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USING AIR EXCHANGE TO REDUCE THE PROBABILITY OF SPREADING CORONAVIRUS INFECTION

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Statement of the problem. Ventilation processes have a significant impact on the spread of airborne infections. It is necessary to use air exchange to reduce the likelihood of spreading such infections.
Mathematical model. Using the Wells - Riley model of airborne transmission of infectious diseases, a mathematical model has been developed for the spread of coronavirus infection in a medical institution, consisting of a group of communicating rooms in which both healthy and infected people are constantly located and moved. The mathematical model makes it possible to take into account the movement of people around the premises and the settling of quanta of the generation of infection by a sick person when air moves.
Results. The general solution of the mathematical model is obtained, which allows calculating the concentration of quanta of generation of infection in the premises during the operation of a medical institution.
Conclusions. The developed mathematical model of a medical institution allows a deeper understanding of the possibilities of the spread of coronavirus infection and taking these risks into account when designing medical institutions.

Keywords: ventilation, hospitals, coronavirus.

Introduction. A lot of studies indicate that ventilation affects the likelihood of contracting airborne infections such as influenza and rhinovirus infections [18]. The latter include the coronavirus (COVID-19), the pandemic which is currently gaining momentum.

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The process of transmission of infection by airborne droplets has multiple aspects to it. This paper discusses the use of air exchange created by ventilation systems to reduce the likelihood of the spread of infections transmitted by airborne droplets.

The concept of the spread of airborne infections was first described by Wells [21] and then by Riley [14, 15, 20]. The Wells-Riley equation [20] was employed to assess the effect of ventilation, filtration, and other physical processes on transmission through droplet nuclei [7, 9, 11, 12]. The detection of pathogenic microorganisms in the air of the premises of medical institutions in the absence of patients indicates a possible indirect connection between the processes of room-to-room air flow and the transmission of diseases [13, 16, 18, 19].

The objective of this work is to assess the effect of general ventilation on the spread of coronavirus infection in a medical institution.

1. Mathematical model of the transmission of the coronavirus infection by means of general ventilation. For individuals in the same room, the probability of contracting an infectious disease transmitted by means of airborne droplets is estimated by the Wells-Riley dependence [20]:

$$P = \frac{D}{S} = 1 - e^{-\frac{Ipqt}{Q}}, \tag{1}$$

where D is the number of cases; S is the total number of people in the room; I is the number of sick people in the room; p is the human respiration rate, m^3/sec; q is the rate of quantum generation of infection by a sick person (quantum/sec); t is the total time spent in the room, sec; Q is the amount of clean air entering the room per unit of time, m^3/sec.

The rate of generation of quanta of infection by a sick person was introduced by Wells [21]. He argued that not all inhaled core droplets would lead to infection and defined the infection quantum as the number of infectious droplet nuclei required for infection. Equation (1) is based on a Poisson distribution and assumes that only one new infection is likely to occur in a fairly short time period.

Based on the Wells-Riley dependence, the number of cases can be expressed:

$$D = S\left(1 - e^{-\frac{Ipqt}{Q}}\right). \tag{2}$$

Or summing for all the facilities in a building

$$\sum_{i=1}^{n} D_i = \sum_{i=1}^{n} S_i\left(1 - e^{-\frac{I_i pqt}{Q_i}}\right), \tag{3}$$

where n is the total number of rooms in a medical institution.

Let us consider the problem of designing effective ventilation of a medical institution comprised of a group of adjoining rooms where both healthy and infected people constantly reside and move around.

Indoor air quality is predominantly defined by air exchange. General exchange ventilation of premises can be mixing ventilation (MV), displacement ventilation (DV) or a combination of both [1, 3]. While mixing ventilation is employed, turbulent air flows are intensively mixed, and highly dispersed particles are relatively evenly distributed along the room [4, 5, 17]. While displacement ventilation is employed, laminar air flows are designed which enables highly dispersed particles to be directed through the room. Displacement ventilation generates a more efficient air exchange in the room, but its use increases capital and operating costs considerably [6, 8, 10, 22].

Let us compose the equation of air exchange of a medical institutio, consisting of a group of communicating rooms with MV ventilation type. The number of quanta of infection generated by a sick person coming from adjoining rooms will be:

$$c_j L_{ji} \psi_{ji} dt\ , \tag{4}$$

where c_j is the concentration of quanta of the generation of infection by a sick person in the j-th room, quantum·m^{-2}; L_{ji} is the amount of air supplied from the j-th room to the i-th room, m^3·sec^{-1}; ψ_{ji} is the coefficient of settling of quanta of infection generated by a sick person when air moves from the j-th room to the i-th room; dt is a time increment, sec.

The number of quanta of infection generated from patients entering the room will be:

$$I_i pq \cdot dt\ , \tag{5}$$

where I_i is the number of patients in the i-th room.

The amount of quanta of infection generated by a sick person into adjoining rooms and into the surrounding air will be:

$$c_i L_{ij} dt + c_i L_{i0} dt\ , \tag{6}$$

where L_{ij} is the amount of air flowing from the i-th room into the j-th room, m^3·sec^{-1}; L_{io} id the amount of air flowing from the i-th room into the environment, m^3·sec^{-1}.

Then we obtain the differential equation of the material balance for the quanta of infection generation by a sick person:

$$\sum_{\substack{j=1 \\ j\neq i}}^{n} c_j L_{ji} \psi_{ji} dt + I_i pqdt - \sum_{\substack{j=1 \\ j\neq i}}^{n} c_i L_{ij} dt - c_i L_{i0} dt = V_i dc_i\ , \tag{7}$$

where V_i is the area of the i-th room, m^3.

By means of transformation we get

$$\sum_{\substack{j=1 \\ j\neq i}}^{n} c_j L_{ji} \psi_{ji} + I_i pq - \sum_{\substack{j=1 \\ j\neq i}}^{n} c_i L_{ij} - c_i L_{i0} = V_i \frac{dc_i}{dt}. \tag{8}$$

For the entire medical institution consisting of a group of communicating rooms, we obtain a system of ordinary differential equations:

$$\begin{cases} \sum\limits_{\substack{j=1 \\ j\neq 1}}^{n} c_j L_{j1} \psi_{j1} + I_1 pq - \sum\limits_{\substack{j=1 \\ j\neq 1}}^{n} c_1 L_{1j} - c_1 L_{10} = V_1 \dfrac{dc_1}{dt} \\ \sum\limits_{\substack{j=1 \\ j\neq 2}}^{n} c_j L_{j2} \psi_{j2} + I_2 pq - \sum\limits_{\substack{j=1 \\ j\neq 2}}^{n} c_2 L_{2j} - c_2 L_{20} = V_2 \dfrac{dc_2}{dt} \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \sum\limits_{\substack{j=1 \\ j\neq n}}^{n} c_j L_{jn} \psi_{jn} + I_n pq - \sum\limits_{\substack{j=1 \\ j\neq n}}^{n} c_n L_{nj} - c_n L_{n0} = V_n \dfrac{dc_n}{dt} \end{cases}. \tag{9}$$

Transforming a system of ordinary differential equations (9), we get

$$\begin{cases} \dfrac{dc_1}{dt} = \sum\limits_{\substack{j=1 \\ j\neq 1}}^{n} c_j \dfrac{L_{j1}\psi_{j1}}{V_1} - c_1 \left(\sum\limits_{\substack{j=1 \\ j\neq 1}}^{n} \dfrac{L_{1j}}{V_1} + \dfrac{L_{10}}{V_1} \right) + \dfrac{I_1 pq}{V_1} \\ \dfrac{dc_2}{dt} = \sum\limits_{\substack{j=1 \\ j\neq 2}}^{n} c_j \dfrac{L_{j2}\psi_{j2}}{V_2} - c_2 \left(\sum\limits_{\substack{j=1 \\ j\neq 2}}^{n} \dfrac{L_{2j}}{V_2} + \dfrac{L_{20}}{V_2} \right) + \dfrac{I_2 pq}{V_2} \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \dfrac{dc_n}{dt} = \sum\limits_{\substack{j=1 \\ j\neq n}}^{n} c_j \dfrac{L_{jn}\psi_{jn}}{V_n} - c_n \left(\sum\limits_{\substack{j=1 \\ j\neq n}}^{n} \dfrac{L_{nj}}{V_n} + \dfrac{L_{n0}}{V_n} \right) + \dfrac{I_n pq}{V_n} \end{cases} \tag{10}$$

The initial condition for the system of ordinary differential equations (10) is the concentration of quanta of infection generation by a sick person in the premises at the initial moment of time.

By means of transformion of the system of ordinary differential equations (10), we get

$$\begin{cases} \dfrac{dc_1}{dt} = m_{12}c_2 + m_{13}c_3 + \cdots + m_{1n}c_n + \dfrac{I_1 pq}{V_1} \\ \dfrac{dc_2}{dt} = m_{21}c_1 + m_{23}c_3 + \cdots + m_{2n}c_n + \dfrac{I_2 pq}{V_2} \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \dfrac{dc_n}{dt} = m_{n1}c_2 + m_{n2}c_2 + m_{n3}c_n + \cdots + \dfrac{I_n pq}{V_n} \end{cases}. \tag{11}$$

Let us denote:

$$M = \begin{pmatrix} 0 & m_{12} & \cdots & m_{1n} \\ m_{21} & 0 & \cdots & m_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ m_{n1} & m_{n2} & \cdots & 0 \end{pmatrix}, \; C = \begin{pmatrix} c_1 \\ c_2 \\ \cdots \\ c_n \end{pmatrix}, \; C' = \begin{pmatrix} \frac{dc_1}{dt} \\ \frac{dc_2}{dt} \\ \cdots \\ \frac{dc_n}{dt} \end{pmatrix}, \; I = \begin{pmatrix} \frac{I_1 pq}{V_1} \\ \frac{I_2 pq}{V_2} \\ \cdots \\ \frac{I_n pq}{V_n} \end{pmatrix}. \tag{12}$$

2. General solution of the equations of the mathematical model of the transmission of the coronavirus infection by means of general ventilation. Matrix M characterizes the movement of infection generation quanta in the air of the premises of a medical institution considering their settling. I characterizes the release of quanta of infection generation in the facilities of a medical institution depending on the number of patients.

Let us write the system of ordinary differential equations (12) in the following form:

$$C' = MC + I. \tag{13}$$

With the initial condition in the form of initial concentrations of quanta of infection generation in the j-th room, quantum·m^{-2}:

$$C_0 = \begin{pmatrix} c_{10} \\ c_{20} \\ \cdots \\ c_{n0} \end{pmatrix}. \tag{14}$$

Generally, the solution to the system of ordinary differential equations (13) can be represented as

$$C = \int_{t_0}^{t} e^{(t-\tau)M} I d\tau + e^{(t-t_0)M} C_0. \tag{15}$$

The opening of doors between rooms is modeled by a sequence of matrices M_1, M_2, M_1 ... The movement of sick people releasing quanta of infection generation is modeled by means of a sequence of vectors:

$$I_1 = \begin{pmatrix} \frac{I_{11} pq}{V_1} \\ \frac{I_{21} pq}{V_2} \\ \cdots \\ \frac{I_{n1} pq}{V_n} \end{pmatrix}, \; I_2 = \begin{pmatrix} \frac{I_{12} pq}{V_1} \\ \frac{I_{22} pq}{V_2} \\ \cdots \\ \frac{I_{n2} pq}{V_n} \end{pmatrix}, \; I_3 = \begin{pmatrix} \frac{I_{13} pq}{V_1} \\ \frac{I_{23} pq}{V_2} \\ \cdots \\ \frac{I_{n3} pq}{V_n} \end{pmatrix} \ldots. \tag{16}$$

Using the developed mathematical model, it is possible to calculate the concentration of infection generation quanta in the premises of a medical institution for any air exchange with variable air flow between rooms, which makes it for the risks of infection in healthy people to be assessed.

Conclusions. Mass exchange processes caused by the operation of general exchange ventilation systems have a significant impact on the spread of infections transmitted by airborne droplets. The room-to-room air movement must be controlled to reduce the risk of spreading such infections.

Based on the Wells-Riley model of airborne transmission of infectious diseases and the differential equation of air exchange, a mathematical model of the spread of the coronavirus infection in a medical institution comprised of a group of adjoining rooms with air overflow has been developed. Both healthy and infected people can be in the facilities permanently or temporarily. The mathematical model allows for the possibility of settling of quanta of infection generation by a sick person in the facilities.

Based on the resulting general solution of the equations of the mathematical model, the concentration of quanta of the generation of infection in the premises during the operation of a medical institution can be calculated.

The mathematical model provides a deeper insight into the possibilities of the spread of the coronavirus infection and considering these risks in the process of designing medical institutions.

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