BUILDING MECHANICS

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METHOD OF CONSTRUCTION OF THE LIMIT BEARING CAPACITY OF IDEALLY PLASTIC COMPOSITE SECTIONS OF RODS IN A COMPLEX STRESS STATE

Statement of the problem. The purpose was to create a numerical method for designing fluid hypersurface for an arbitrary rigid-rod section consisting of any number of materials in a complex stress state. Wy identified an algorithm of solving the problem, based on the method of limit equilibrium and extreme principles of maximum load power and minimum dissipation rate. The above numerical method can be applied to arbitrary cross-sections. While implementing the method, a section is divided into areas for which equations of equilibrium and yield conditions are composed. Yield conditions can be arbitrary. In order to solve the problem the method of linear programming is used.

Results. Based on the presented method fluid hypersurfaces of the most common composite cross sections used in bridge construction were designed. For reinforced concrete sections hypersurfaces were designed with and without consideration of the concrete in tension.

Conclusions. The algorithm for designing the surface is quite universal and effective. Due to a small dimension of the problem no significant computing resources are needed to address it. This method can be applied to the analysis of structures for the first group of limit states.

Keywords: fluid hypersurface, rigid plastic rod, method of limit equilibrium, composite section.

Introduction

In order to determine limit load parameters [1—3] or to optimize a bar structure using the limit balance method for design sections, fluidity conditions need to be in place. As for a complex size of a section made up of several materials with different fluidity limits, determining limit efforts can be a daunting task commonly having no analytical solution [4].

1. Statement of the problem

The paper introduces and tests a numerical method of designing a hyper surface for random sections of bars for a complex state of stress. In the proposed method, a section area is split into the final subareas for each of which the fluidity conditions are in place based on the properties of the material of a particular section area. An assumption is made that in determining limit efforts for sections of thin-walled bars there are no stresses generated by a bimoment and flexural-bending moment according to the theory by V.Z. Vlasov. Further on, for a group of section areas, a balance equation matrix is designed. Based on the conditions of fluidity and balance equations, mathematical models are designed for limit balance of a section: a static and kinematic one.

The static model is as follows

$$\begin{cases} p_0 \to \max, \\ \Phi \cdot S \le N_0, \\ A \cdot S = p_0 \cdot \eta, \end{cases} \tag{1}$$

where p_0 is a limit load parameter; S is a vector of actual efforts during plastic failure; N_0 is a vector of limit efforts of final subareas; A is a balance equation matrix of final subareas; η is a vector of load distribution; $\hat{O}$ is a matrix of fluidity conditions.

The purpose function of linear programming (1) is a maximum power of load.

As a result of the linear programming problem (1), a limit load parameter p_0 and effort distribution S is universally determined over the final subareas of the section. In order to determine the boundaries of possible limit states of the section, it is necessary that using a vector η all possible combinations of the examined efforts are detailed. As a result of joining the points in the graph obtained while solving the linear programming problem (1) for each effort combination, we get a boundary of the area of possible limit states of the section. The kinetic model is as follows

$$\begin{cases} \left\{\dot{\lambda}^{+} + \dot{\lambda}^{-}\right\}^{T} \cdot S \to \min, \\ \Phi^{T} \cdot \left\{\dot{\lambda}^{+} + \dot{\lambda}^{-}\right\} - A^{T} \cdot \dot{U} = 0, \\ \dot{U}^{T} \cdot \eta = 1, \\ \dot{\lambda}^{+} \ge 0, \dot{\lambda}^{-} \ge 0, \end{cases} \tag{2}$$

where $\dot{\lambda}^{+}, \dot{\lambda}^{-}$ are vectors of positive and negative deformation rates; $\dot{U}$ is a vector of deformation displacement rates. The solution of the kinetic model (2) using linear programming allows two questions to be answered. The first one is about how plastic hinges are formed: the greater the deformation rate is, the faster a plastic hinges forms in the final subarea that corresponds to a particular element. The second one is about how compression and tension zones are divided following the formation of a plastic hinge in the section. Whether the final subarea belongs to any of the zones depends on the sign of the corresponding element of the deformation rate vector. For a more detailed description of the kinetic model see the paper [5].

The balance equation for the section (Fig. 1) is as follows

$$\begin{gathered}
N = -\int_A \sigma_x dA \approx -\sum_{i=1}^{k} N_i, \\
Q_y = -\int_A \tau_{xy} dA \approx -\sum_{i=1}^{k} q_{xyi}, \\
Q_z = -\int_A \tau_{xz} dA \approx -\sum_{i=1}^{k} q_{xzi}, \\
M_x = \int_A (\tau_{xy} \cdot z - \tau_{xz} \cdot y) dA \approx \sum_{i=1}^{k} (q_{xyi} \cdot z_i - q_{xzi} \cdot y_i), \\
M_y = -\int_A (\sigma_x \cdot z) dA \approx -\sum_{i=1}^{k} (N_i \cdot z_i), \\
M_z = \int_A (\sigma_x \cdot y) dA \approx \sum_{i=1}^{k} (N_i \cdot y_i),
\end{gathered} \qquad (3)$$

where A is the area of the section; dA is the area of the section segment; k is a number of segments in the section; σ_x is a normal stress; τ_{xy}, τ_{xz} are stress tangents; x, y are coordinates.

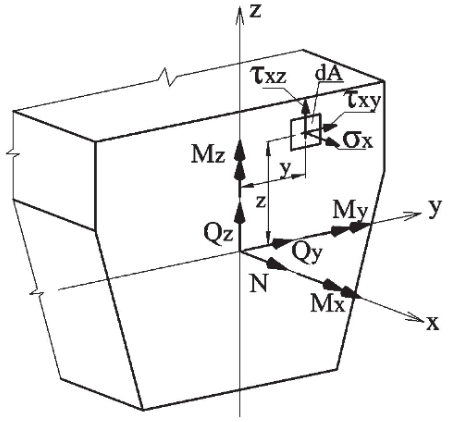


Fig. 1. Section of the bar

The structure of the balance matrix for a section where 6 effort component is takes the following form

$$A=\begin{bmatrix} -EE & 0 & 0 \\ 0 & -EE & 0 \\ 0 & 0 & -EE \\ 0 & HH & -BB \\ -HH & 0 & 0 \\ BB & 0 & 0 \end{bmatrix}, \tag{4}$$

where $EE=\{1\cdots1\}$ is a single vector; $HH=\{z_1\cdots z_k\}$ is a coordinate vector of the segments along the axis *z*; $BB=\{y_1\cdots y_k\}$ is a coordinate vector of the segments along the axis *y* (see Fig. 1).The dimensionality of the vectors *EE, HH, BB* equals the number of the segments in a section. In the balance matrix (4) the first column relates to the balance of normal forces along the axis *x*, the second column relates to the balance of tangent forces along the axis *y*, the third one corresponds to the balance of tangent forces along the axis *z*. The lines in the matrix (4) corresponds to the efforts in a section: the first line is tension-compression along the axis *x*, the second one is a displacement along the axis *y*, the third one is a displacement along the axis *z*, the fourth is torsion around the axis *x*, the fifth one is a curve along the axis *y,* the sixth is a curve along the axis *z*.

Depending on the strain state of the section, a matrix (4) undergoes transformations. During a curve in two planes accompanied by tension-compression a balance matrix is

$$A=\begin{bmatrix} -EE \\ -HH \\ BB \end{bmatrix}, \tag{5}$$

during torsion accompanied by tension-compression a balance matrix is

$$A=\begin{bmatrix} -EE & 0 & 0 \\ 0 & HH & -BB \end{bmatrix}. \tag{6}$$

The expression of a fluidity matrix is dependent on the acting forces in a section as well as the chosen fluidity conditions.

As a rule, the dimensionality of the linear programming problem is not high and the algorithm is sufficiently effective in the calculation plan.

2. Numerical study

In order to put the algorithm to test, let us consider most common longitudinal section employed in bridge construction. This kind of sections commonly has a complex shape and are composite ones.

1. Section of a ferroconcrete I-beam of 24 m according to a standard project «3.503.1-81. Manufacture 7-1. Span beams of 12, 15, 18, 21, 24 and 33 m in length, wholly transportable, post-tension». The geometric sizes and reinforcement of the section are presented in Fig. 2.

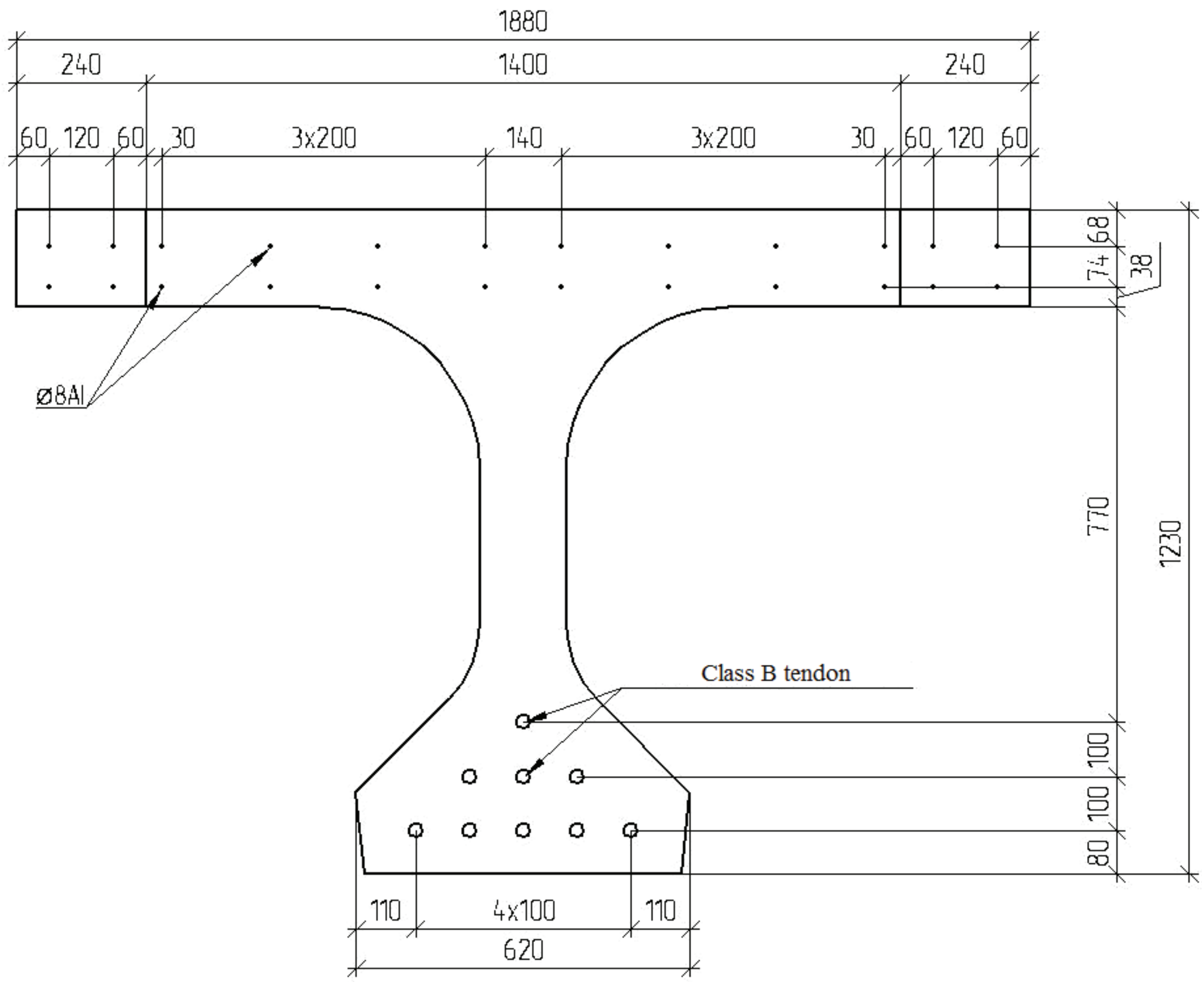


Fig. 2. Sizes and reinforcement
of a section of a ferroconcrete beam

The section is composed of two concrete types: a beam section made from concrete class B35; monolith segments from concrete B40. The division of a concrete section into segments is shown in Fig. 3.

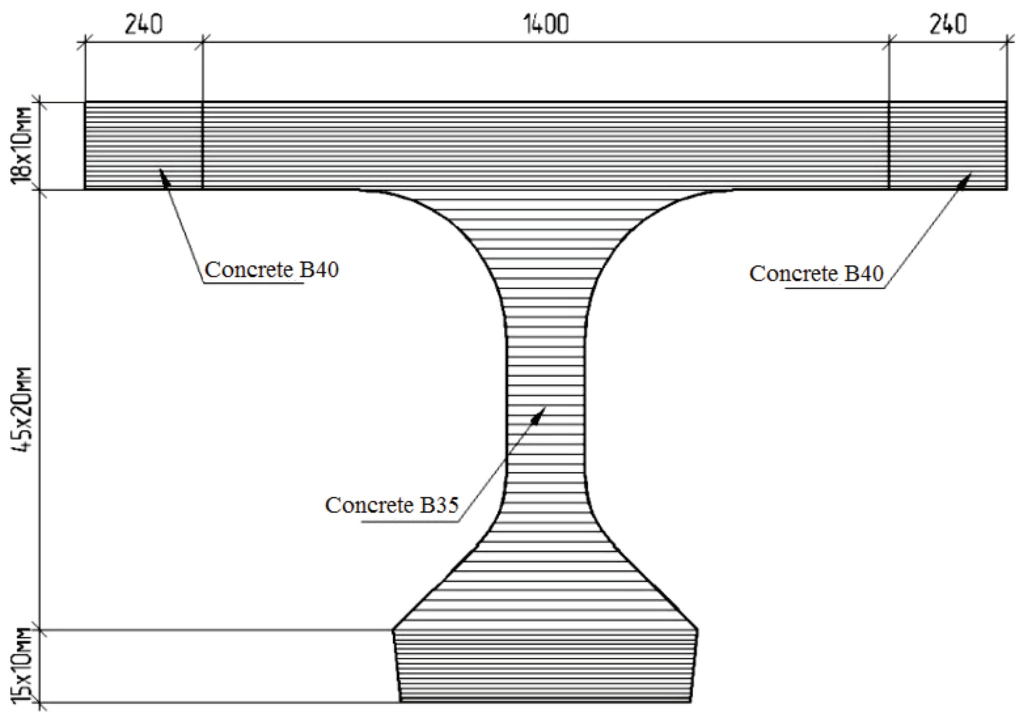
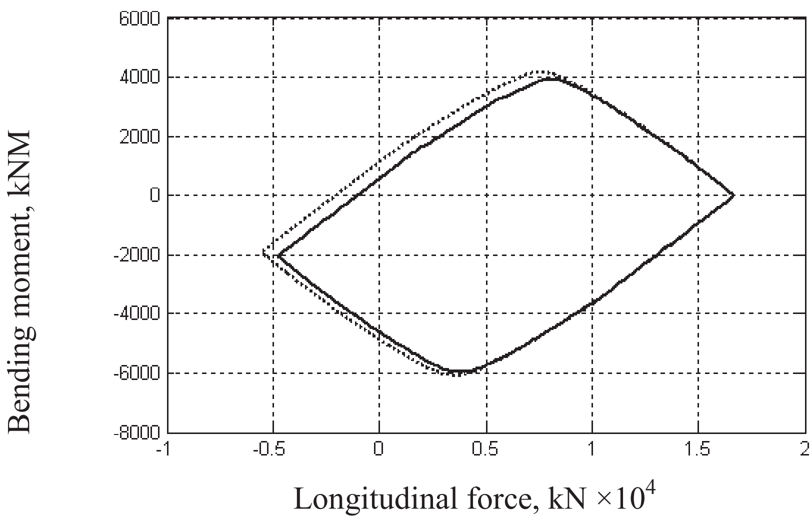


Fig. 3. Division of a concrete section into elements

This method allows for the tension resistance of concrete. Fig. 4 shows the graphs of bearing capacity of a section considering and not considering the tension resistance of concrete.



— Not considering tension operation of concrete

... Considering tension operation of concrete

Fig. 4. Hypersurfaces of the fluidity of a section of a ferroconcrete beam

2. Section of a ferroconcrete beam. The geometric sizes of the section and reinforcement are in Fig. 5. It is assumed that a part of a ferroconcrete beam of the section is composed of different types of concrete: a console part of the changeable section is made from concrete B35, the rest is from concrete B30. Fig. 6 shows the boundaries of possible limit states of the section considering the tension resistance of concrete.

3. Section of a box ferroconcrete beam. The geometric sizes of the section and division of the section into elements are in Fig. 7. A schematic of the location of a strained reinforcement and reinforcement following the reinforcement of the section is shown in Fig. 8.

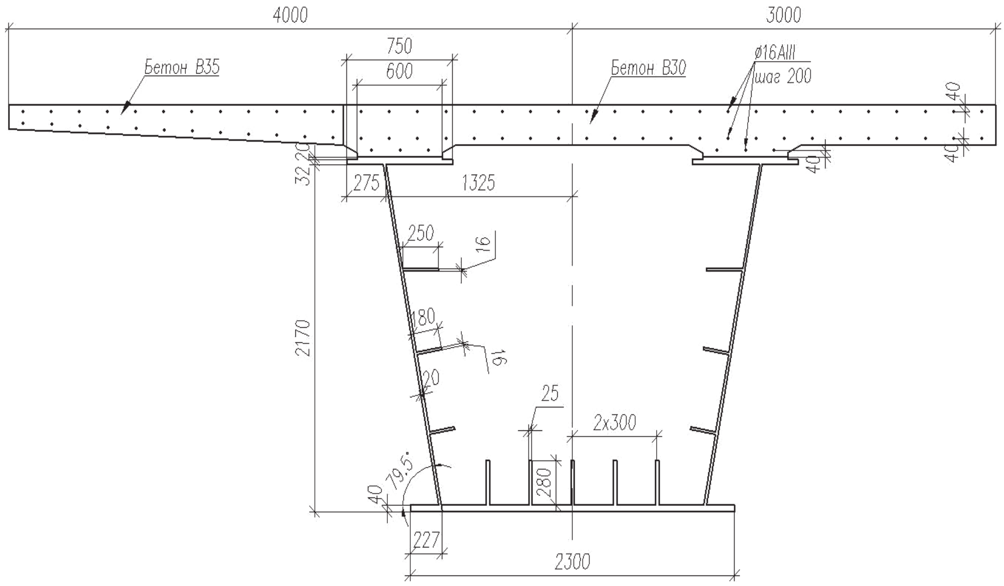
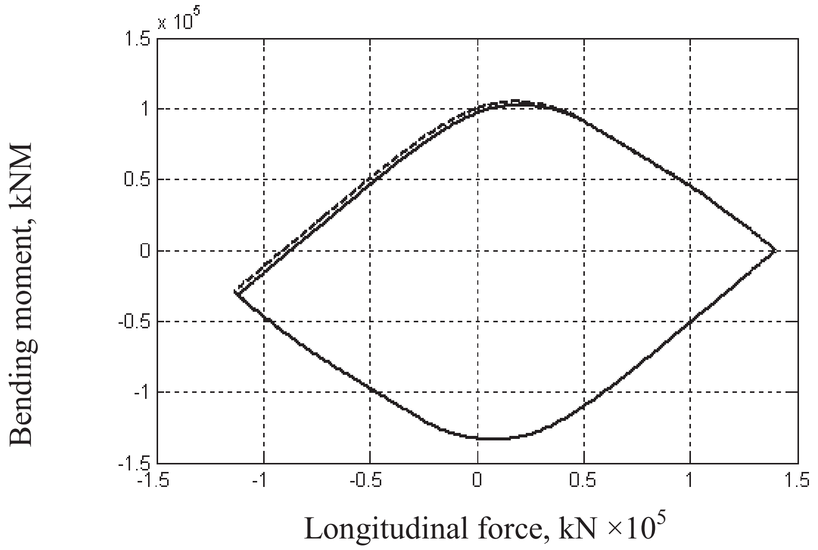


Fig. 5. Section of a steel ferroconcrete beam



— Not considering tension operation of concrete

... Considering tension operation of concrete

Fig. 6. Hypersurface of the fluidity of the section of a steel ferroconcrete beam

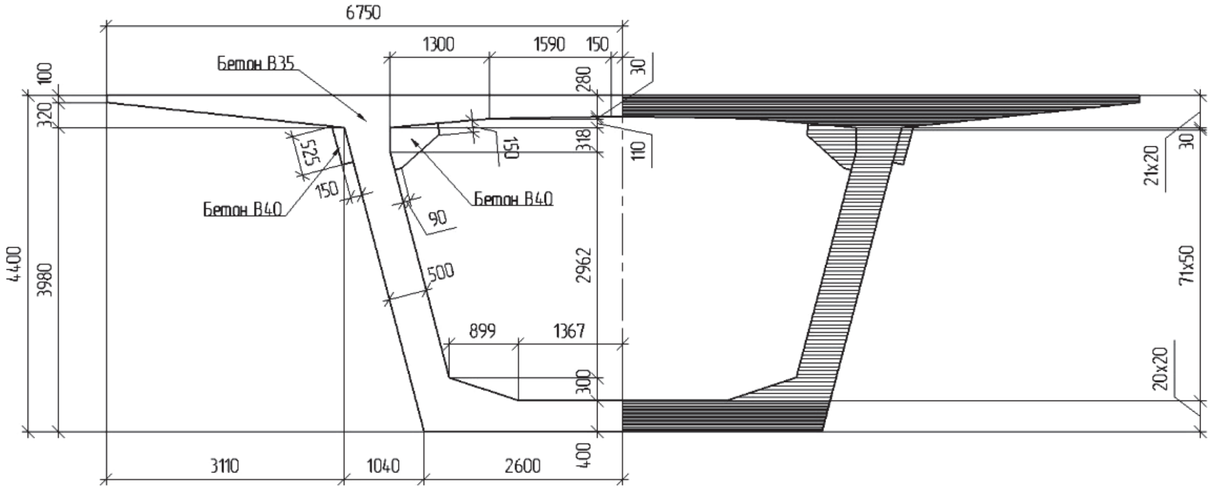


Fig. 7. Sizes of the section of a ferroconcrete beam and division of the section into segments

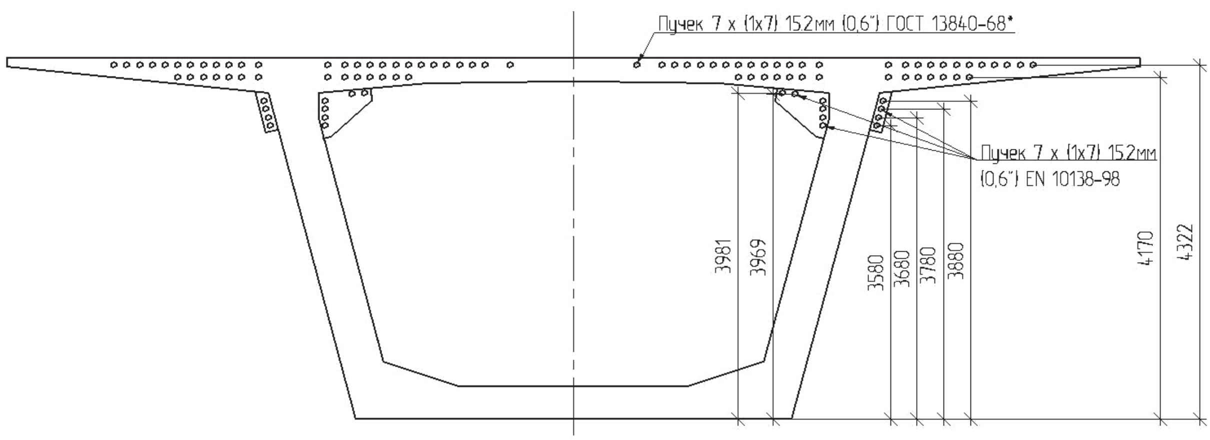


Fig. 8. Schematic of the location of a preliminary strained reinforcement

A preliminary strained reinforcement according to GOST (ГОСТ) and EN have different fluidity limits.

Fig. 9 presents the boundaries of possible limit states of the section of a ferroconcrete box before and after the reinforcement as well as considering and not considering tension operation of concrete.

Conclusions

1. Therefore the suggested numerical method of designing a hypersurface of the fluidity is sufficiently universal and efficient.

2. The method enables a numerical solution of designing the fluidity conditions for random sections composed of materials with different plastic properties with a specific degree of accuracy. There might be different fluidity criteria in place for different areas of a composite section.

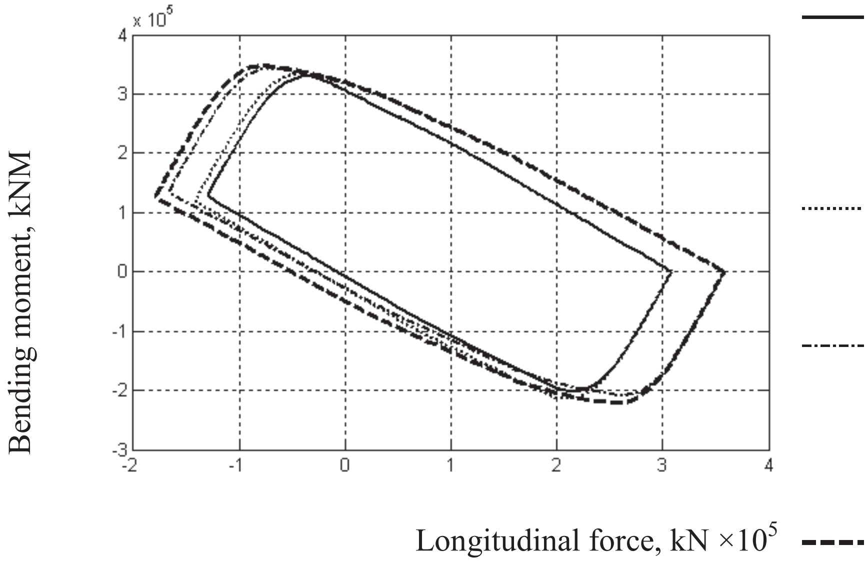


Fig. 9. Hypersurface of the fluidity of the section of a ferroconcrete box beam

3. Using this method, different defects can be modelled in sections with changes made to the characteristics of specific areas of a section (change in the area, fluidity limit). The algorithm can be applied for a numerical analysis of a spatial plastic failure of bar systems [6, 7, 8].

4. The method was tested on simplest sections and proved to be in good agreement with the analytical solutions [5].

References

1. **Efryushin, S. V.** Issledovanie nesushhej sposobnosti sterzhnevyx sistem, primenyaemyx v mostostroenii po metodu predel'nogo ravnovesiya / S. V. Efryushin, M. A. Vikulov // Stroitel'naya mexanika i konstrukcii. — 2010. — Vyp. № 1. — S. 7—15.

2. **Efryushin, S. V.** Chislennoe issledovanie gruzopod«emnosti proletnogo stroeniya avtodo-

rozhnogo mosta s defektami na osnove prostranstvennoj modeli predel'nogo ravnovesiya / S. V. Efryushin, M. A. Vikulov // Stroitel'naya mexanika i konstrukcii. — 2011. — Vyp. № 2 (3). — S. 82—97.

3. **Efryushin, S. V.** Predel'noe ravnovesie zhestkoplasticheskix setchatyx plit / S. V. Efryushin, M. A. Vikulov // Stroitel'naya mexanika i konstrukcii. — 2012. — Vyp. № 1 (4). — S. 7—30.

4. **Erxov, M. I.** Teoriya ideal'no plasticheskix tel i konstrukcij / M. I. Erxov. — M.: Nauka, 1978. — 352 s.

5. **Efryushin, S. V.** Primenenie linejnogo programmirovaniya dlya postroeniya giperpoverx-nosti predel'nyx usilij sechenij zhestkoplasticheskix sterzhnej / S. V. Efryushin, M. A. Vikulov, S. V. Cherkasov // Stroitel'naya mexanika i konstrukcii. — 2011. — Vyp. № 1 (2). — S. 5—11.

6. **Chiras, A. A.** Matematicheskie modeli analiza i optimizacii uprugoplasticheskix sistem / A. A. Chiras. — Vil'nyus: Mokslas, 1982. — 112 s.

7. **Gvozdev, A. A.** Raschet nesushhej sposobnosti konstrukcij po metodu predel'nogo ravno-vesiya / A. A. Gvozdev. — M.: Strojizdat, 1949. — 280s.

8. **Rzhanicyn, A. R.** Raschet sooruzhenij s uchetom plasticheskix svojstv materialov / A. R. Rzhanicyn. — M.: Gosudarstvennoe izdatel'stvo literatury po stroitel'stvu i arxitekture, 1954. — 288 s.