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STRESS AND STRAIN OF MULTIPLY CONNECTED PRISMATIC STRUCTURES, MOUNTED ON A SKEWED CROSS-SECTION

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Statement of the problem. The distribution of pressure in building elements of bridge constructions in the period of installation can differ considerably from a field of pressure during the operation of a complete object. In some cases bridge flights during this period are a console with a fixed cross-section. The calculation of the deflected mode of such elements allows one to provide safe installation during the erection of bridges.
Results. Stress and strain in thin-walled multiply connected prismatic structures at fixed one of the reference circuit on the diagonal section and the second circuit is identified. Unlike in some known works, variable thickness of panels and supporting walls-longerons along a design is considered. The bend from the distributed loading and cross-section force, torsion from the distributed and concentrated moments are considered.
Conclusions. Using the law of variation of a thickness along a construction, it is possible to obtain the redistribution of the pressure under the influence of various power factors.

Keywords: stress and strain, building constructions, variable rigidity.

Introduction

Modern thin-walled structures are commonly made up of complex and generally piecewise-smooth surfaces. Therefore designing their calculation models is closely associated with studies of complex geometric shapes. Their development unlike that of shapes of simplest canonical shapes is way behind current engineering demands. This might be why in designing new types of geometrically complex spatial structures experimental studies and costly natural tests are crucial. Designing practical methods of calculating such structures is one of the most current issues that involves saving materials in industrial production, improving reliability and reducing costs of engineering structures. This paper attempts to address this problem.

In previous papers [1—3] thin-walled prismatic structures with an anchorage along a cross section normal is investigated. But in some cases a structure is fitted along a skewed contour. It is special because under any load in sections parallel to an embedding plane there are bending and rolling moments.

Therefore bending is always accompanied by rolling thus describing a rigidity axis of a structure actually makes no sense. Therefore in calculating skewed thin-walled spatial systems of six generalized displacements corresponding with that of a contour of a solid body, it is necessary to maintain V_2, V_4, V_6 joining them to the three remaining V_1, V_3, V_5 for an asymmetrical profile as well when an external load is such that they are different from zero or Q_z (a longitudinal force acting along the axis z), or Q_x (a bending force acting along the axis x), or M_y (a rolling moment in relation to the axis y).

Generally the equilibrium equation are not generally met.

$$\Sigma X = 0,\ \Sigma Y = 0,\ \Sigma M_y = 0,$$

However, for transverse sections of bridge spans and under their typical loads this lack of equilibrium is insignificant and does not actually affect the results.

Due to a bending of a structure of a skewed edge closely associated with rolling, a solving system of differential equations becomes connected making it somewhat difficult to solve the problem. In this case general displacements V_j of a bending and rolling can be obtained together.

An approach [4] used in calculating multi-connected straight wall-thinned structures allows one to correctly and sufficiently identify stress-strains of shells only remote from the skewed edge. In order to accurately describe stress-strains inside a root triangle this method requires maintaining a lot of members making it daunting to perform calculations. It can be avoided if a skew coordinate system is used instead. In orthogonal coordinates the skewed edge is not a coordinate line and accurate natural conditions cannot be obtained using variational solutions. Using such an orthogonal system where the skewed edge is a coordinate line allows one to design boundary conditions in this line using a variational method and thus providing equal accuracy without having to introduce extra members for displacement functions. In this case while maintaining one or two extra expansion members for these spatial systems solution can be designed and reduced to calculation formulas which are effective overall. If there are more expansion members, stress strain of skewed systems can be investigated in detail using PC.

1. Solving system of differential equations. Fig. 1 shows a shell loaded with moments, force and distributed load concentrated at the section $z = l$

$$q(z) = q_0(1 - z / l),$$

where q_0 is the load intensity at $z = 0$. The thickness of a wall and panels can be assumed to be a variable using a power law

$$h(z) = (b - \beta z)^k,$$

where

$$b = \sqrt[k]{h_1},$$

As bend warping is relatively small and does not have a great effect on normal strains [5], in order to obtain an approximate analytical solution a bend warping of the contour can be neglected. Let a shell be loaded with a concentrated force Q_y, bending M_x and rolling M_z moments at the section $z = l$. In this case generalized displacements V_j and thus deformations and strains are identified using the solution of four differential equations:

$$a_{22}V_2' + b_{24}V_4 = Q_y,\ \ a_{44}V_4' + a_{46}V_6' + b_{47}V_7 = Q_y(z - l) + M_x,$$
$$a_{66}V_6' + a_{64}V_4' + b_{67}V_7 = M_z,\ \ (a_{77}V_7')' - b_{47}V_4' - b_{67}V_6' - c_{77}V_7 = 0,$$

where a_{ij}, b_{ij}, c_{ij} are variable coefficients. The generalized displacement V_7 corresponding with rolling warping is expressed using modified Bessel I_p, K_p [6] and Struve Λ_p functions in an identical fashion [2].

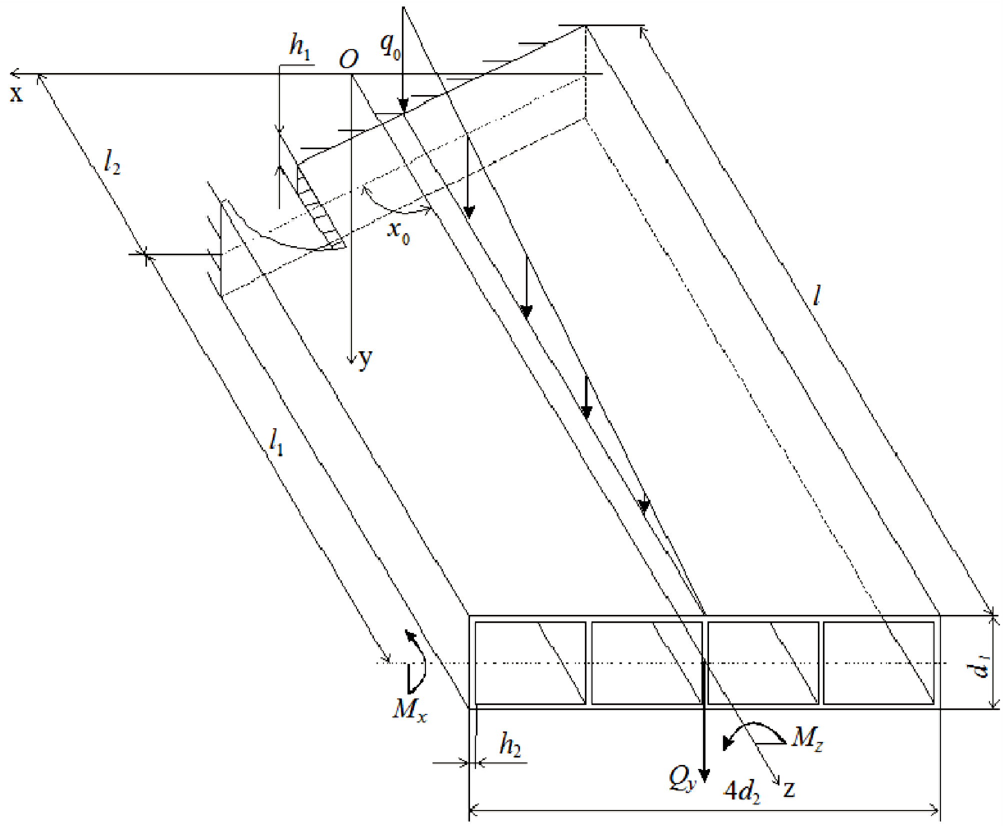

Fig. 1

Using PC a prismatic four-connected thin-walled structure was calculated in bending by a distributed load $q(z)$ and rolling concentrated at the section $z = l$ with a rolling moment M_z.

2. Numerical results. Fig. 2 shows dependency graphs

$$\overline{\sigma}_1 = \sigma_1 / E = f(\overline{z})$$

for linear load bending $q(z)$; it is accepted that

$$l_2 = 0{,}425 \text{ m}, \ d_2 = 0{,}16 \text{ m},$$

$$\chi_0 = 56^{\circ}, \ q_0 = 24{,}5 \text{ kN/m}, \ l = 1{,}6 \text{ m};$$

the dimensionless parameters are

$$\overline{d} = d_1 / 4d_2, \ \overline{h} = h_1 / h_2, \ \overline{z} = z / l .$$

Continuous curves are designed using a short one and the dotted ones along long ribs of the upper panel which corresponds to the coordinates: $x = 2d_2$, $y = -d_1/2$ и $x = -2d_2$, $y = -d_1/2$.

The analysis of the graphs in Fig. 2 shows that the distribution $\overline{\sigma_1}$ largely depends on $\overline{h}$ (curves 1 and 2). The relative height of a structure considerably influences its stress and strain (curves 2 and 3).

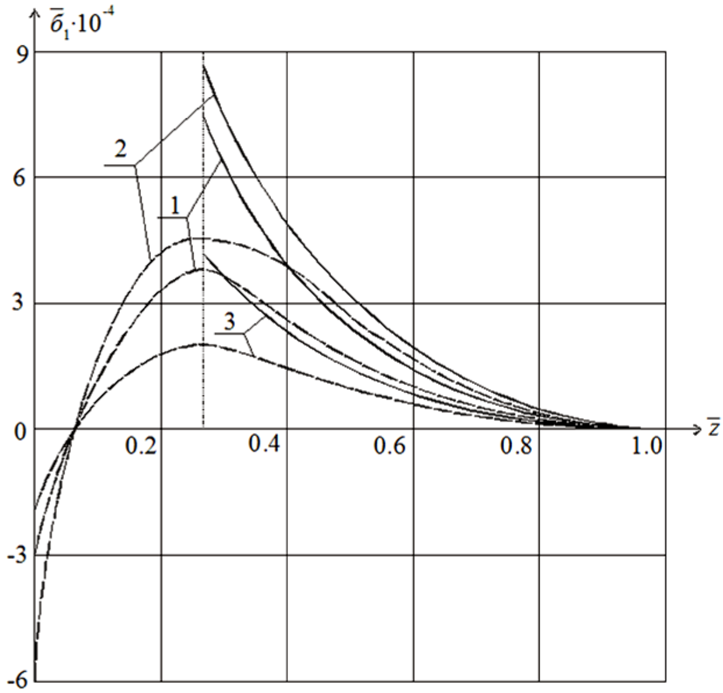

Fig. 2

Table 1

Curve	1	2	3
$\overline{d}$	0,125	0,125	0,25
$\overline{h}$	1,25	5	5

Fig. 3 shows the distribution $\overline{\sigma_1}$ under the effect of a concentrated rolling moment M_z on a structure applied at the end section $z = l$. In the calculations we assume that M_z = 1,57 kN·m, $\chi_0 = 56^{\circ}$, d_2 = 0,16 m, l_2 = 0,425 m. As previously, continuous curves are designed along a short rib with the coordinates $x = 2d_2$, $y = -d_1/2$, the dotted ones along a long one: $x = -2d_2$, $y = -d_1/2$. Dimensionless parameters of a thin-walled spatial system are shown in Table 2 where $\tilde{d} = 4d_2/l$.

It is clear to see that during rolling of a structure the distribution $\overline{\sigma_1}$ considerable differs in the short and long ribs of the slabs.

In all the cases at $\bar{h} \to 1$ we have a graph (curve 5) known from the literature [4]. In the section z = 0 there is a clear edge effect due to the restriction of warping of a skewed section. A surge in strains at $\bar{h}$ = 5 is at $\bar{z}$ = 0,83. It is obviously due to a dramatic decrease in the area of a transverse section of a structure at a large $\bar{h}$. It is proved by the fact that at $\bar{h} \to$ 1 there is no more surge in strains in a section $\bar{z}$ = 0,83.

Fig. 4 shows the distribution $\overline{\sigma_1} = f(\bar{x})$ in transverse sections of the upper slab corresponding with the coordinate $y = -d_1 / 2$, $\bar{x} = x / 4d_2$.

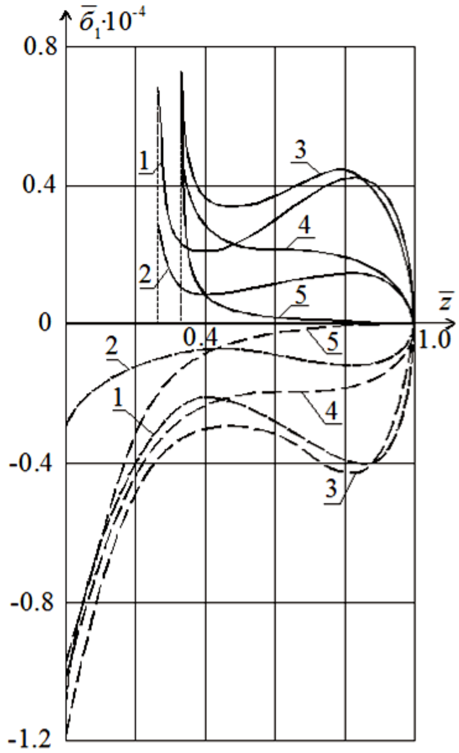

Fig. 3

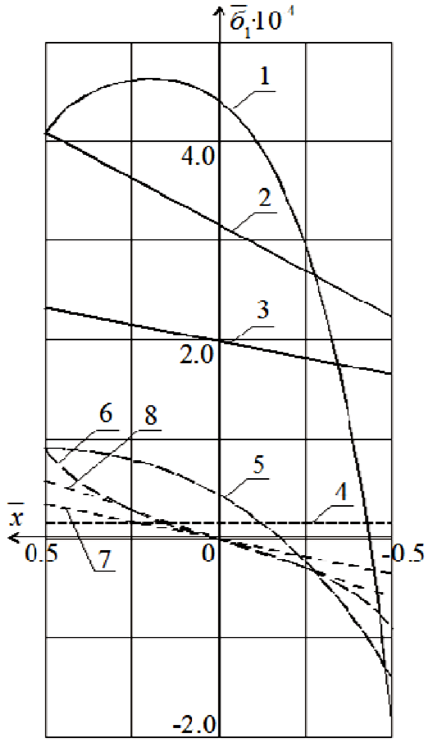

Fig. 4

For the calculations we assume that l_2 = 0,425 m, $\bar{d}$ = 0,25, $\tilde{d}$ = 0,4, $\bar{h}$ = 5. The curves 1—4 are designed for a bending under a distributed load $q(z)$ at q_0 = 24,5 kN/m; the curves 5—8 — for a bending under a concentrated moment M_z = 1,57 kN·m. The curves 1 and 5 correspond with an embedding section ($\overline{z}$ = 0), 2 and 6 are designed at $\overline{z}$ = 0,26, 3 and 7 — at $\overline{z}$ = 0,4, 4 and 8 — at $\overline{z}$ = 0,8.

Table 2

Curve	1	2	3	4	5
$\bar{d}$	0,25	0,5	0,25	0,25	0,25
$\tilde{d}$	0,4	0,4	0,5	0,5	0,5
$\overline{h}$	5	5	5	20/7	1

In addition a function $K_p\left(\sqrt{a}\xi\right)$ is a sign-alternating continuous row and in PC calculations there is also loss of accuracy, arithmetic disruptions when the row is overfilled or gone. This is what was observed in calculating using formulas containing special functions for $\overline{h} = h_1/h_2$ approaching one. This is due to the fact that at $\overline{h} \to 1$ an argument of special functions increases significantly causing inconsistencies in PC calculations. Therefore in order to use PC formulas containing modified functions, it was not the dispersion of these functions into degree rows but their integral representation which is free of disadvantages associated with rows was used.

However the equations describing warping displacements can be integrated using one of the numerical methods, i.e. to solve a linear edge task for second-order differential equations. For that a differential run method was used that helped to solve a number of tasks on PC for a small range of the thickness of embedding and at the free end of the shell ($\overline{h}$ = 1,1). The obtained solutions are in good agreement with solutions for shells with a constant thickness [4]. Besides for $\overline{h} \geq 2$ displacements and strains determined using numerical integration of warping equations are in agreement with similar values calculated using the formulas containing special functions.

In conclusion it should be noted that cone-shaped thin-walled structures designed along a normal contour of a transverse section or a skewed edge can be examined in an identical fashion. In [7, 8] there are analytical solutions and results of a numerical calculation for a spatial cone-shaped structure of a varying thickness.

Conclusions. The analysis of the graphs shows that in the linear law of changes in the thickness the distribution $\overline{\sigma_1}$ for $\overline{h}$ = 1,1 ranges identically to a structure with a constant thickness [4]. However, as $\overline{h}$ is on the rise, $\overline{\sigma_1}$ changes its strain modulus at h = const.

Therefore for the above loading schemes of stress and strain of prismatic thin-walled structures of a constant and variable thickness are different, which should be considered in calculating elements of actual building structures. By varying the law of changes in the thickness of a structure and thus its stiffness, a new distribution of strains occurring under the effect of different force factors can be obtained.

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