

## TECHNOLOGY AND ORGANIZATION OF CONSTRUCTION

UDC 624

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### STRUCTURAL OPTIMIZATION OF BEIJING GYMNASIUM SUSPEN-DOME WITH CARBON FIBRE REINFORCED POLYMER (CFRP) CABLE

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**Statement of the problem.** A study on the structural behavior of carbon fibre reinforced polymer cable as a constitute material for the tensegrity system for Beijing gymnasium suspen dome compared with results of studies on of steel as the constitute material for the Beijing gymnasium structure is presented.

**Results.** Non linear software known as ANSYS was adopted for the analysis in this study. Based on the results of the analyses, the structural behavior of the carbon fibre reinforced polymer tensegrity system was compared with that of steel tensegrity system for the Beijing gymnasium suspen dome reported from other studies, taking into account the internal forces, displacement and frequency.

**Conclusions.** The comparison of these results indicates the efficiency of CFRP tensegrity system for the suspen dome, over that of steel tensegrity system.

**Keywords:** Suspen dome; CFRP tensegrity system, Steel tensegrity system, Finite element model, Static, Modal.

### Introduction

The evolution of long span structures has made engineers to search for more reliable and dependable materials for such structure. Suspen dome which falls in such category has evolved tremendously over the past few years in roof design. The tensegrity system which makes up a part of the suspen dome is one of the most promising solutions for self equilibrium. However, the need to improve such system is required in order to be more dependable and controllable. The tensegrity system was developed by Fuller (Fu, 2005), (Wojciech, 2015) and the idea to incorporate the system into a single layer reticulated dome was created by Prof. Kawaguchi

and his team to form a system called suspen dome (Subramanian, 2006) which is attractive due to its structural properties. The construction of the structure includes Fureai dome and Hikarigaoka dome in Japan, Kiewitt suspen dome in Tianjin, Olympics badminton gymnasium suspen dome in Beijing, China, are few examples.

Cables are one of the main parts of a tensegrity system of a suspen dome; the choice of cables types depends mainly on the mechanical properties, structural properties and economical criteria. The evolution of Carbon fiber-reinforced polymer (CFRP) has impacted in the application of civil engineering structures due to its great strength, flexibility, light weight, fatigue and resistance to corrosion. However, its disadvantages include high cost, unfamiliarity with contractors, ductility, sensitivity to impact damage and difficulty in connections, even though CFRP cables are higher in some qualities compared to steel cables such as in some mechanical aspects; namely, relaxation and creep (Xu et al, 2015).

The stated reasons have made CFRP cables attain heights for its use in long span structures such as bridges. In order to extend the lifespan of a suspen dome, which is also a long span structure, a new material has to be introduced, the structural behavior of which would differ significantly compared to steel cable.

Static and dynamic research both on numerical and experimental results can be found in literature based on the application of steel material for suspen and even construction (Kitipornchai et al, 2005), (Wenjiang et al, 2003), (Behnam et al, 2012); no literature results are found on the application of carbon fibre reinforced polymer cables where such structure has been addressed. With the current tremendous studies involving carbon fibre by researchers, it is believed that optimizing the structure with carbon fibre material can lead to solutions where members can exceed their capabilities. In fact, the instability of a suspen dome can disappear because of the presence of carbon fibre reinforced polymer cables.

The analysis of the prototype suspen dome was through a non-linear software known as ANSYS (ANSYS, 2008). Based on the analysis, the structural behavior of the CFRP tensegrity system for the prototype is summarized. A comparison between the literature results by Zhang et al [9] on Beijing Olympic badminton suspen dome with steel tensegrity and the CFRP tensegrity system proposed by the authors of this study is made to validate the use of CFRP cables.

### **Theoretical Background**

This section introduces the finite element formation for static and modal analysis. Static analysis predicts the structures deformation whereas the modal analysis predicts the most reliable design against vibration.

### An overview on Static Analysis

The fundamental equation for static analysis is given as:

$$[K]\{U\} = -\{P\} + \{R\}, \quad (1)$$

where  $[K]$ = total stiffness;  $\{U\}$ = displacement vector of the node;  $\{P\}$ =load vector;  $\{R\}$ = residual force.

Investigating the static behavior of the system is very vital; this is attained by analyzing the force and deformation in the structure.

Incorporating Newton Raphson approach in solving the equation is given as

$$[K]\{\Delta U\} = \{\Delta P\}, \quad (2)$$

where  $[K]$  = stiffness matrix;  $\{\Delta U\}$ = displacement increment;  $\{\Delta P\}$ = Load imbalance.

### An overview on Modal Analysis

The modal analysis can be expressed in matrix form as:

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = \{F\}, \quad (3)$$

where  $[M]$  = mass matrix;  $[C]$  = damping matrix;  $[K]$  = stiffness matrix;  $\{u\}$  = displacement vector of each node;  $\{\dot{u}\}$  = velocity vector of each node;  $\{\ddot{u}\}$  = acceleration vector of each node, and  $\{F\}$ = force vector

Ignoring resistance, a dynamic equilibrium equation can be obtained as:

$$[M]\{\ddot{u}\} + [K]\{u\} = 0 \quad (4)$$

Assume a harmonic motion

$$\{u\} = \{\phi_i\} \sin(\omega_i t + \theta_i), i = 1, 2, \dots, n \quad (5)$$

$$\{\ddot{u}\} = -\omega_i^2 \{\phi_i\} \sin(\omega_i t + \theta_i) \quad (6)$$

Where

$n$  = number of degree of freedom,  $\{\phi_i\}$ =eigenvectors,  $\omega_i^2$  = eigenvalues,  $\theta_i$  = the phase angle for  $i$  mode of vibration.

The eigenvectors and eigenvalues represent the mode shapes and the square of the natural circular frequency.

Substituting  $\{u\}$  and  $\{\ddot{u}\}$  in the governing equation gives

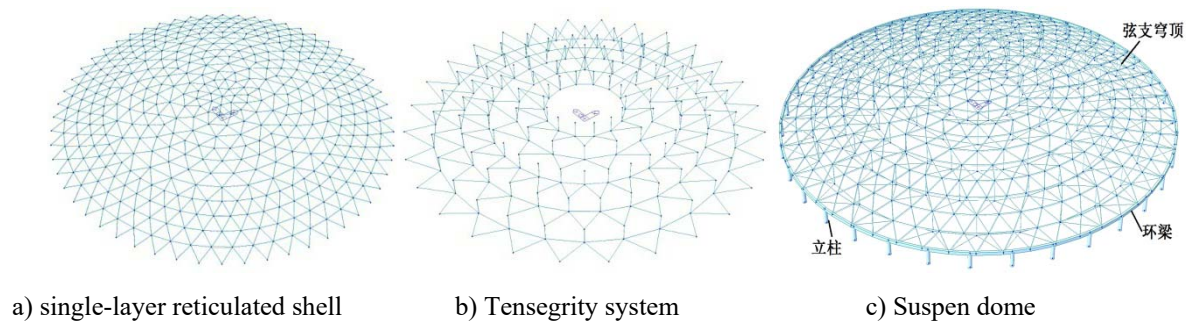
$$([K] - \omega^2 [M])\{\phi\} = 0. \quad (7)$$

Equality is given if  $\{\phi_i\} = 0$ .

Where  $\omega$  = natural frequency of the system, and  $\{\phi\}$  = vibration modals of the structure.

### Layout and material used for Beijing Olympic Suspen dome

Based on appropriate simplification, the physical dimensions of Beijing Olympic Badminton Gymnasium suspen dome are given as follows (Ge et al, 2007a), Ge et al, 2007b), (Liu, 2010), the span the model is 93m and 8m high, fixed and hinged as shown in Fig. 1.



**Fig. 1.** Suspen dome model

The single layer reticulated shell is a  $\Phi 219 \times 10$  circular steel pipe with Q345 steel circular tube of  $\Phi 168 \times 8$  for strut and a yielding strength of  $345 \text{ N/mm}^2$ . The permanent and live load of the roof are  $0.85 \text{ kN/m}^2$  and  $0.5 \text{ kN/m}^2$  respectively. Based on static equivalent principle, the uniformly distributed load is equivalent to a vertical concentrated load on each node of the upper single layer of the reticulated shell. The area and material properties are illustrated in tables 1 and 2 respectively.

Table 1

Area of ring cables and radial cables

	Ring cable			Cable diameter	
	HS1	HS2~3	HS4	JS1	JS2~4
Cable	7658	2730	1179	1179	726
Cross section area	7658	2730	1179	1179	726
The equivalent axial stiffness	9898	3258	1524	1524	938
Strength	6950	2478	1070	1070	659

Table 2

Material Properties

	Modulus of elasticity $E/\text{GPa}$	density $/\text{kg/m}^3$	tensile strength $/\text{MPa}$	Design strength $/\text{MPa}$	Poisson ratio	The temperature coefficient of expansion
Section steel	206	7850	550	315	0.3	$12 \times 10^{-6}$
Steel Cable	190	7850	1670	835	0.3	$12 \times 10^{-6}$

### Material Type Proposed by the Authors

As hinted earlier the authors are proposing CFRP cables to replace steel ones. Based on the structural dimension of Beijing Olympic Badminton stadium suspen dome, the structural scheme of the tensegrity system made up of CFRP cables would involve material properties illustrated in Table 3

Table 3

Material properties proposed by the Authors

	Modulus of elasticity $E$ /GPa	density /kg/m <sup>3</sup>	tensile strength /MPa	Design strength/MPa	Poisson ratio	The temperature coefficient of expansion
CFRP Cable	160	1600	2300	920	0.3	$6.8 \times 10^{-7}$

### Model Analysis

A finite element model of Beijing gymnasium suspen dome was built and its behavior under concentrated load was investigated. Based on (Jiamin et al, 2012), it is realized that the overall stability under full span load defines the structural behavior better than half span load. Hence, only full span loading was considered in the investigation to determine the global stability of the system.

The structure is made up of series of beam and truss elements. Element type inputted to simulate the beam, rod and cable on ANSYS are: beam 188, link 8 and link 10 respectively.

The assumptions implemented throughout the investigation are as follows:

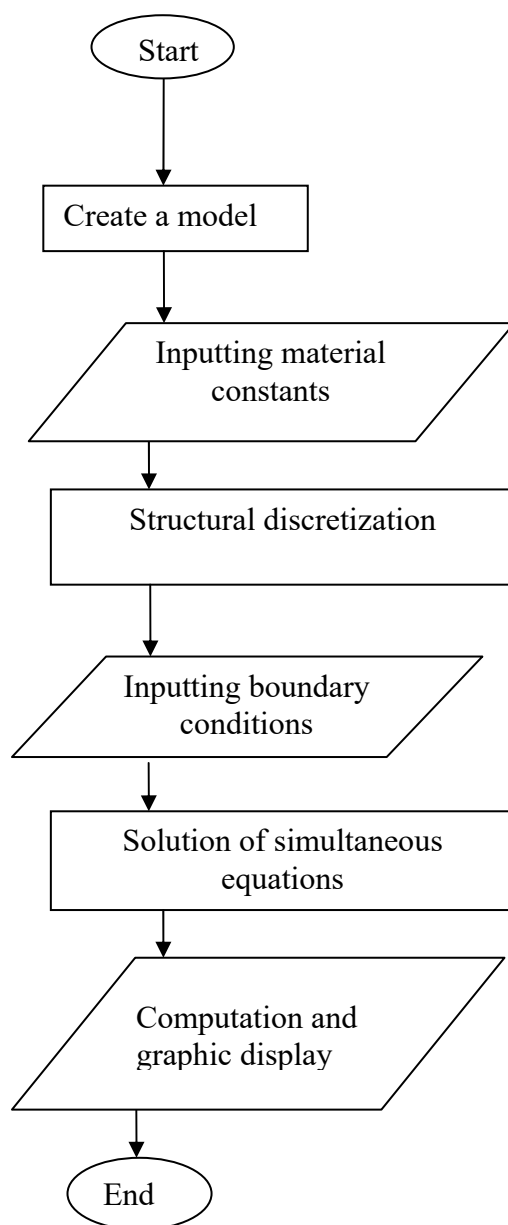
- External loads are applied at nodes
- Self weight are transferred to nodes as point loads, the loads are applied on the top of the dome structure in the z-direction
- Cables are elastic
- The tensegrity members are connected by pin-joint

The behavior of the cable system under uniformly distributed loads as well as point loads were analyzed by studying the deformation of their components, since structures would only deform because of their elasticity under external load.

### Finite Element Method

Finite element analysis (FEA) software package ANSYS was employed for the structural analysis in the study due to its consideration for geometric non-linearity. Building the FEM

model, the geometric, material properties and load condition of components were inputted using the algorithm described in Fig. 2. Due to the complexity of the suspen dome structure, results for a three dimensional FEM analysis are considered to be more comprehensive and reliable than results of empirical formula.



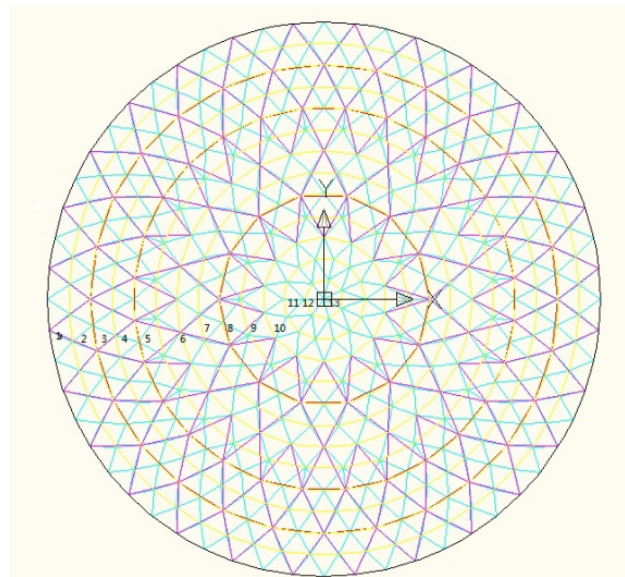
**Fig. 2.** Flowchart for the analysis

### Comparison of Structural Types

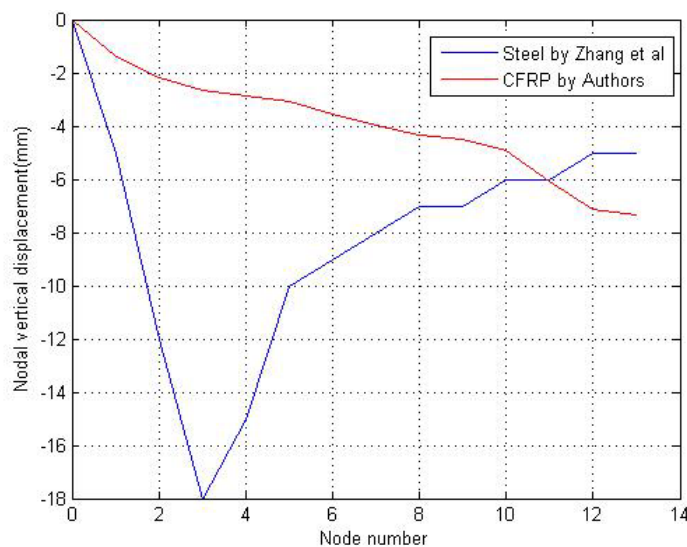
The material type proposed by the author is compared with the prototype-Beijing Olympic suspen dome by (Zhang et al, 2007) in terms of internal forces, displacement and frequency.

### Static analysis

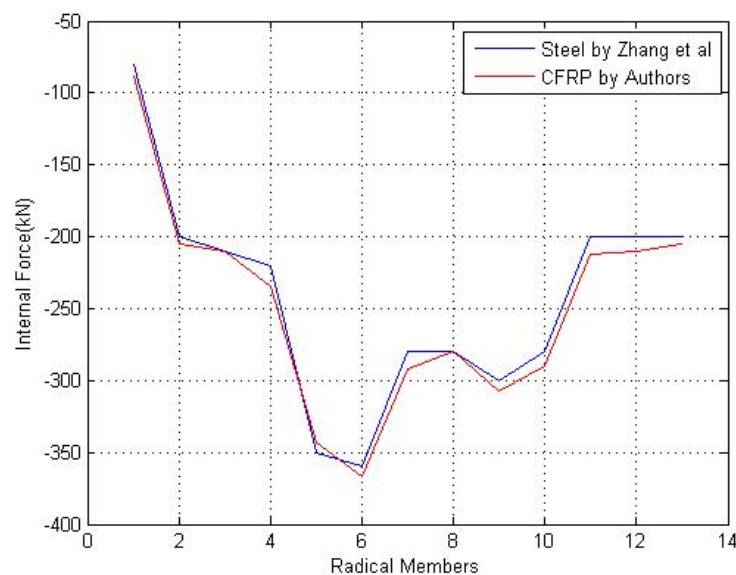
Due to the symmetrical nature of the dome structure, nodes and elements can be represented by one typical node or element. Fig 3 illustrates the nodes and elements of the dome, Fig 4 illustrates the nodal displacement for the nodes as stated in fig 3 for the suspen dome. The nodal displacement of the CFRP tensegrity are much lower than the steel tensegrity system which implies that the CFRP tensegric system of the suspen dome is more effective than the steel tensegric results obtained by (Zhang et al, 2007), it can be observed that the outer position of the nodes had smaller nodal displacement, the farther the nodes are away from the mid rib, the larger is the displacement.



**Fig. 3.** Numbering of nodes



**Fig. 4.** Comparison of nodal displacement



**Fig. 5.** Comparison of Internal force of radical members

Fig 5 illustrates the internal forces in the radical members for the two dome structures which are similar; it implies that CFRP tensegric system has little effect on the internal forces generated in the radical members. In addition, it was observed that the internal force was at its largest at the outermost ring and lowest at the centre.

Furthermore, Table 4 illustrates the result obtained from the computation and comparison of the interior force in the loop truss. It was observed that there was a slight decrease in CFRP cables compared to steel cables by (Zhang et al, 2007) which shows the reduction of interior forces.

Table 4

Comparison of Interior Force Analysis in the Loop Truss

Outer ring truss chord			Inner ring truss chord		Lower ring truss chord	
(kN)	Minimum value	Maximum value	Minimum value	Maximum value	Minimum value	Maximum value
Zhang et al	18.6	55.29	-131.04	148.73	15.94	-5.57
Authors	16.1	51.45	-120.87	139.74	13.41	-4.70
Percentage difference(%)	13.4	6.9	7.7	6.0	15.8	15.6

### Modal analysis

The natural frequencies for the first mode carried out through modal analyses are shown in Table 5. The frequency of the structure predicts the ability of the structure to avoid collapses caused by earthquake and wind storm.



Table 5

## Comparison of natural frequency

Material type	Steel by Zhang et al(Hz)	CFRP by Authors(Hz)
Frequency	5.70	6.50

From table 5 it can be observed that the frequency of CFRP tensegrity system is much lower than that of steel with a percentage difference of 23.8% which presents a better resistance capacity.

### Conclusion

In this study, a finite element model of Beijing gymnasium suspen dome was established. Analyses were carried out to investigate the behavior of carbon fibre reinforced polymer (CFRP) cable as a constitute material. From the comparison, the following conclusions are drawn:

- 1) The application of CFRP by the authors is practical based on the satisfactory results obtained from the numerical analysis.
- 2) The concept for the application of CFRP cable is to strengthen the dome; this was proven with CFRP cables having lower nodal displacement than that of steel cables which implies that CFRP has more stiffness capability.
- 3) The frequency result proves that CFRP tensegrity system would successfully overcome fatigue and other harmful effect of forced vibration such as earthquake and wind storms over steel cables.

Thus, the application of CFRP cables demonstrates an outstanding performance and the results show an efficient design.

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