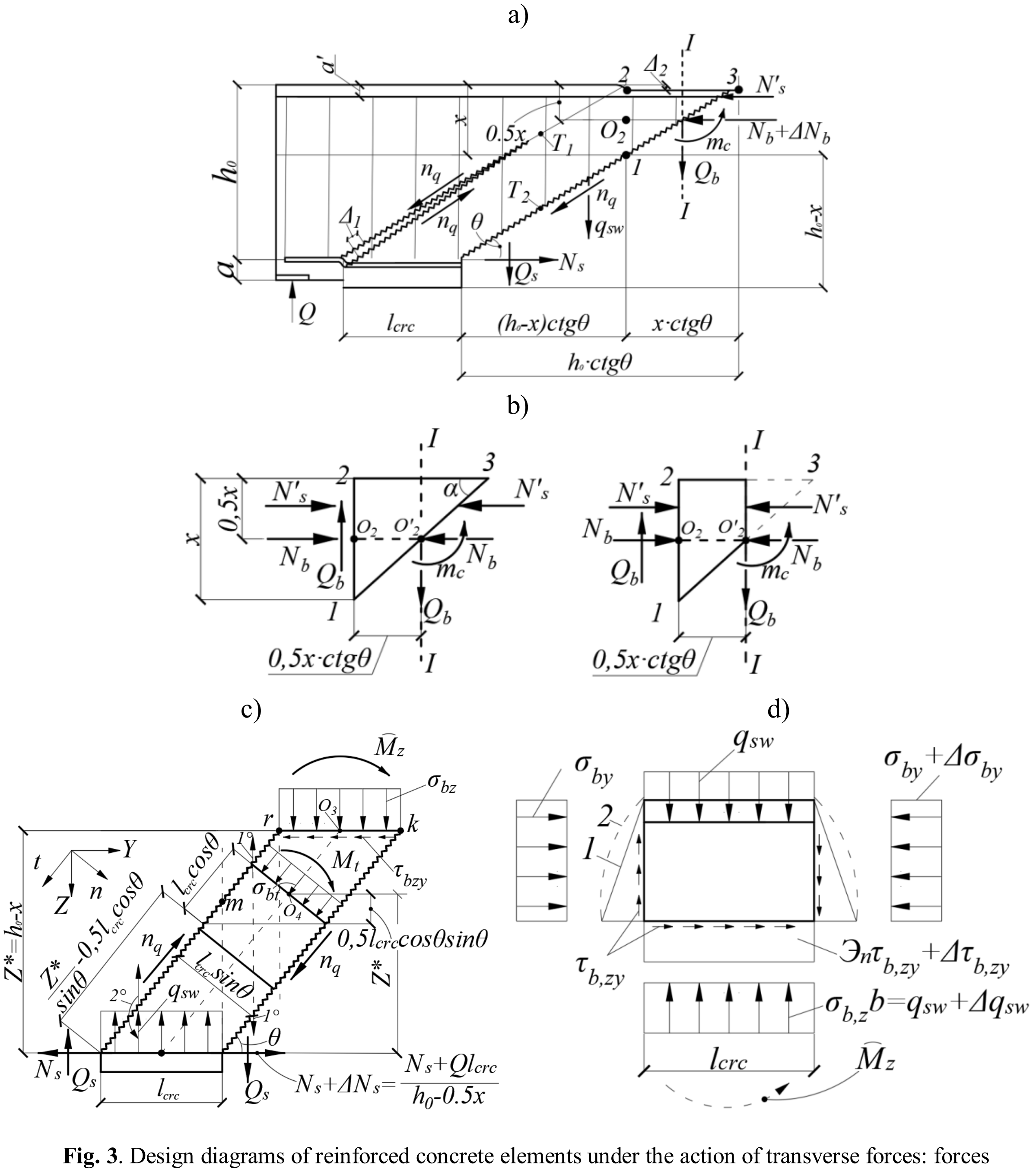

Fig. 3. Design diagrams of reinforced concrete elements under the action of transverse forces: forces in the working reinforcement in an inclined section (a); transition to through and partially through inclined sections (b); strips of concrete between cracks (c); diagrams of forces in transverse reinforcement and concrete of a compressed inclined strip (d); 1° is the transverse rebars in the mr section, the forces in which are balanced and do not affect the stress state of the strip, 2° is the transfer of forces in the transverse reinforcement to the anchorage level in the longitudinal reinforcement

The second feature is that some efforts are mutually balanced (Fig. 3, d) and do not affect the general equations.

Comparing the dependences (11) with a similar one by M. S. Borishansky [24], two of its differences can be obtained:

1) the equation (11) directly contains the shear forces n_q and shear forces in the longitudinal tensile reinforcement Q_s;
2) the equation does not directly contain the shear force component in concrete Q_b. It is expressed in terms of Q and the other components of the stress strain (n_q, q_{sw} and Q_s).

As the basis, we have taken a simpler strength condition (12) which includes the unknown quantities n_q, Q_s, θ. The shear forces n_q have been conditionally carried over the crack edges (Fig. 3, a). One shear is associated with obstacles to the bending of concrete strips, and the second one with a pure shear of the crack edges, $n_q = n_{q1} + n_{qII}$.

Bending of the strip causes increments of the effort ΔN_s. In adjacent strips, separated by a crack, one crack edge is stretched and the other one is compressed, which leads to a shear of the concrete edges (Δ_1 in Fig. 3, a), $n_{q1} = g_{crc}\Delta_1$. g_{crc} is the linear shear rigidity of the concrete edges of the crack, which largely depends on the width of the crack opening. $n_{q2} = g_{crc}\Delta_2$ is the shear for an oblique crack.

The analysis of the experimental data of the basic approximation shows that

$$n_q = R_{bt} b\left(k_1 + k_2 tg^2\theta + k_3\chi_m^2\right). \quad (13)$$

Here χ_m is a new variable that takes into account the influence of the moment M (M_2 in Fig. 3, a in the critical crack T_2) on the value of the shear forces n_q ($\chi_m = Qh_0\beta_m / M$); k_1, k_2, k_3 are experimental coefficients [18—20]; β_m is a parameter that takes into account the form of the moment diagram (single-valued, two-digit) in the area of strength verification by Q (with a single-digit moment diagram $\beta_m = 1$, with a two-digit one $\beta_m = 0,71$).

The displacements of concrete cracks Δ_{crc} and Δ_Q take the form:

$$\Delta_{crc} = \Delta_1 + \Delta_2 = \frac{n_{q1}}{g_{crc}} + \frac{n_{q2}}{g_{crc}} = \frac{n_q}{g_{crc}} \le \Delta_{\exp,crc} = e^{-0.91127\left(\frac{a}{h_0} - 3.06419\right)} + 0.43976, \quad (14)$$

$$\Delta_Q = \Delta_{crc}\sin\theta, \quad (15)$$

where $n_q = n_{q1} + n_{q2}$; n_q is the linear shear force; q_{sw} is the linear load in clamps; Q is the ultimate shear force; θ is the angle of inclination of cracks to longitudinal reinforcement; g_{crc} is the linear shear rigidity of the crack.

The transverse (dowel) forces Q_s in longitudinal tensile reinforcement have been identified based on the analysis of horizontal, i.e., normal (U_s) and vertical, i.e., tangential (V_s) displacements of the reinforcing bars of the lower tensile reinforcement during the opening of an inclined crack a_{crc}. The displacement (U_s) corresponds to the normal forces $N_s = \sigma_s A_s$, and the displacement (V_s) corresponds to the tangential $Q_s = \tau_s A_s$ forces in the longitudinal tensile reinforcement. According to [18], the normal displacements are

$$U_s = 0.5 l_{crc} \varepsilon = 0.5 l_{crc} \sigma_s \psi_s / E_s = 0.5 l_{crc} N_s \psi_s / A_s E_s , \qquad (16)$$

where ε is the average relative deformation of the reinforcement in the sections between cracks; l_{crc} is the distance between cracks, ψ_s is the coefficient by V. I. Murashev taking into account the effect of adhesion of reinforcement to concrete on its deformation. In (16), the group of quantities represents the horizontal pliability of the reinforcement B_s is

$$B_s = 0.5 l_{crc} \psi_s / E_s A_s . \qquad (17)$$

The dependence can be simplified:

$$U_s = N_s B_s . \qquad (18)$$

Experimental studies show that the pliability of tangential movements (V_s) is significantly lower than that of normal movements. B_s can be determined using the transition coefficient η_τ:

$$V_s = Q_s B_s \eta_\tau . \qquad (19)$$

According to [18—20]:

$$\left.\begin{aligned} U_s &= 0.5 a_{crc} \sin\theta - 0.5 \Delta_s \cos\theta; \\ V_s &= 0.5 a_{crc} \cos\theta + 0.5 \Delta_s \sin\theta. \end{aligned}\right\} \qquad (20)$$

Denoting $\varphi_\Delta = \Delta_s / a_{crc}$ (with allowance for $\Delta = \Delta_1 + \Delta_2$), we have obtained:

$$\left.\begin{aligned} U_s &= 0.5 a_{crc} (\sin\theta - \varphi_\Delta \cos\theta); \\ V_s &= 0.5 a_{crc} (\cos\theta + \varphi_\Delta \sin\theta). \end{aligned}\right\} \qquad (21)$$

According to the formulas (19), (20), (21):

$$\Delta_s = \frac{0.5 a_{crc,s} \sin\theta - N_s B_s}{0.5 \cos\theta} \le \Delta_{\exp,s} . \qquad (22)$$

From the dependence (19) we get:

$$Q_s = \frac{V_s}{\eta_\tau B_s} = \frac{V_s N_s}{\eta_\tau U_s} = \frac{V_s \sigma_s A_s}{\eta_\tau U_s}. \tag{23}$$

Taking into account the approximate dependence taking the form

$$U_s = 0.5a_{crc}\sin\theta; \quad V_s = 0.5a_{crc}\cos\theta, \tag{24}$$

we have obtained the "dowel" force:

$$Q_s = \frac{\sigma_s A_s ctg\theta}{\eta_\tau}. \tag{25}$$

From [18—20], the transition coefficient prior to the yield of the reinforcement is equal to $\eta_\tau = 13 - 17$ (on average $\eta_\tau = 16$), and following the yield (at $\sigma_s = R_s$) $\eta_\tau = 20 - 25$.

As a result, we have obtained the rigid physical characteristics (Fig. 4) D_{pq} (p, q – 1, 2, 3, 4) in the system of equations:

$$\left| \begin{array}{l} M_x = D_{11}\dfrac{1}{r_{x,j,i,*}} + D_{12}\cdot\varphi_{j,i,*} + D_{13}\cdot\varepsilon_{0,j,i,*} + D_{14}\cdot\Delta_{Q,j,i,*}; \\ M_t = D_{21}\dfrac{1}{r_{x,j,i,*}} + D_{22}\cdot\varphi_{j,i,*} + D_{23}\cdot\varepsilon_{0,j,i,*} + D_{24}\cdot\Delta_{Q,j,i,*}; \\ N = D_{31}\dfrac{1}{r_{x,j,i,*}} + D_{32}\cdot\varphi_{j,i,*} + D_{33}\cdot\varepsilon_{0,j,i,*} + D_{34}\cdot\Delta_{Q,j,i,*}; \\ Q = D_{41}\dfrac{1}{r_{x,j,i,*}} + D_{42}\cdot\varphi_{j,i,*} + D_{43}\cdot\varepsilon_{0,j,i,*} + D_{44}\cdot\Delta_{Q,j,i,*}. \end{array} \right| \tag{26}$$

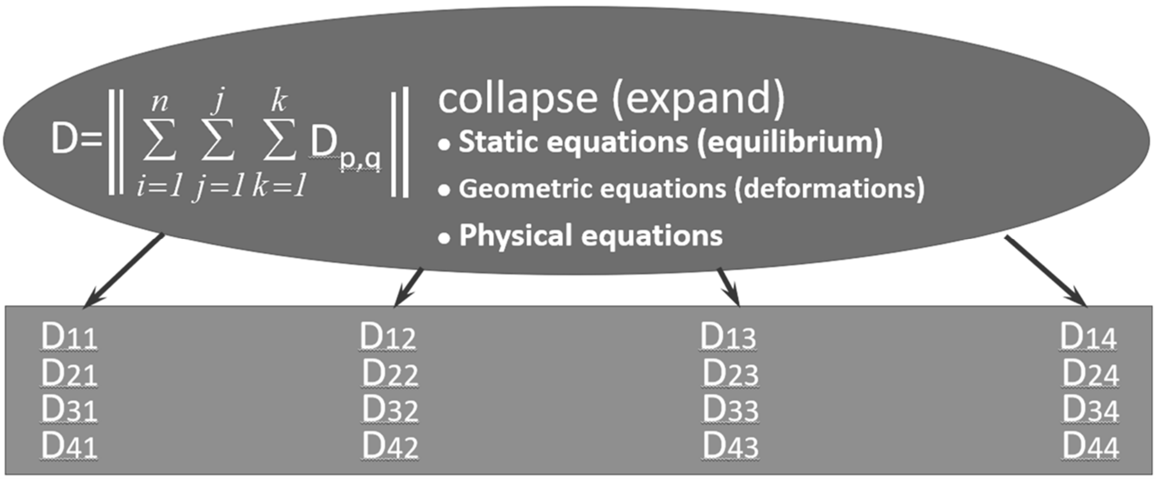

Fig. 4. Matrix of rigidity rectangular cross-sections using small squares for physical characteristics from D_{11} to D_{44}: *n* is the total number of small squares; m is the number of squares of the compressed area; *n-m* is the number of squares of the stretched area of the longitudinal reinforcement; *k* is the tensile reinforcement, compressed reinforcement, lateral reinforcement with normal cracks and with lateral cracks; *j* are cross sections, *j* = 1 – 6

Here $D_{11} - D_{44}$ are the matrix elements for the function of the parameters $\eta_{hor,b}; a'_{sc};$ $G_b(\lambda)$; $A_{b,x_k}; A_{b,j,i}; A_{b,*};$ $A_{b,*,ad}; x_{B,k}; l_1; l_2;$ $\alpha_{Q_b}; \alpha_{Q_{sw}}$.

Additionally, for norms and documents with eccentric compression or tension of elements and distribution in the cross-section of concrete of an element of the deformation diagram of only one sign, the limiting values of the relative deformations of concrete $\varepsilon_{b,ult}$ ($\varepsilon_{bt,ult}$) are determined depending on the ratio of concrete deformations on the opposite faces of the section of the element ε_1 and ε_2 ($|\varepsilon_2| \geq |\varepsilon_1|$) by the formulas:

$$\varepsilon_{b,ult} = \varepsilon_{b2} - (\varepsilon_{b2} - \varepsilon_{b0}) \cdot \frac{\varepsilon_1}{\varepsilon_2}, \tag{27}$$

$$\varepsilon_{bt,ult} = \varepsilon_{bt2} - (\varepsilon_{bt2} - \varepsilon_{bt0}) \cdot \frac{\varepsilon_1}{\varepsilon_2}. \tag{28}$$

Here ε_{b0}, ε_{bt0}, ε_{b2}, ε_{bt2} are the deformation parameters of the design diagrams of the concrete state ($\varepsilon_{b0} = 0.002$ – under axial compression; $\varepsilon_{bt0} = 0.0001$ – under axial tension; $\varepsilon_{b2} = 0.035$ – for concrete of compressive strength B60 and below; from $\varepsilon_{b2} = 0.033$ for high-strength concretes of strength class B70 to $\varepsilon_{b2} = 0.028$ – B100; $\varepsilon_{b0} = 0.00015$ – with a short load).

The limiting values of the relative deformation of the reinforcement are assumed to be equal: 0.025 – for reinforcement with a physical yield point; 0.015 – for reinforcement with a conditional yield point.

Conclusions

1. The approximation has been performed for rectangular cross-sections using small squares in the elements of a matrix for rigidity characteristics from $D_{11,****}$ to $D_{44,****}$, and direct (for internal forces) and reverse transition is also has been obtained for a known curvature, twist angle, linear deformation and displacement from shear force, including elastic and plastic compressed concrete, spatial crack opening widths and longitudinal and transverse shear.

2. In the strips between the cracks, for the shear of the crack edges for their linear rigidity along the entire length of the crack, there is force “meshing” or a new effect of reinforced concrete in the works by Vl.I. Kolchunov. Based on the analysis of horizontal and vertical displacements, the forces in the longitudinal tensile reinforcement are obtained in the works by N. I. Karpenko, S. I. Karpenko. The parameters of crack spacing l_{crc}, coefficient by

V. I. Murashev ψ_s, "dowel" force for the ratio of normal stress and reinforcement area to the experimental coefficients η_τ have been determined.

3. Experimental studies show that the pliability of tangential displacements V_s and normal displacements can be expressed through a certain transition coefficient η_τ. Experimental studies show that the pliability of tangential displacements V_s and normal displacements U_s can be expressed using the certain transition coefficient η_τ. The analysis shows that the transition coefficient prior to the yield of the reinforcement is equal to $\eta_\tau = 13 - 17$ (on average $\eta_\tau = 16$), and following the yield (at $\sigma_s = R_s$) $\eta_\tau = 20 - 25$. The value of the shear forces is also determined. k_1, k_2, k_3 are experimental coefficients by N. I. Karpenko and S. N. Karpenko.

4. The concrete crack displacements Δ_{crc} are obtained as a ratio of shear forces n_q to shear rigidity g_{crc} where Δ_Q equals $\Delta_{crc} \sin\theta$. The experimental dependence $\Delta_{crc,exp} - a/h_0$ of the crack shear for the exponential coupling is plotted.

5. As a result, the rigidity physical characteristics D_{pq} have been determined in the system of equations for the function $D_{pq}(\eta_{hor,b}; a'_{sc};\ G_b(\lambda); A_{b,x_k}; A_{b,j,i}; A_{b,*};\ A_{b,*,ad}; x_{B,k}; l_1; l_2;\ \alpha_{Q_b}; \alpha_{Q_{sw}})$.

Then the matrix takes the form $D = \left| \sum_{i=1}^{n} \sum_{j=1}^{j} \sum_{k=1}^{k} D_{pq} \right|$ in simple expressions and in full expanded form for static (equilibrium), geometric (deformation) and physical equations.

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