BUILDING MECHANICS

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STATISTICAL MODELING OF A DYNAMIC RESPONSE OF A STADIUM GRANDSTAND TO HUMAN LOAD

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Statement of the problem. The aim of this paper is the estimation of the amplification factor of stadium grandstands in response to synchronous and non-synchronous activities of people. Human actions are modeled by concentrated half-sine force; nonsynchronous motions are caused by lag each person's actions and spatial distribution of people with different weights. The grandstand is modeled as a pivotally supported I-beam.

Results. Deflections of the model are determined by means of direct integration of dynamic equations. Estimating a random amplification factor and inconsistency factor are performed by means of the statistical modeling method. Resonant and non-resonant modes of the synchronous and non-synchronous movements of people are considered.

Conclusions. The results showed that the values of the amplification factors obtained by using quasi-static approach by summing the response from each spectral component of the load, are higher than experimental values by 21—23 %, and nonsynchronous movements can significantly (up to 63 %) reduce the amplification factor.

Keywords: human-structure interaction, amplification factor, inconsistency factor, dynamic response, non-synchronous motions.

Introduction. Consistent loads from human-structure interaction occur from synchronous movements of crowds of people that are common during sports and cultural events. Synchronous movements of spectators during a football match or a concert might cause perceivable vibrations, disruption of operational criteria and sometimes local failures as well [5—7, 9—12, 14—21]. Stands made of suspending structures or consoles with a long are particularly sensitive to these loads with a long gibbet.

A load from spectators who are moving in synchrony is modeled with a sequence of semi-sinusoid impulses [13—21]:

$$F(t) = \begin{cases} K_p G \sin\left(\dfrac{\pi t}{t_p}\right), & 0 \le t \le t_p, \\ 0, & t_p \le t \le T_p, \end{cases} \tag{1}$$

where G is a static load on stands from spectators (spectators' weight); t_p is time of contact; T_p is a period of impulses; $K_p = \pi / (2\alpha)$ is a shock factor where $\alpha = t_p / T_p$ is a contact ratio; for pedestrian movement and low-rhythm aerobics $\alpha = 2/3$, for rhythmic movements and high-rhythmic aerobics $\alpha = 1/2$, for regular jumps $\alpha = 1/3$, for high jumps $\alpha = 1/4$ [17]. The frequency of impulses is $f_p = 1 / T_p$; it was experimentally found that the frequency of impulses can range from 1 to 4 Hz [14—16, 18, 19].

The expansion of the function $F(t)$ in a Fourier row [5] is known:

$$F(t) = G\left[1 + \sum_{n=1}^{\infty} a_n \cos\frac{2\pi n}{T_p}t + \sum_{n=1}^{\infty} b_n \sin\frac{2\pi n}{T_p}t\right] = G\left[1 + \sum_{n=1}^{\infty} r_n \sin\left(\frac{2\pi n}{T_p}t + \varphi_n\right)\right], \tag{2}$$

where a_n, b_n are Fourier coefficients, at $2n\alpha = 1$, $a_n = 0$, $b_n = \pi / 2$, otherwise

$$a_n = 0.5\left[\frac{\cos(2n\alpha - 1)\pi - 1}{2n\alpha - 1} - \frac{\cos(2n\alpha + 1)\pi - 1}{2n\alpha + 1}\right],$$

$$b_n = 0.5\left[\frac{\sin(2n\alpha - 1)\pi - 1}{2n\alpha - 1} - \frac{\sin(2n\alpha + 1)\pi - 1}{2n\alpha + 1}\right],$$

$$r_n = \sqrt{a_n^2 + b_n^2}, \quad \varphi_n = \operatorname{arctg}(a_n / b_n).$$

Six or eight members of the row yield a good approximation of the function (1).

The ways of determining a dynamic reaction of a structure to consistent loads (1) were detailed in [5, 9] where two of them are described, i.e. quasistatic method in a deterministic and probabilistic context, and checked by means of direct integration of movement equations. Bear in mind that for the quasistatic method external impulse forces are applied to a structure as static loads, internal efforts do not depend on time, which makes construction calculations easier. Dynamic effects in the quasistatic method are accounted for using dynamism coefficients. At determined loading all the parameters of a calculation model and loading are considered strictly determined and dynamism coefficients are identified by means of the methods of the oscillation theory. In the probabilistic approach loading is seen as a random process and dynamism coefficients depend on spectral density of an input random process [1, 2].

The studies in [5, 9] revealed a typical feature of loading from consistent movement of spectators, i.e. extraordinarily high dynamism coefficients when a spectrum of own frequencies is in the range of possible frequencies of forced vibrations (1—4 Hz). It turned out that for high jumps a maximum dynamism coefficient is 14 at 5 % damping and 27 at 2.5 % damping. The authors of the study accounted for these dynamism coefficients with a narrow-band nature of loading when all the impulse energy is focused on certainly areas and spectral density of impact is a sequence of impulse functions at these frequencies; a detailed discussion and findings can be found in [9].

Note that high values of dynamism coefficients correspond to absolutely synchronous movements of people with the same phase, frequency, amplitude, which is obviously unlikely. It might be sensible to reduce dynamism coefficients considering inconsistency of people's movements. In foreign methods it is suggested that the parameters of a dynamic reaction to a nonsynchronous coefficient which is 0.67 [13]. We were not able to find any references as to why an inconsistency coefficient is important in any available foreign scientific literature and regulations. It is not known how this coefficient was obtained and whether it was theoretical or experimental.

The problem might become more certain by introducing random parameters of a dynamic load such as amplitude, movement phase (it is pointed out how necessary it is to consider a random phase parameter in [12, 20]) as well as by taking into account uneven spatial distribution of impulses from each spectator on a model of a stand. Statistic modeling of a dynamic reaction will allow evaluation of coefficients of dynamism and inconsistency.

The objective of the paper is to obtain the justified coefficients of dynamism and inconsistency by means of a statistic modeling method considering a random nature of amplitude values and phase of a spatial vector on people's consistent movements.

1. Coefficient of dynamism for a random impact. A way to determine a dynamic reaction for consistent movement of spectators is given in [5, 9]; in [3, 5] it can be found that a similar conclusion is made for formulas for a dynamism coefficient. When the problem started being addressed, it was assumed that an impact is simultaneous, synchronous and the forces have the same amplitudes, i.e. a one-dimensional impact was being dealt with. The solution of the movement equation in an established mode of forced vibrations was obtained when loading was decomposed in a Fourier row (2) followed by the use of the superposition method. A formula for maximum modal dynamism coefficient corresponding with the k-th form of oscillations with own frequency f_k was obtained:

$$\beta(f_k)=1+\sum_{n=1}^{N_F}\frac{r_n}{\sqrt{\left(1-\frac{n^2 f_p^2}{f_k^2}\right)^2+\left(\frac{2\xi_k n f_p}{f_k}\right)^2}},\qquad(3)$$

where N_F is a number of confined members of the row (2); ξ_k is a modal damping coefficient; f_p is an excitement frequency, Hz. In Fig. 1 there is a graph of a dynamism coefficient in the axis of own frequencies $\beta(f_k)$ at the frequency of impulses $f_p = 2$ Hz for high jumps ($\alpha = 1/4$) and damping 6.53 % ($\xi_1 = 0.0653$ is a damping coefficient for the lowest own frequency corresponds with the below model of a stand).

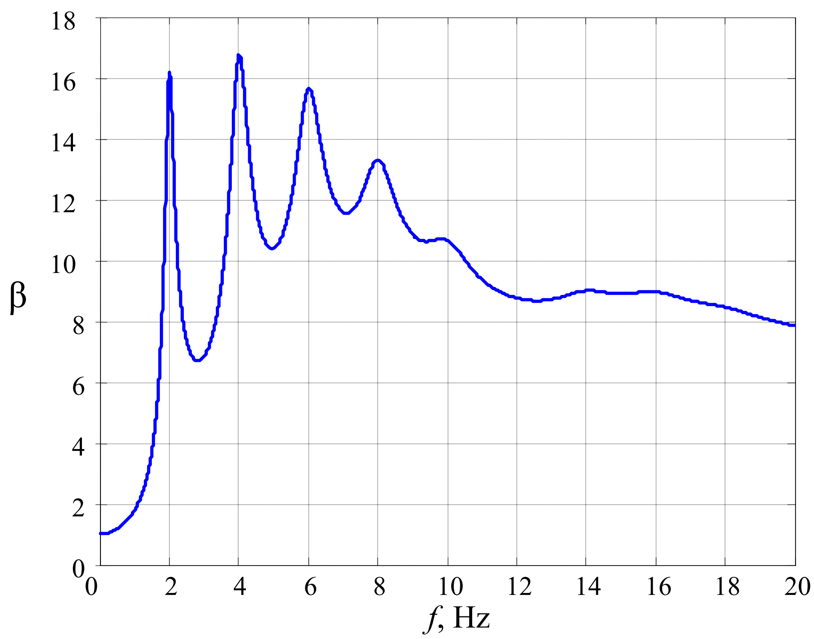

Fig. 1. Dynamism coefficient

It should be emphasized that in the formula (3): first, a simultaneous maximum response to all the harmonic components of an external load not considering their phases is employed; secondly, possible inconsistency of the process is not considered; thirdly, the damping coefficient is accepted to be constant and equals ξ_1.

2. Stating the problem and designing static modeling. As the simplest model of a stand, let us accept a hinged beam (Fig. 2). A beam with the length of 6 m has a double-tee profile chosen based on the density for a maximum static load. A load from 9 spectators is specified as concentrated forces with the step of 0.6 m. Each force changes in time according to the semi-sinusoid impulse law with a random amplitude and phase (i = 1, …, 9):

$$F_i(t)=\begin{cases}\dfrac{\pi}{2\alpha}G_i\sin\left(\dfrac{\pi t}{\alpha T_p}+\varphi_i\right), & 0\le t\le \alpha T_p,\\ 0, \quad \alpha T_p\le t\le T_p. & \end{cases} \tag{4}$$

During a series of numerical experiments it was assumed that the amplitude G_i (a spectator's weight) and a phase φ_i are evenly distributed random values. равномерно Let a spectator's mass be able to vary from 70 to 100 kg, then $700 \le G_i \le 1000$ H. Besides, for each spectator there was allowed to be a delay. With a delay from 0 to 1 sec the phase φ_i changed randomly from 0 to $(\pi\cdot 1)/\alpha T_p$. The random values $_i$ and φ_i, which are different for each spectator, are generated using a sensor of random numbers before the start of each numerical experiment. A more dangerous variant of spectators' activity was considered, i.e. high jumps (α = 1/4) with the frequency of 2 Hz.

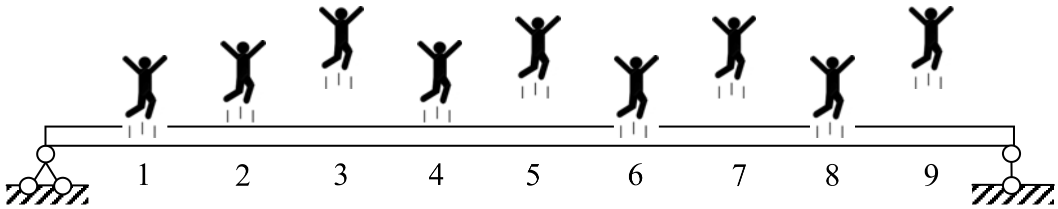

Fig. 2. Model of a stand with spectators

The objective of the numerical experiment was to identify displacement of a central section of the beam under a static load (the spectators are standing) and a dynamic one (the spectators are jumping high) as well as to evaluate coefficients of dynamism and inconsistency. A coefficient of inconsistency will be defined as a ratio of dynamism coefficients for an inconsistent (obtained as a result of a numerical experiment considering random amplitudes and phase) $\beta_{эксп}$ and completely consistent processes:

$$K=\beta_{эксп}/\beta. \tag{5}$$

In order to solve the task, the method of direct integration of a movement equation was employed in a time range in a simulation environment *Simulink* in the *MatLab* software.

The plan of the numerical experiment is as follows:

1. Designing a finite-element model, specifying node transformations **u**, forming a matrix of rigidity **K** and inertia **M** using the finite element method. Specifying a matrix of damping

B based on a model of internal friction. Determining own frequencies of the damping models and parameters;

2. Specifying the amplitude and phase for each concentrated force (a sensor of random numbers is used); designing a vector of node static loads from the spectators' weight **G** and a vector of random phases **φ**;

3. Designing a calculation static model of the beam. The solution of the equation of static balance considering own weight of the beam is the following:

$$\mathbf{Ku} = \mathbf{G}.$$

Determining a static bend u_{cm} in the centre of the beam;

4. Designing a calculation dynamic model of the beam. The movement equation:

$$\mathbf{M\ddot{u}} + \mathbf{B\dot{u}} + \mathbf{Ku} = \mathbf{F}(t),$$

where **F**(*t*) is a vector of a dynamic load with the elements (4) is solved by direct numerical integration in *Simulink*. Identifying a maximum dynamic displacement for transitional and established modes;

5. Identifying a dynamism coefficient as a ratio of a dynamic displacement to a static one. Determining an inconsistency coefficient (5).

Numerical modeling included a series of 10 experiments (steps 1—4). As a result, implementation of random coefficients of dynamism and inconsistency was obtained and their mathematical expectation and standard was identified.

3. Modeling results. The following cases were considered as part of the numerical experiment.

1. Non-resonance mode: beam (double-tee № 16) with the length of 6 m is divided into 10 bar finite elements with the length of 0.6 m each; for determining the matrices of rigidity, inertia and damping the method of finite elements was employed. A finite-element mdoel of the stand includes 20 node degrees of freedom: 9 linear displacements and 11 node ones (considering the boundary conditions); the lowest own frequency of the beam is 14.6 Hz (91.7 rad/sec), which is considerably higher than the impact frequency of 2 Hz, damping is 6.53 %. The dynamism coefficient not considering inconsistency according to the formula (3) is β(14.6 Hz) = 9.02 (see Fig. 1). The results of statistical modeling for the series of 10 numerical experiments can be found in Table 1.

Here are the displacements of a central section for static loading, dynamic displacements for the established and non-established forced vibrations, experimental coefficients of dynamism and inconsistency. Fig. 3a shows one of the obtained functions of displacements in time.

Table 1

Results of the series of 10 experiments. Non-resonance mode.
Displacements of a central section (according to a module), coefficients of dynamism and inconsistency

Number of the experiment	U_{st}, m	U_{dyn}, m		B_{exp}		*K* (formula (5), β = 7.15)	
		Н	*У*	*Н*	*У*	*Н*	*У*
1	0.015	0.039	0.038	2.639	2.558	0.38	0.37
2	0.015	0.047	0.037	3.170	2.524	0.45	0.36
3	0.015	0.041	0.035	2.788	2.377	0.40	0.34
4	0.014	0.036	0.036	2.640	2.640	0.38	0.38
5	0.015	0.051	0.049	3.473	3.342	0.50	0.48
6	0.016	0.028	0.028	1.781	1.781	0.25	0.25
7	0.015	0.033	0.033	2.255	2.255	0.32	0.32
8	0.014	0.044	0.035	3.107	2.507	0.44	0.36
9	0.015	0.031	0.031	2.087	2.081	0.30	0.30
10	0.016	0.043	0.043	2.732	2.732	0.39	0.39
Average value	0.015	0.039	0.038	2.667	2.480	0.38	0.35
Standard	0.015	0.047	0.037	3.170	2.524	0.07	0.06

Note: *Н* is a non-established movement; *У* is an established movement.

2. A resonance mode: beam (double-tee № 16) with the length 16 m is divided into 10 bar finite elements with the length 1.6 m each; for determining the matrices of rigidity, inertia and damping the method of finite elements was employed. A finite-element model of the stand had 20 node degrees of freedom; the lowest own frequency of the beam is 2.0 Hz (12.9 rad/sec), which coincides with the impact frequency, damping is 6.53 %. The dynamism coefficient not considering inconsistency according to the formula (3) is β(2 Hz) = 16.10 (see Fig. 1).

The results of statistical modeling for the series of 10 numerical experiments are presented in Table 2.

Here are the displacements of a central section for static loading, dynamic displacements and experimental coefficients of dynamism and inconsistency. Fig. 3b shows one of the obtained functions of displacements in time in a resonance mode.

Table 2

Results of the series of 10 experiments for a resonance mode.
Displacements of a central section (based on the module), coefficients of dynamism and inconsistency

Number of the experiment	u_{st}, m	u_{dyn}, m	B_{exp}	K (formula (5), β = 12.33)
1	0.306	1.033	3.381	0.27
2	0.309	1.282	4.150	0.34
3	0.308	1.566	5.087	0.41
4	0.307	1.070	3.487	0.28
5	0.288	1.755	6.099	0.49
6	0.308	1.651	5.362	0.43
7	0.307	2.085	6.792	0.55
8	0.323	1.148	3.558	0.29
9	0.312	1.554	4.984	0.40
10	0.295	1.208	4.092	0.33
Average value	0.306	1.435	4.699	0.38
Standard	0.009	0.342	1.164	0.09

3. Completely consistent movements. The mass of each spectator is 100 kg, a delay is zero. The modeling results are as follows:

— a non-resonance mode: β_{exp} = 7.15;

— a resonance mode: β_{exp} = 12.33.

Fig. 3c and 3d show the functions of displacements in time for completely consistent movement in a non-resonance and resonance modes.

4. Uneven spatial distribution. The weight of each spectator is a random value, a delay is zero. The more accurate results coincide with those in section 3.

According to the results of numerical modeling, the following is concluded:

1. The values of the dynamism coefficients found using the results of the numerical experiment turned out to be lower than those calculated using the formula (3): 7.15 and 9.02 for the non-resonance mode 12.33 and 16.10 for the resonance mode (experiment 4, section 3). The difference is 21 and 23 % respectively. The reasons for that are, first of all, losses of the phases of the Fourier harmonics of the input impact in the solution (3), secondly, in the specification of modal coefficients of damping ξ_k in (3) as they were specified as constants for any own frequency.

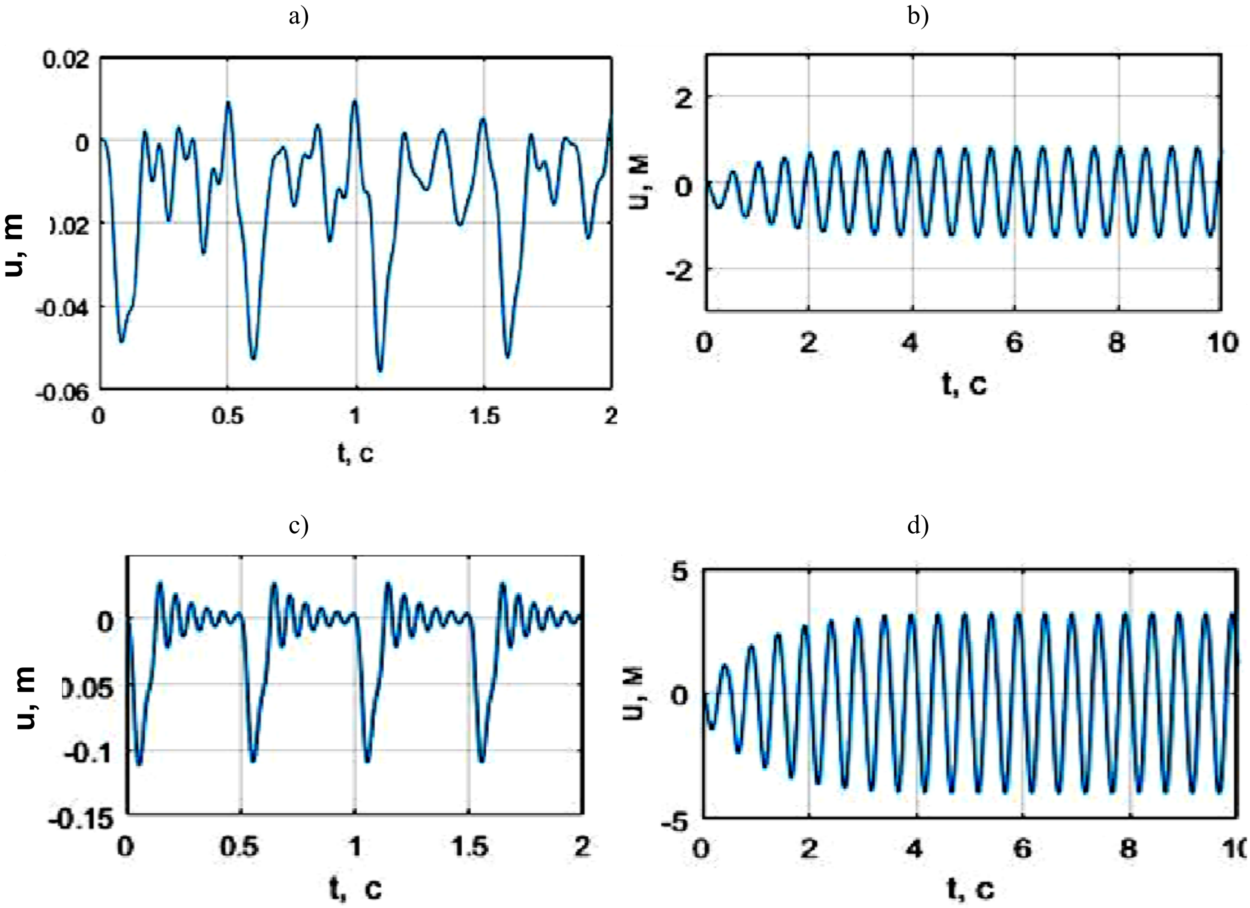

Fig. 3. Results of the experiments. Typical implementations:
a) inconsistent movement, non-resonance mode; b) inconsistent movement, resonance mode;
c) consistent movement, non-resonance mode; d) consistent movement, resonance mode

2. Oscillations in the resonance mode: inconsistent movements of the spectators reduce the dynamism coefficient to 4.70 (a 62 % reduction compared to a 12.33 one for consistent movement) with a standard deviation of 1.16, average value of the inconsistency coefficient is 0.38, standard deviation is 0.09 (experiment 2, section 3, Table 2);

3. Oscillations in the non-resonance mode: inconsistent movements of the spectators reduce the average value of the dynamism coefficient to 2.48—2.68 (a 63—65 % reduction compared to a 7.15 one for consistent movement), average value of the inconsistency coefficient is 0.35—0.38, standard deviation is 0.06—0.07 (experiment 1, section 3, Table 1);

4. Experiment 4, section 3, showed that the reduction in the dynamism coefficient in the resonance mode is determined with a significant delay, phase φ_i in the formula (4);

5. Spatial positioning of the spectators with a different weight (in a specified range from 70 to 100 kg) has no effect on the coefficients of dynamism and inconsistency (experiment 4, section 3);

6. Experiment 1, section 3, showed that regardless of whether a mode is resonance or not, inconsistent movements of the spectators have a similar effect on the dynamism coefficients;
7. The effect of the transitional mode of forced vibrations can be neglected for the model in question, the average values of the dynamism coefficients of the established and non-established oscillations differ by around 7.5 %.

Conclusions

1. The papers shows the results of some series of numerical experiments that allowed us, first of all, to compare them with the results of an analytical approach (3) and secondly, to evaluate the effect of inconsistency of people's movements on a dynamism coefficient.
2. An analytical approach showed extremely high values of a dynamism coefficient, i.e. up to 23 %. It must be due to a very simplified damping model when modal damping coefficients ξ_k in (3) were assumed to be constant for any own frequency.
3. A reduction in a dynamism coefficient was found to be largely determined by people's nonsynchronous, inconsistent movements, i.e. a random phase in (4). Inconsistent movements can dramatically reduce a dynamism coefficient up to 62—65 %. Spatial positioning of spectators with different mass has almost no effect on a dynamism coefficient.
4. The current research might be of interest to designing engineers for strength calculations of sports structures. The above results allow more insight into dynamism coefficients and the factors that might contribute to their reduction.

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