BUILDING MECHANICS

UDC 624.04:624.131

Voronezh State University of Architecture and Civil Engineering

D. Sc. in Engineering, Prof. of Dept. of Structural Mechanics D. M. Shapiro

Ph. D. in Engineering, Senior Lecturer of Dept.

of Structural Mechanics A. V. Agarkov

Ph. D. in Engineering, Senior Lecturer

of Structural Mechanics Department N. N. Melnichuk

Ph.D. of Dept. of Structural Mechanics Chan Tkhi Tkhui Van

Russia, Voronezh, tel.: +7(4732)71-52-30;

e-mail: agarkov3@yandex.ru; thuyvan@mail.ru

D. M. Shapiro, A. V. Agarkov, N. N. Melnichuk, Chan Tkhi Tkhui Van

THE NON-LINEAR METHODS OF ANALYSIS IN MODERN DESIGNING (BY THE EXAMPLE OF GEOTECHNICS FACILITIES AND BRIDGES)

Problem statement. Non-linear solutions are widely used within the framework of solution of the problems of development of the areas which were earlier unsuitable for construction (wetlands, slope areas; bases consisting of weak overwet soils).

Results and conclusions. Design models and examples of practical implementation of two groups of non-linear analysis in design of construction objects are considered. These are spatial deformation analysis of reinforced concrete decks of road bridges and elastoplastic numerical analysis of the bases, soil structures and structures interacting with soil structures.

Keywords: non-linear method of analysis, designing, bridge span, geotechnical facilities.

Introduction. In modern building design, non-linear methods of analysis on the mathematical basis become widely used. This is connected with increasing number of

 ISSN 2075-0811

the objects which can be designed properly only with the use of rigorous theory solutions. Non-linear solutions are widely used within the framework of solution of the problems of development of the areas which were earlier unsuitable for construction (wetlands, slope areas; bases consisting of weak overwet soils). Non-linear (elastoplastic) methods of analysis are of great utility because of their availability related to the progress in computer and computational technique.

The important distinction of the problems of rigorous theory is their statement as deformation problems. Deformation problems are related to the description of development of stress-strain state in accordance with the sequence of loads up to the exhaustion of bearing capacity.

Unlike simplified methods of analysis, this method permits one:

- not to use restriction on acting loads to provide the correctness of the technique for determining design stresses;
- not to use conditional coefficients which indirectly take into account physical nonlinearity of material or increase in bearing capacity at the cost of certain (limited) development of plastic domains;
- to obtain physically feasible stress state at all points. The stress state can be prelimit or limit one with boundaries of appropriate subdomains which can be calculated.

In present paper, two groups of solutions of nonlinear problems related to the design of bridges and geotechnical objects are considered.

1. The nonlinear analyses of reinforced concrete bridge decks. Beam-slab systems (Fig. 1a) are the most commonly encountered type of reinforced concrete bridge decks of length from 9 to 33 m. At present, such structures accounts for more than 90 % of all structures of operated decks of road bridges. Operated decks and decks which are built to standard designs developed by Союздорпроект (they have been developed since 1957 up to the present) are constructions composed of 5 types of beams (Fig. 1b-g):

- tee beams of height 80—108 cm with standard reinforcement of length 11.36—16.76 and 12—18 m without diaphragms (standard design «выпуск 56д», types 3.503-14, 3.503.1-73);
- tee beams of height 120—123 cm of length 12—24, 28 m without diaphragms and beams of length 33 m, depth of beams 150—153, 170—173 cm with prestressed reinforcement (standard designs of types 3.503-12, 3.503.1-81 and their more recent versions developed after 1990);
- tee beams of height 75 cm of length 9—18 m without diaphragms with prestressed reinforcement (designs dated by 2001—2003);
- tee beams of height 80—105 cm with diaphragms and standard reinforcement, length 8.66—16.76 m (standard design «выпуск 56»);

– tee beams of height 120 cm without diaphragms, length 22.16 cm with pre-stressed reinforcement («выпуск 122-62»).

Standardized design loads on bridge structure increase 20—30 % (Н-30, НК-80 in 1962, А11 instead of Н-30 in 1984, А14, НК-102.8 in 2007), which is caused by real increase in intensity of movement, weights of cars. In this connection, bridge decks are operated or will be operated in the period of action of more heavy loads than that on which they are designed.

The method of analysis applied in design and determination of bearing capacity of bridge decks consists of two parts:

1) spatial linear (elastic) analysis of ribbed slab system to determine design bending moments and shearing forces in beams under the action of permanent and temporary loads;

2) comparisons of design forces in sections of most loaded or damaged beam with corresponding limiting values.

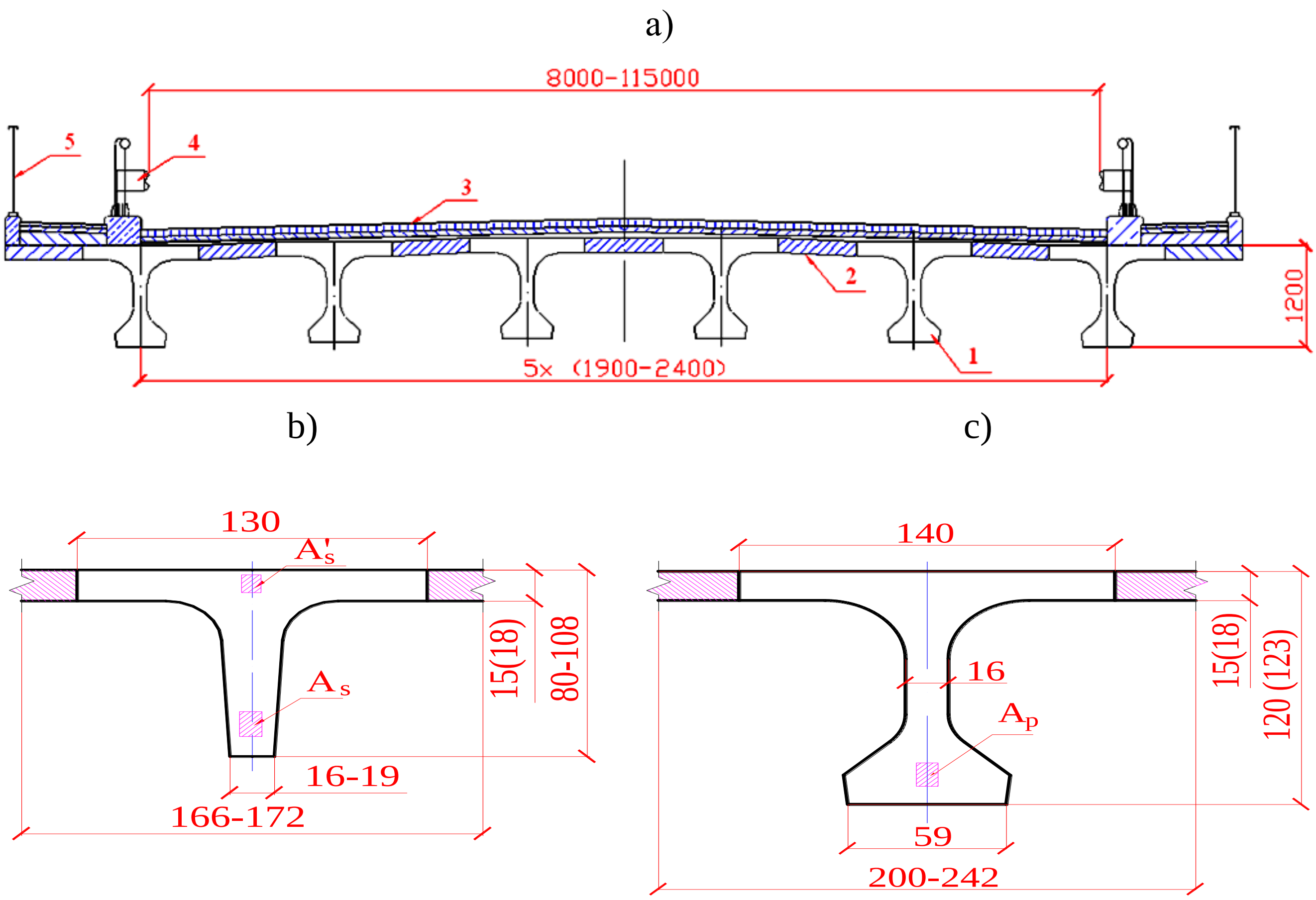


Fig. 1. Cross-sections of the deck (a), of tee beams with standard reinforcement (b, e), of I-beam (c, f), tee beams (d) with high-strength reinforcement:
1 — I-beam with prestressed reinforcement; 2 — longitudinal grouting joints;
3 — multilayer road surface; 4 — metal barrier fences; 5 — ferroconcrete fences with metal railing

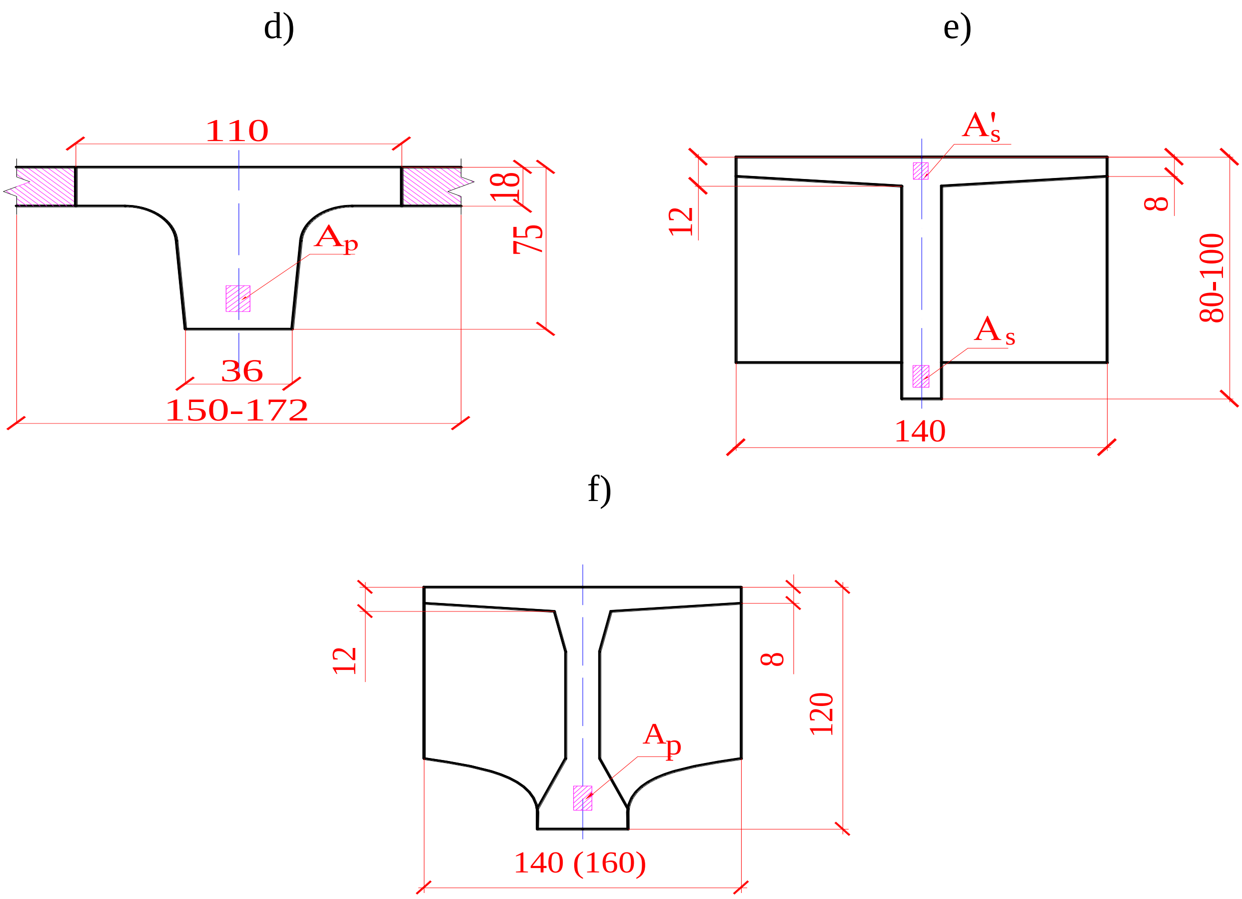


Fig. 1 (end). Cross-sections of the deck (a), of tee beams with standard reinforcement (b, e), of I-beam (c, f), tee beams (d) with high-strength reinforcement: 1, 2, 3, 4, 5 — the same (see above)

Such approach is technologically justified in design of new bridge decks, particularly in standard design. However, the attainment of limiting forces in one of the most loaded beam does not result in exhaustion of bearing capacity of the whole deck. This one of the reason of retention of operation suitability of the old decks under loads which exceed design ones.

To use bearing capacity of the decks, criterion of limiting state with respect for the «strength with consideration for stressed state before destruction» (СП 52-101-2003, СП 52-102-2004) can be applied. For this purpose, department of building mechanics of Voronezh State University of Architecture and Civil Engineering has developed the method of analysis [1] which enables one to substantiate higher bearing capacity of beam ribbed slab decks. The method consists of two analysis procedures:

– deformation nonlinear analysis which describe the development of stress-strain state of beams with standard or high-strength reinforcement;
– spatial analysis of the decks by means of finite element method with step-wise application of temporary load.

The theoretical footing for deformation nonlinear analysis of reinforced concrete beam is kinematic conditions of plane-sections hypothesis, assumption of unimpeded deformation of concrete subjected to tension; three-linear diagram of state of concrete subjected to compression, bilinear (three-linear) diagram of state of standard (or high-strength) reinforcement subjected to tension in accordance with Fig. 2a, b, c, on which all designated parameters of relative deformations and stresses are standardized parameters (СП 52-101-2003, СП 52-102-2004): $\sigma_{b1} = 0{,}6R_b$, $\sigma_{b0} = R_b$, $\varepsilon_{b1} = 0{,}6\sigma_{b1} / E_b$, $\varepsilon_{b0} = 0{,}002$, $\varepsilon_{b2} = 0{,}0035$; $\sigma_{s0} = R_s$, $\varepsilon_{s0} = \sigma_{s0} / E_s$, $\varepsilon_{s2} = 0{,}025$; $\sigma_{p1} = 0{,}9R_p$, $\sigma_{p0} = R_p$, $\sigma_{p2} = 1{,}1R_p$, $\varepsilon_{p1} = \sigma_{p1} / E_p$, $\varepsilon_{p0} = R_p / E_p + 0{,}002$, $\varepsilon_{p0} = 1{,}1R_p / E_p + 0{,}004$, where E_b, R_b are initial modulus of deformation and design strength on concrete compression, R_s, R_p, E_s, E_p are design strengths and moduli of deformation of standard and prestressed reinforcement.

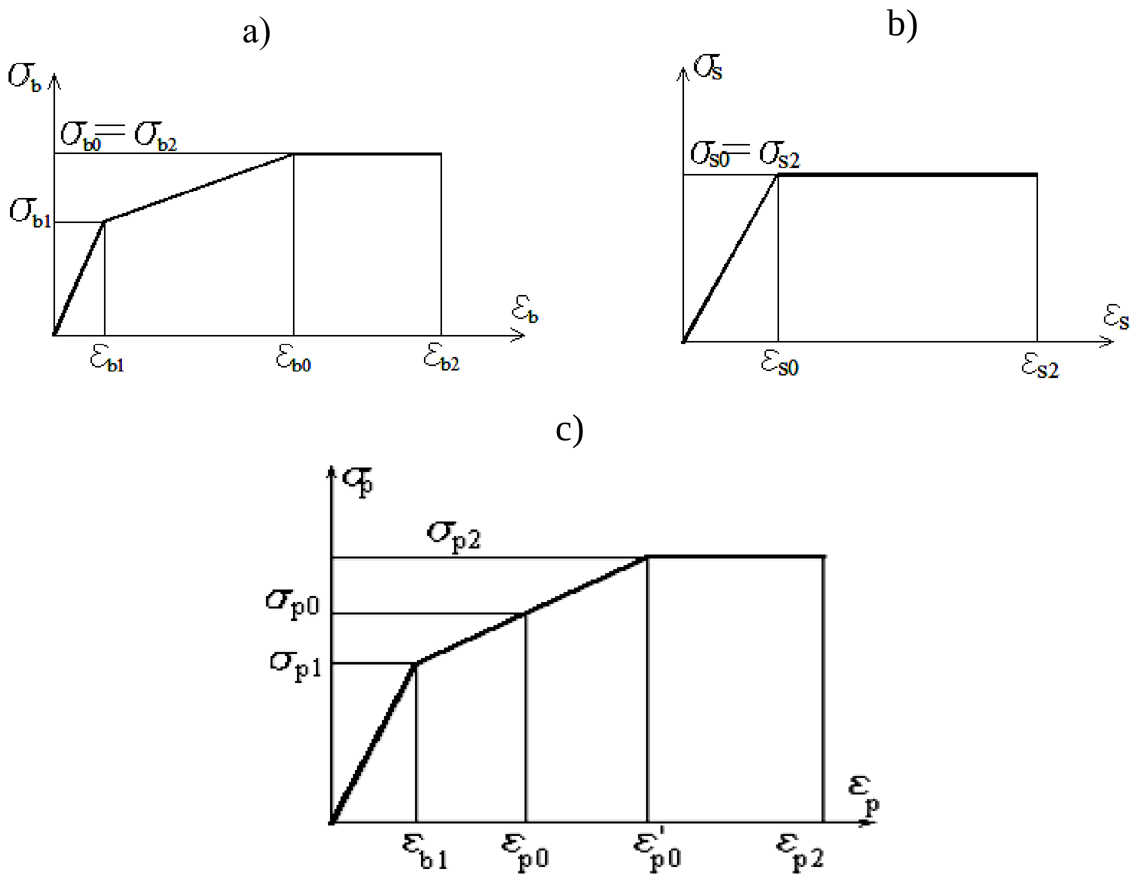


Fig. 2. Diagram of state:
a — of compressed concrete, b — of rod reinforcement, c — of high-strength reinforcement

The problem on distribution of stresses and strains in the section of bendable reinforced concrete is solved as inverse problem. Relative strain of concrete $\varepsilon_{b,\ z=x}$ of compressed side of the section is taken as independent variable (Fig. 3a, b), where x is the height of compressive zone of the concrete, z is the coordinate reckoned from the neutral axis. In accordance with plane-sections hypothesis used in analysis, distribution of relative strains over the height in concrete ε_b, in tension and compression reinforcement ε_s, ε_{sc}, ε_p is taken to be linear. Stresses σ_b, σ_s, σ_{sc}, σ_p, corresponding to the strains referred above are determined with the use of diagrams (Fig. 2). Described concepts in combination with equilibrium conditions of normal forces allow one to perform design description of stress-strain state of the section at all stages of loading.

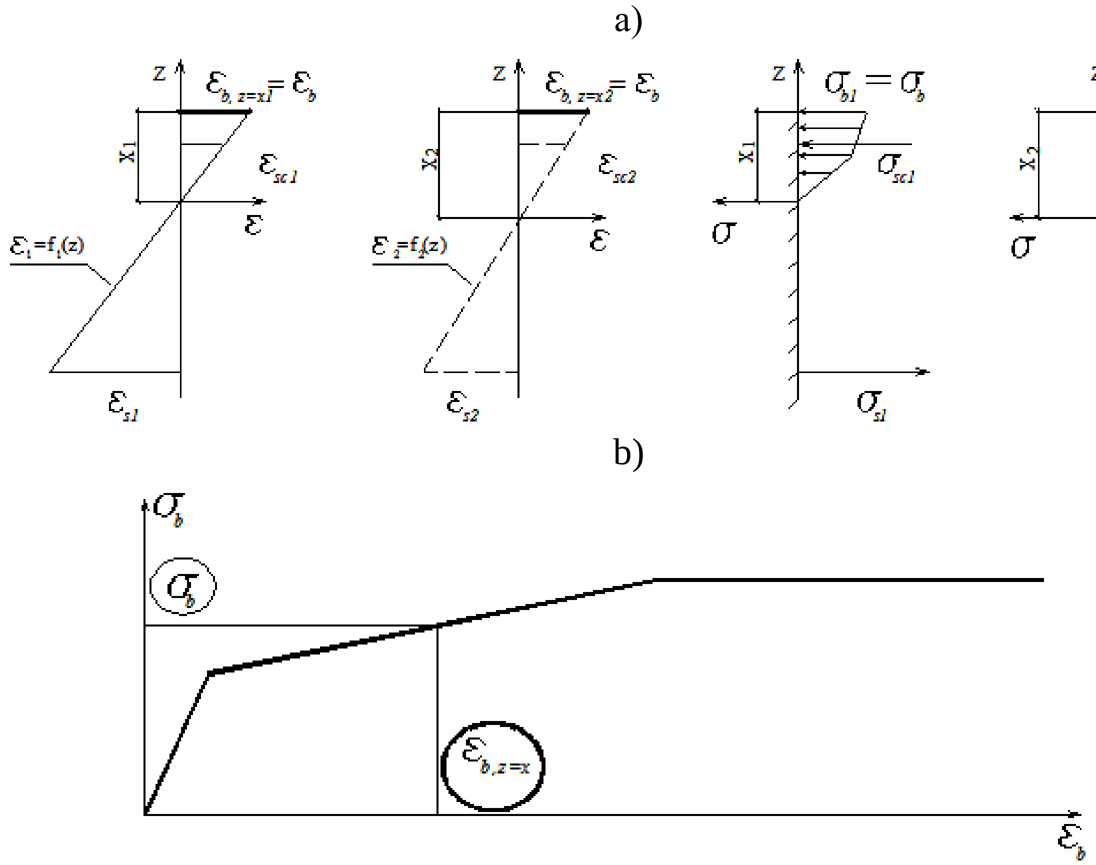


Fig. 3. The schemes to the deformation analysis of reinforced concrete beam:
a — diagrams of distribution of stresses and strains over the height of beam at two values of the height of compression zone x; b — diagram of state of the concrete subjected to compression, relationship of σ_b and $\varepsilon_{b,\,z=x}$

Based on results obtained we construct the diagram (or prepare the table) where each value of $\varepsilon_{b,\,z=x}$ corresponds to the unique values of the moment M of outside (inside) forces and height x of compression zone of the section. Interrelated $\varepsilon_{b,\,z=x}$, M, x allow one to obtain distribution of relative strains and stresses over the height of the section, curvature of the bend $1/\rho = (\varepsilon_{b,\,z=x} - \varepsilon_s)/h_0$, current flexural stiffness $B = M/(1/\rho) = f\,(M)$. The last parameter ($M\rho = M/(1/\rho)$) is used to perform spatial analysis aimed at stiffness distribution over the height of the beam at stepwise increase in temporary load.

Spatial analysis of ribbed slab decks is performed with the use of modern general-purpose certified program complexes implementing finite element method (*LIRA*, *SCAD*, etc.). To implement the analysis, we use structural model in the form of ribbed slab (Fig. 4) [1].

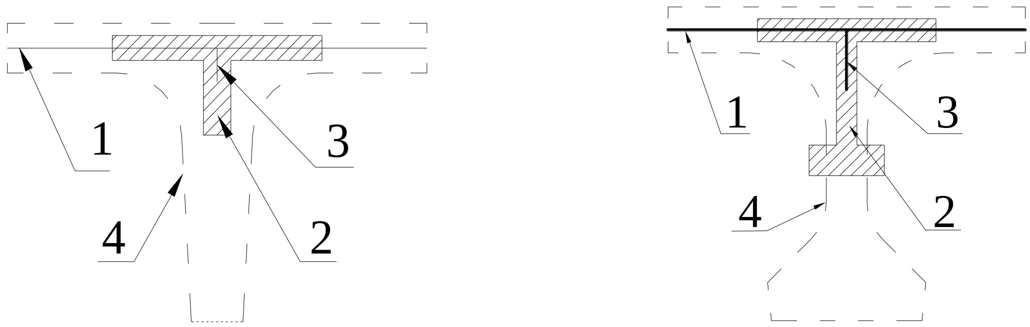


Fig. 4. Structural models of beams of decks of ribbed slab system:
1 — finite element slabs, 2 — rod finite elements modelling the lengths of the beam,
3 — rigid fixing, 4 — outline of deck beam section

The model reflects the stiffnes of the beams variable over the length and consists of the following finite elements:

- right-angled slab finite elements with 3 degrees of freedom in the joint; these elements model the slab;
- rod finite elements with 6 degrees of freedom in the joint; these elements model the lengths of the beam.

Spatial nonlinear deformation analysis of beam decks involves the formation of the structural model and representation in the form feasible for analysis with the use of finite elements method, determination and description of constant and temporary loads; deformation nonlinear analysis of the sections of all beams, determination (construction of diagrams) of moments M, parameters x, $M\rho$ in relation to $\varepsilon_{b,\, z\, =\, x}$; spatial finite element analysis with stepwise application of temporary load. The shares of temporary loads АК = А1 and 0.05÷0.10 of load НК-102.8 (Н14) are taken as the stages of temporary loads at the final stage of analysis.

Beam deflections versus temporary load АК or НК curve is constructed with the use of the results of the analysis. Plastic hinges generated in beam cross-sections are located. The decision on attainment of limiting state is made with respect for the following criteria: the attainment of the limit deflection in the most loaded beam (1), the attainment of the limit average deflection of the deck (2); the attainment of the limit deflection gradient in the most loaded beam (3); the number of the plastic hinges generated in the deck.

2. Example. Fig. 5a-d show the cross-section and the results of analysis of the deck of length 12 m consisting of prestressed T-beams of height 75 cm. Design characteristics of materials are as follows: modulus of concrete deformation (of grade B35) E_b = 31000 MPa, modulus of reinforcement deformation (of class B) E_p = 177000 MPa, design strength of concrete R_b = 17.5 MPa, design strength of reinforcement R_p = 1055 MPa. Sectional area of working reinforcement in the middle of the deck of the beam A_p = 18.84 cm^2. Design loads are taken from the project A11, НК-80. Load АК is applied in the most unsuitable position with displacement to the left edge of the roadway (including safety lines).

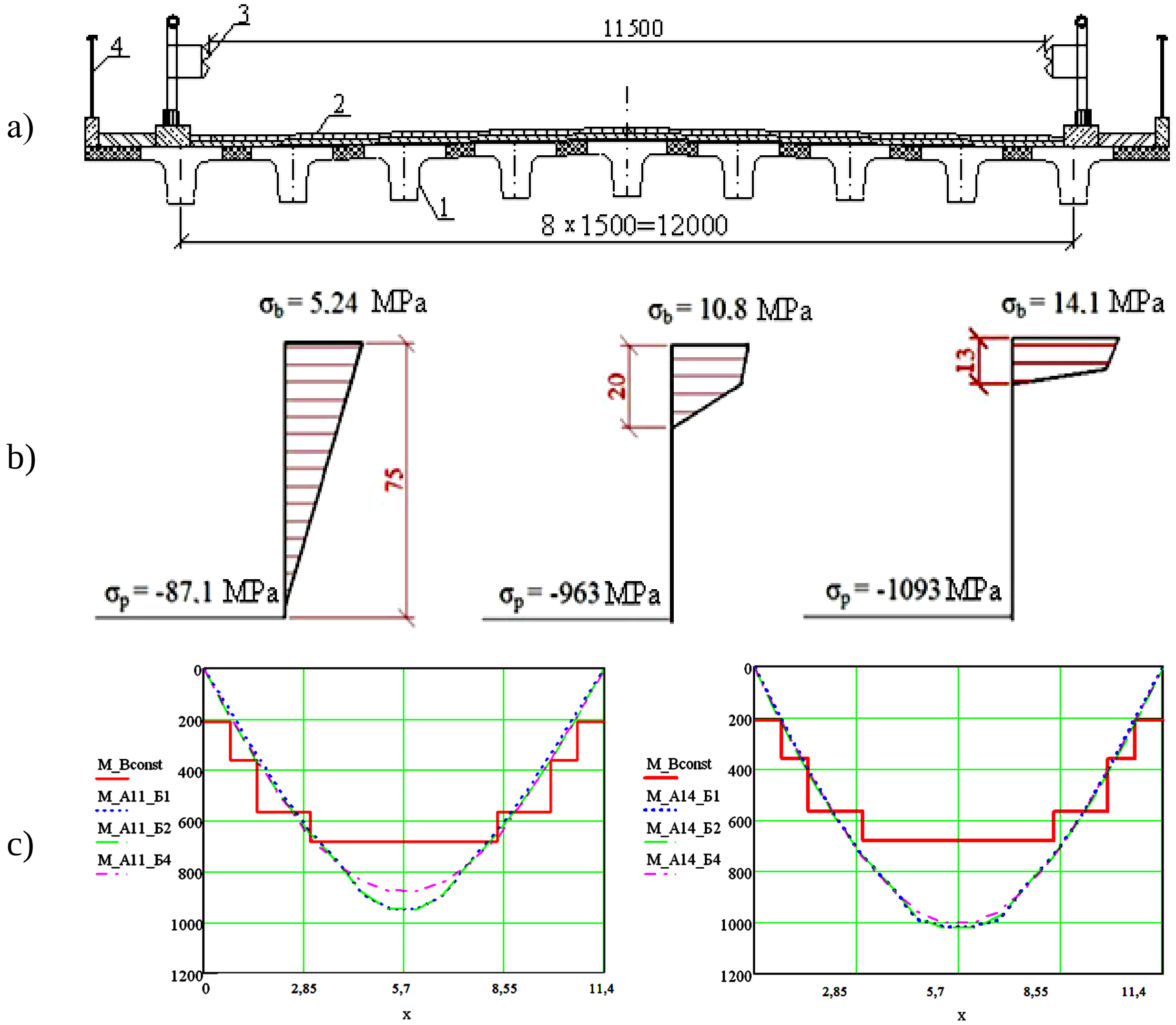
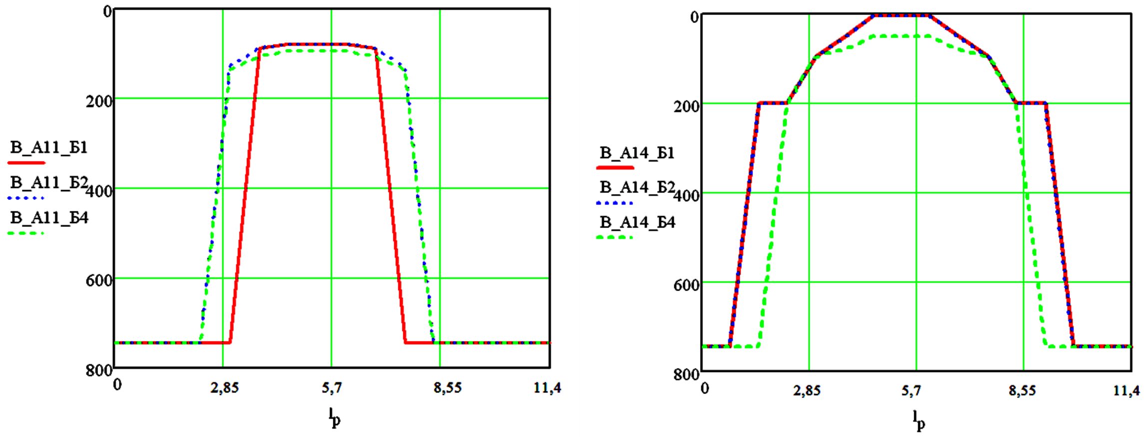


Fig. 5. Cross-section and results of spatial deformation analysis of the deck of length 12 m: a — cross-section of the deck; b — stress diagram σ_b, σ_p in the average section of the beam at values of external moment, respectively:
M = 683; 913; 1062 kNm ($\varepsilon_{b,\,z=x}$ = 1,7×10^{-4}; 4×10^{-4}; 12×10^{-4});
c — diagrams of moments in beams Б-1, Б-2, Б-4 at loads A11 and A14;
1 — T-beem with prestressed reinforcement, 2 — multilayer road pavement,
3 — metal barriers, 4 — metal railing

d)



e)

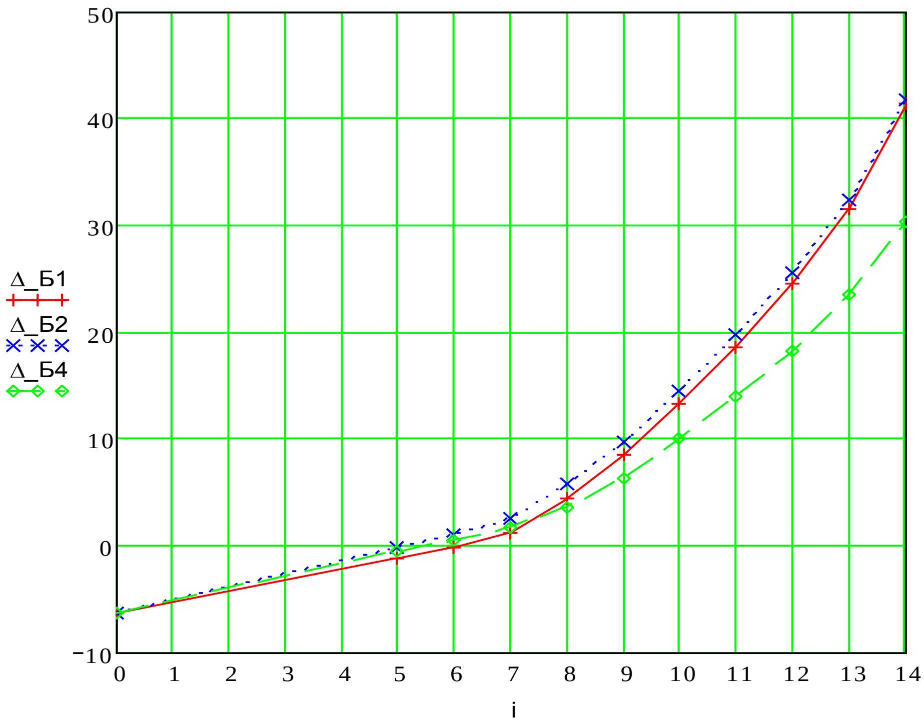


Fig. 5 (end). Cross-section and results of spatial deformation analysis of the deck of height 12 m:
d — diagram of distribution of stiffnesses over the height of beams Б-1, Б-2, Б-4 at loads A11 and A14;
e — diagram of dependence of beam deflection Б-1, Б-2, Б-4 on load AК at К from 0 to 14;
1, 2,3, 4 — the same (see above)

The results of analysis of the deck show that the first plastic hinges were formed in beams Б-1, Б-2 under the load A13. Under the load A14 deflections of the beams were as follows: Б-1 — 41.4 mm, Б-2 — 41.8 mm, Б-3 — 38.8 mm, Б-4 — 30.3 mm. The average deflection of the beams of the deck is 18.7 mm. Load A13 is taken as limit load with respect for the strength with consideration for deformation state before destruction.

3. The elastoplastic analysis of geotechnical facilities. In geotechnics, preference is given to the structural models which are clear to the users and which require standard parameters of mechanical characteristics of soils. A shear dilatancy model of incremental plasticity theory and solution of the combined problem of theories of elasticity and plasticity for soils ([2, 3, 4]) satisfy these conditions. Fig. 6 shows the structural model of considered design model which illustrates the set and relations of applied equations.

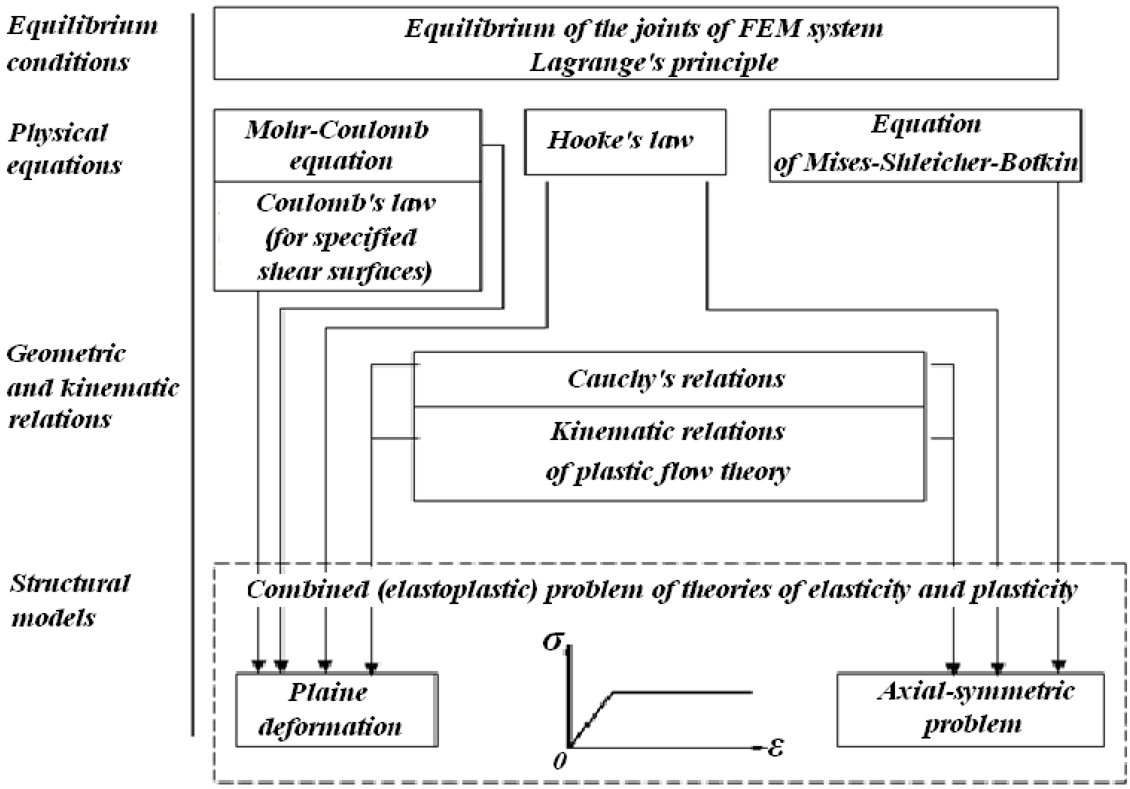


Fig. 6. Structural scheme of elastoplastic model of soil

The computer implementation of obtained solution and analysis technique developed on its basis is performed in programs *START* [3] (for computers of generation EC) and *УПРОС*-2000 [4] (for modern PC).

Both programs implement algorithms developed on the basis of equations described in [2] and contain plane and axisymmetric versions. Besides, the program “Foundation” was developed on the basis of the same algorithm in *Poltava National Technical University* named after Yuri Kondratyuk. The program contains plane version of solution of elastoplastic problem.

The solution under consideration and programs stated above were applied in design of various facilities in difficult engineering and geological conditions as well as for the solution of scientific and technical problems, for example:

– development of the method of design modeling of drilled pile indentation by stepwise increasing axis force which involves determination of bearing capacity, specification of limit state criteria [5];

- development of numerical methods for justification of additional bearing capacity (strengthening) of bases under shallow foundations after prolonged exposure to the tightening operation load [6];
- development of the technique to design bases in construction of new buildings in the area of existing site development [7].

4. Example. Drilled pile of 1.7 m in diameter and of 26.8 m in length was made in construction of pile foundation of the large bridge in 1992. In the course of well-drilling, three types of sand ($\phi = 28 \div 33°$) and four layers of dusty clay soils ($\phi = 19 \div 27°$, $c = 15 \div 54$ kPa) were passed through. The pile was embedded in the layer of dusty dense sand to a depth of 1.6 м (E=28 MPa, $\phi = 32°$). Bedding courses are stiff loam (depth = 1.2 m, $E = 17$ MPa, $\phi = 20°$, $c = 17$ kPa), semisolid clay ($E = 23$ MPa, $\phi = 18°$, $c = 40$ kPa). Specified weight of soils is assumed to be $\gamma = 19$ kN/m^3.

The pile was tested in design position. The test was brought up to the load 12.5 MN and was stopped because of exhaustion of anchor system capacity. The bearing capacity (limit resistance) of the pile was not attained.

In elastoplastic analysis modelling the test of drilled pile, finite element structural model with radius 10.2 m, height 37 m was accepted. The analysis was brought up to the load 20 MN.

Data on Fig. 7a show good agreement of « settlement-loading» relations by the results of elastoplastic analysis and static test. Design curve obtained is smooth up to the end of design. Fig. 7b shows the zones of limit stress state at three values of load P = 15.0; 17.0 and 18.0 MN. At indentation force P = 17,5 MN, the zone of limit stress state of the soil under the drilled pile foot cuts the axis of symmetry of design field and attains the height of 0.9 m (0.5 of pile diameter). Design soil settlement is 80 mm (0.047 of pile diameter). Elastic component of settlement is 30 mm (37.5 percent of total settlement) (line 3 on Fig. 7a).

a)

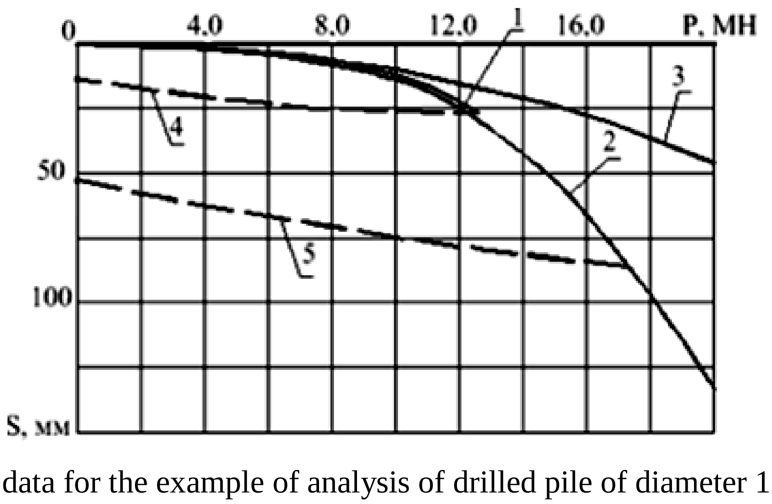


Fig. 7. Graphic data for the example of analysis of drilled pile of diameter 1.7 m:
a — $s = f(P)$ diagrams: 1 — by the results of static test,
2 — by the results of elastoplastic analysis, 3 — elastic solution, 4, 5 — at pile unloading;

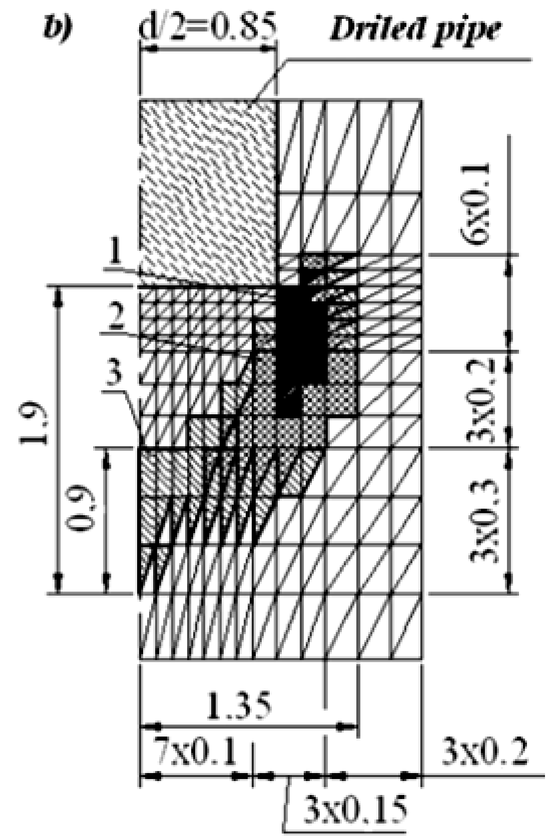


Fig. 7 (end). Graphic data for the example of analysis of drilled pile of diameter 1.7 m:
b — fields of limit stress state: 1, 2, 3 — at loads *P* equal to 15.0, 17.0 and 18.0 MN, respectively

Conclusions. Considered analysis was performed in the course of the study which allowed us to justify criteria of limit resistance of drilled piles by the results of design modeling.

Nonlinear methods of analysis are instruments of modern applied studies and building design. These methods cannot substitute simplified methods of analysis, however, they can supplement simplified methods.

References

1. **Шапиро, Д. М.** Пространственный нелинейный деформационный расчёт пролётных строений автодорожных мостов / Д. М. Шапиро, А. В. Агарков, Чан Тхи Тхюи Ван // Научный вестник ВГАСУ. Строительство и архитектура. — 2008. — № 2. — С. 29—37. = **Shapiro, D. M.**, Agarkov, A. V., and Chan Thi Thui Van. Spatial non-linear deformation analysis of road bridge decks. *Scientific Herald of Voronezh State University of Architecture and Civil Engineering. Construction and Architecture (in Russian)*. 2008. N 2, pp. 29—37.

2. **Шапиро, Д. М.** Практический метод расчета оснований и грунтовых сооружений / Д. М. Шапиро // Основания, фундаменты и механика грунтов. — 1985. — № 5. — С. 19—21. = **Shapiro, D. M.** Practical method for analysis of bases and soil structures. *Foundation engineering and soil mechanics (in Russian).* 1985. N 5, pp. 19—21.

3. **Шапиро, Д. М.** Внедрение нелинейного метода расчета при проектировании оснований и грунтовых сооружений / Д. М. Шапиро, Г. В. Полторак // Межвуз. сб. науч. трудов. — Йошкар-Ола: Изд-во Марийского политехн. ин-та, 1990. — С. 24—273. = **Shapiro, D. M.**, and Poltorak, G. V. Introduction of a non-linear method of analysis in bases and soil structures design. *Coll. papers.* Yoshkar-Ola, 1990, pp. 24—27.

4. **Гузеев, Р. Н.** Упругопластический расчет МКЭ при проектировании и исследовании геотехнических объектов / Р. Н. Гузеев // Современные методы статического и динамического расчета сооружений и конструкций: сб. науч. тр. — Воронеж, 2000. — С. 84—91. = **Guzeev R. N.** Elastoplastic analysis with the use of FEM in designing and study of geotechnical facilities. *Modern methods of statistical and dynamic analysis of facilities and structures.* Voronezh, 2000, pp. 84—91.

5. **Шапиро, Д. М.** Расчётное моделирование нагружения буронабивных свай осевой силой / Д. М. Шапиро, Н. Н. Мельничук // Проблемы механики грунтов и фундаментостроения в сложных грунтовых условиях: тр. междунар. науч.-техн. конф.: в 2 т. Т. 1. — Уфа: БашНИИстрой [и др.], 2006. — С. 155 — 164. = **Shapiro D. M.,** Melnichuk N. N. Design modeling of bored piles loading by axial force. *Problems of soil mechanics and foundation engineering in difficult soil conditions. Proceeding of International Scientific and technical conference.* In 2 vol. Vol. 1. Ufa, 2006, pp. 155—164.

6. **Шапиро, Д. М.** Упругопластический анализ несущей способности оснований реконструируемых объектов методом конечных элементов / Д. М. Шапиро, Н. Н. Мельничук // Основания, фундаменты и механика грунтов. — 2007. — № 2. — С. 18—21. = **Shapiro, D. M.,** Melnichuk, N. N. Elastoplastic analysis of bearing capacity of the bases of reconstructed objects with the use of FEM. *Foundation engineering and soil mechanics (in Russian)*, 2007, N 2, pp. 18—21.

7. **Борт, О. В.** Моделювання геотехнічних процессів при будівництві і реконструкціі у стиснених умовах забудови / О. В. Борт, О. О. Голов, М. Л. Зоценко // Ресурсоекономні матеріали, конструкціі, будівлі та споруди: зб. наук. пр. — Рівне, 2003. — Вип. 9.— С. 424—430. = **Bort, O. V.**, Golov, O. O., and Zotsenko, M. L. Modeling of geotechnical processes in construction and reconstruction in difficult conditions. *Resource-saving materials, structures, buildings, and facilities: coll. papers.* Rivne, 2003, Iss. 9, pp. 424—430.