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MODELLING OF DISTRIBUTION OF VELOCITIES AND PRESSURES IN THE VORTEX FLOW IN THE COUNTER-FLOWMETER OF HEATING SYSTEM

Problem statement. The absence of possibility of registration of small heat-carrier discharges is revealed on the basis of the analysis of well-known methods of heat supply control.

Results and conclusions. The vortex type measuring device is suggested for which the mathematical model describing interaction of velocity and pressure fields applied to specific movement conditions of two-component medium (a liquid and solid particles of rust and scale) is developed. Mathematical modeling of the force action of the heat carrier made it possible to consider the vortex core as a solid body acting on recording element of the flowmeter, which provides for more lower sensitivity level, for example, of the counter-flowmeter of the heating system.

Keywords: velocity and pressure distribution**,** heating3 system, counter-flowmeter.

Introduction

In crisis, problem of energy consumption reduction is urgent. One of the solutions of the problem is an optimization of heat energy consumption in heat supply system by increasing the control range up to minimum permissible discharge registered by a counter.

Widely used turbine type flowmeters hardly register small discharges of moving liquid because of design features (mandatory clearance between the bottom surface of the rotating element of flow movement registration and internal surface of the counter frame bottom).

The other solution of the problem is an optimization of heat carrier discharge in changing environmental conditions of a residential building. To increase instrument sensitivity at low liquid discharges vortex gauges with embedded swirling jet apparatus in the shape of axially tangential swirlers mounted in the form of the nozzles at the outlet and inlet of the measuring tube are applied.

Liquid discharge measurement is performed with the use of the strain gauge flow meter (Fig. 1).

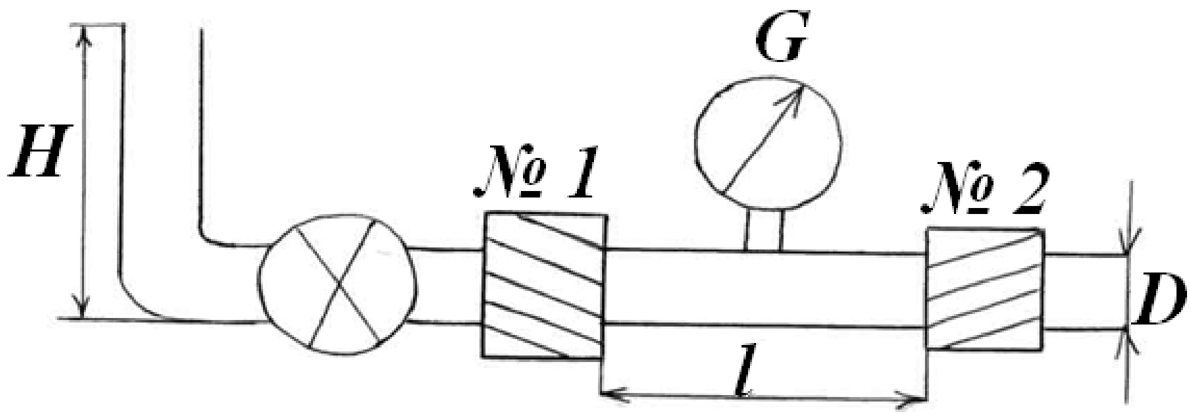

Fig. 1. Experimental arrangement diagram:
№ 1, № 2 — swirling jets apparatus at the inlet and at the outlet, respectively;
ℓ — length of branch pipe; D — diameter of branch pipe

Rotating flow produced by the inlet swirling jets apparatus № 1 has the central zone which do not move translationally. The presence of this zone greatly influences on the output characteristics of the device which affects the change in physicotechnical characteristics of the measured medium, for instance, in its viscosity, pressure, etc.

To remedy this flaw, swirling jets apparatus № 1 is used which swirls the incoming flow in the opposite direction, which makes it possible to compensate the influence of viscosity on the reading of the instrument and reduce natural head losses. An effective area of the second swirling jets apparatus is less than that of the first apparatus, therefore, when inside and outside rotating jets mix, total flow velocity and direction of rotation are formed by the outside jet with larger power obtained from nozzle № 1. Inside (reflected) jet from nozzle № 2 inhibits the rotation of the outside jet. It is obviously that viscosity of the measured medium change the rotation velocity of both jets.

However, inasmuch as decrease in the rotation velocity of the inside jet causes decrease in inhibition of the resultant rotating flow, its angular velocity remains stable and viscosity-independent [1]. Such diagram involves swirlers in the form of branch pipe nozzles in which the liquid is fed tangential to the apparatus surface in inverse directions from each nozzle.

If nozzles № 1 and № 2 are intercommunicated with each other through the common branch pipe of length ℓ and diameter D, with

$$\ell/D=1.42\div6.42,$$

frequency of vortex oscillations remains constant in a wide range of discharges. Length of the branch pipe and its diameter D significantly influence the frequency $f_в$ and stability of vortex formation. The frequency of vortex oscillations $f_в$ is in linear connection with the flow velocity $\omega_в$ on condition that diameter of vortex core diameter $d_в$ is constant in the whole range of the vortex core. This condition is true if the length ℓ of the forming branch pipe is so that the thickness of a boundary layer δ is less than thicknesses of the circular element of the exterior of the rotating flow:

$$\delta \le \frac{d_0 - d_в}{2}, \tag{1}$$

branch pipe length ℓ is determined by empirical formula

$$l = \frac{d_0 - d_в}{2a}\sqrt{\mathrm{Re}}, \tag{2}$$

where d_0 is the inside diameter of the branch pipe; Re is the Reynolds number; a is the const defined from the experiment.

In the range

$$Re=10^3\div2\cdot10^6$$

the length of the forming branch pipe

$$\ell=(1.9\div2.0)D,$$

where D is the inside diameter of the vortex chamber. The length of the smooth transitions from vortex chamber to the forming branch pipe and outlet to the expansion area are chosen from condition

$$\frac{\partial \delta}{\partial \ell} = \min .$$

Considering this condition, the length of confusor № 2 is chosen to be equal to 0.3*D*, the length of confusor, 0.7*D*.

Let us denote angular velocity of the vortex core of resulting rotating flow by $\omega_в$ and determine pressure distribution in its normal section. For this purpose we apply the differential equations of motion of an ideal medium of a liquid particle in the form of Euler equations.

In the case of velocity and pressure field unchanging with time, the differential equations of motion of an ideal medium in Euler's form for flat flow are as follows [2]:

$$\upsilon_x \frac{\partial \upsilon_x}{\partial x} + \upsilon_y \frac{\partial \upsilon_x}{\partial y} = F_x - \frac{1}{\rho} \cdot \frac{\partial p}{\partial x}, \tag{3}$$

$$\upsilon_x \frac{\partial \upsilon_y}{\partial x} + \upsilon_y \frac{\partial \upsilon_y}{\partial y} = F_y - \frac{1}{\rho} \cdot \frac{\partial p}{\partial y}, \tag{4}$$

where ρ is the liquid density, F_x, F_y are the projections of mass gravity forces F acting on the particle in the vortex.

Components of linear velocity of liquid particles in the vortex core υ_x and υ_y are determined from relationship (Fig. 2)

$$\begin{aligned} \upsilon_x &= -\omega_в y; \\ \upsilon_y &= -\omega_в x. \end{aligned} \tag{5}$$

Let us find corresponding values of frequency derivatives:

$$\begin{aligned} \frac{\partial \upsilon_x}{\partial x} &= 0; \\ \frac{\partial \upsilon_x}{\partial y} &= -\omega_в; \\ \frac{\partial \upsilon_y}{\partial x} &= \omega_в; \\ \frac{\partial \upsilon_y}{\partial y} &= 0. \end{aligned} \tag{6}$$

By substituting (5) and (6) in (3) and (4) and considering that

$$F_x = F_y = 0,$$

we obtain

$$-\omega^2 x = -\frac{1}{\rho} \cdot \frac{\partial p}{\partial x};$$
$$-\omega^2 y = -\frac{1}{\rho} \cdot \frac{\partial p}{\partial y}. \quad (7)$$

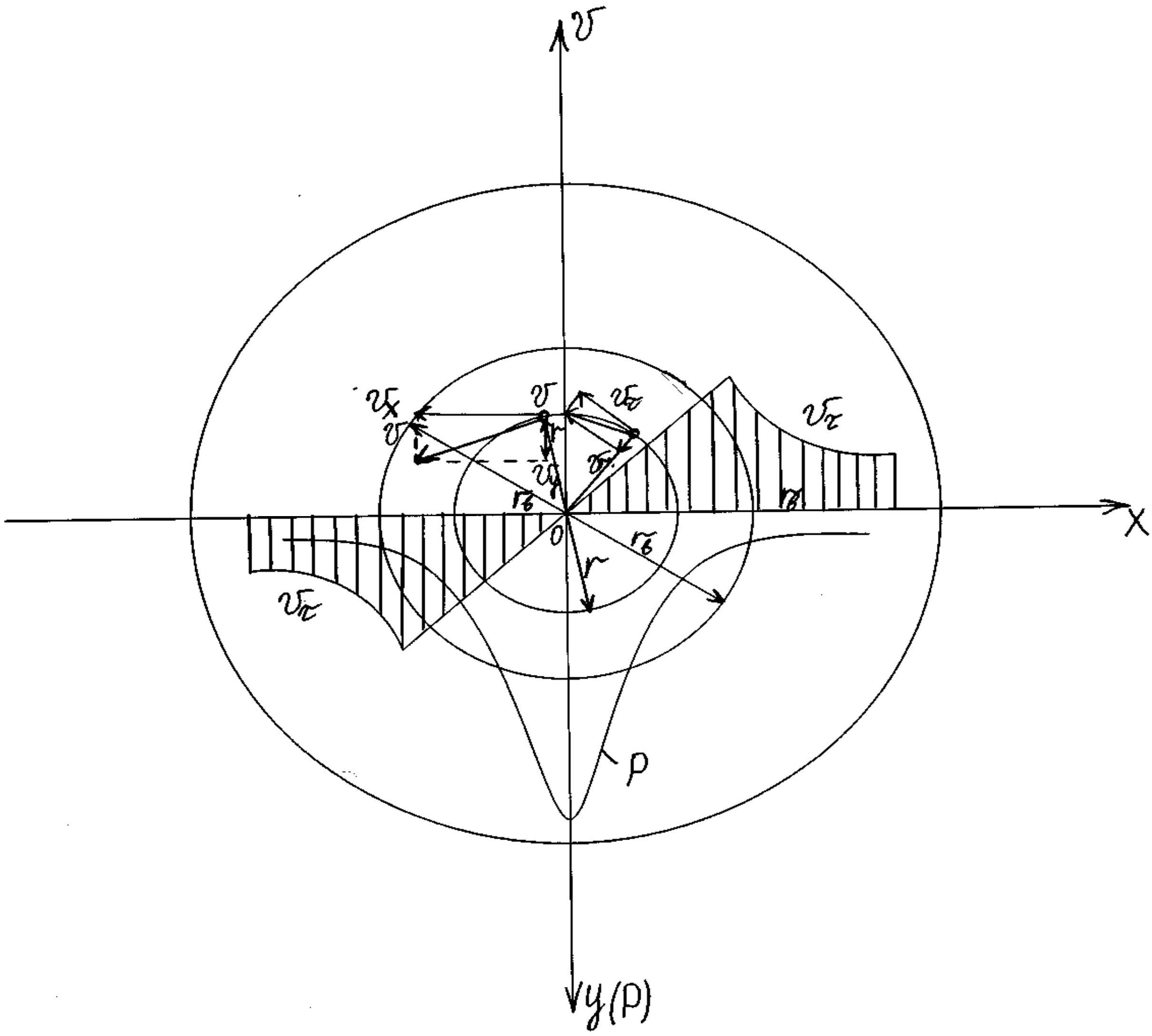

Fig. 2. Distribution of velocities υ_τ and excess static pressure in vortex flat flow

Multiplying both sides of each equation by corresponding differentials and summing the equation result in:

$$dp = \frac{\rho\omega_в^{\ 2}}{2} d\left(x^2 + y^2\right) = \frac{\rho\omega_в^{\ 2}}{2} dr^2, \quad (8)$$

where

$$r = \sqrt{x^2 + y^2}$$

is the distance from the vortex axis to the liquid particle under consideration.
Integrating gives

$$p = \frac{\rho \omega_в^{\ 2}}{2} r^2 + c\,, \tag{9}$$

where c is the constant of integration.

The quantity c can be determined provided that at the boundary of the vortex core, i. e., at $r=r_в$, the outside pressure is equal to the inside pressure.

Let us determine pressure outside the vortex. From Stocks' formula it is known that circulation $C_в$ of linear velocity υ of the liquid particle in the vortex is constant for the circle of any radius r:

$$C_в = C_{1в} = C_{2в} = \ldots = C_{nв} = 2\pi R\upsilon = const\,. \tag{10}$$

Vortex core velocity circulation is determined by the relation in polar coordinates (see Fig. 2):

$$\begin{gathered} Cв = \oint \upsilon_r dr + \oint \upsilon_\tau r \cdot d\alpha = \\ = 0 + \int_0^{2\pi} \omega_в r \cdot r d\alpha = \omega_в r^2 \int_0^{2\pi} d\alpha = 2\pi\omega_в r^2, \end{gathered} \tag{11}$$

where r are distances from the vortex centre to the considered liquid particle in a given point of space; υ_r, υ_τ are normal and tangential components of velocity, respectively υ; α is the angle of rotation of radius r.

Let us consider the nature of distribution of velocity and excess statistical pressure in the vortex field restricted by the circle of radius r.

From (11) follows

$$\upsilon = \frac{C_в}{2\pi r}. \tag{12}$$

Eq. (12) shows that velocity in flat vortex field is in inverse proportion to the distance to the vortex axis.

It should be noted that pressure distribution in the vortex field cannot be determined by the Bernoulli equation because it is applicable only to the points of the same current line.

Therefore, this problem should be solved with the use of Euler integral.

Locating the axis OZ perpendicularly to the drawing plane, we obtain:

$$p+\frac{\rho\upsilon^2}{2}=p+\frac{\rho C_в{}^2}{8\pi^2 r^2}=const, \quad (13)$$

where P is the excess statistical pressure.

Assuming that the pressure at infinity is equal to the atmospheric pressure P_0, and velocity $\upsilon_0=0$, we obtain from Eq. (12) for each flow point:

$$p-p_0=-\frac{\rho C_в{}^2}{8\pi^2 r^2}. \quad (14)$$

From Eq. (14) it follows that pressure in the vortex field decreases inversely as the square of the distance from the considered liquid particle to the vortex axis (see Fig. 2). Taking into account that angular velocity in the vortex field $\omega_в=const$, and current lines are circles, that is, the vortex core rotates according to the laws of a solid body. In the case velocity in each point of vortex core is determined by the formula

$$\upsilon_\tau=\omega_в r. \quad (15)$$

From Eq. (15) it follows that velocity of liquid particles in the vortex core is linearly varying. It is obviously that at small velocities the liquid is incompressible and does not contain hollow spaces, that is, the condition of uniformity and motion continuity is met.

As a result, the angular velocity of vortex core rotation $\omega_в$ at $r=r_в$ is equal to

$$\omega_в=\frac{C_в}{2\pi r_в{}^2}. \quad (16)$$

By equating pressure P in formulae (9) and (14), we obtain constant of integration c:

$$\frac{\rho\omega_в{}^2}{2}r^2+c=p_0-\frac{\rho C_в{}^2}{8\pi^2 r^2}, \quad (17)$$

whence

$$c=p_0-\frac{\rho C_в{}^2}{8\pi^2 r^2}-\frac{\rho\omega_в{}^2}{2}r_в{}^2. \quad (18)$$

Considering that

$$c_в=2\pi\omega_в r_в{}^2,$$

we obtain

$$c = p_0 - \rho\omega_в^2 r_в^2 . \tag{19}$$

By substituting (19) into (9), we obtain

$$p - p_0 = \frac{\rho\omega_в^2}{2}\left(r^2 - 2r_в^2\right). \tag{20}$$

From Eq. (9) it follows that distribution of the excess pressure P in the vortex core varies according to the parabolic law. From Eq. (20) follows that value of $P - P_0$ is negative and decreases at $r \to 0$. In the case value of

$$\Delta p_{изб} = -\rho\omega_в^2 r_в^2$$

is the value of the excess statistical pressure on the vortex axis.

This value of pressure "collapsed" the liquid and forms vortex body.

Summary

1. The motion of heat carrier in the heating system as two-component mix consisting of the dropping liquid and solid particles of contaminants (rust and scale) is considered for the first time. Based on the solution of Stocks and Bernoulli equations, pressure and velocity fields of the components are determined.

2. Mathematical modeling of the force action of the heat carrier made it possible to consider the vortex core as a solid body acting on recording element of the flowmeter, which provides for more lower sensitivity level, for example, of the counter-flowmeter of the heating system.

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